Robert M. Solow’s Neoclassical Growth Model: An Influential Contribution to Economics

Edward C. Prescott*

Federal Reserve Bank of Minneapolis and University of Minnesota, Minneapolis, MN, USA

I. Introduction

In 1956 Robert Solow’s paper “A Contribution to the Theory of Economic Growth” [21] appeared in the Quarterly Journal of Economics. The model this paper describes is a simple one. It has a constant returns to scale aggregate production function with substitution between two inputs, capital and labor. The model is completed by assuming that a constant fraction of output is invested. The discrete time version of the model is

\[ c_t + i_t = f(k_t, n_t) \]
\[ k_{t+1} = k_t + i_t \]
\[ i_t = \sigma f(k_t, n_t) \]

where \( c \) is consumption, \( i \) investment, \( k \) capital, \( n \) labor, \( \sigma \) the fraction of output invested, and \( f \) the neoclassical aggregate production function.

If factors are assumed to be paid their marginal product, then given \( k_t \) and \( n_t \) the date \( t \) national income and product accounts can be computed for this model economy. Since that also determines the capital stock for date \( t+1 \), this model can be used to generate time series which can be compared with those constructed by Simon Kuznets and others.

Why is this neoclassical growth model an important advance for economics? Partly because, as Solow has gone on to show, this model

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† Numbers in brackets refer to publications by Robert M. Solow (see Bibliography) and dates in parentheses to the references listed at the end of this article.
parsimoniously explains the essential growth features of the U.S. economy. And partly because the model’s influence has spread far beyond the study of growth.

II. The Study of Growth

Before the neoclassical growth model was developed, there were fixed coefficient growth models, including those of Evsey Domar, Roy Harrod, Wassily Leontief, and John von Neumann. But Solow introduced substitution between capital and labor. With this key feature and with labor augmenting technological change, his model mimics the basic growth facts without the knife-edge instability of the Harrod–Domar model. These facts (described by Solow in 1970 [76, pp. 8–9) are that

- Real output per worker-hour on average grows at a more or less constant rate.
- The stock of tangible capital grows at more or less that same rate, so the capital/output ratio is more or less constant.
- Capital’s share of income is more or less constant. This fact, along with a constant capital share, implies a constant rental price for capital.
- Output per worker and the real wage grow on average at a more or less common rate.

In 1957, in an equally brilliant paper in the *Review of Economics and Statistics* [26], Solow used his neoclassical growth model to account for growth in the U.S. economy. He found that four-fifths of the growth in this nation’s output per worker is accounted for by changes in the technology coefficient and one-fifth by increases in tangible capital per worker. Subsequent research has somewhat reduced the contribution of technological change, but has not successfully challenged Solow’s basic result that such change accounts for more than half the growth in output per worker.

III. Beyond Growth

The neoclassical growth model has not been limited to the study of growth. This model has played the central role in organizing and guiding subsequent aggregate economic research.
Public Finance

One of the first questions the model led people to ask is, What determines the savings rate? David Cass (1965) addressed this question in his important work on optimal savings. He introduced into the neoclassical growth model an infinitely lived family with time additive preferences for different date consumption goods. His solution to the optimal growth problem effectively endogenized the savings decision since for this economy the optimal allocation is the competitive equilibrium allocation.

The endogenous savings decision implied another important finding: that the equilibrium path converges to the balanced, or steady-state, growth path. At that point, the marginal product of capital equals the steady-state rental price of capital. That price of capital services depends in an explicit way on the features of the tax system. Therefore, the consequence of any of an incredibly rich set of alternative taxing schemes for the steady-state behavior of the economy can be determined. This is the rental price of capital theory developed by Dale Jorgenson and Robert Hall — a theory now central to public finance.

Thus, the neoclassical growth model led to steady-state analysis, which is among the most important constructs in public finance. Besides that, though, steady-state analysis is the theory underlying much of the development literature. This point is convincingly argued in Robert Lucas' 1986 Marshall Lectures, “On the Mechanics of Economic Development”.

More recently, the neoclassical growth model has been used not only to estimate the consequence of taxation policies on steady-state behavior, but also to estimate the effect of a tax policy change on the subsequent path of the economy. Kenneth Judd (1987), for example, has used it to assess the present value of the welfare costs of a number of alternative tax schemes.

Business Cycle Fluctuations

But the influence of neoclassical growth theory has spread even further. While the theory was developed to account for the low-frequency growth observations and for steady-state behavior, it is proving surprisingly useful in organizing and understanding business cycle fluctuations as well. It leads to a focus on the co-movements of a particular set of
variables: consumption, investment, labor input, capital input, factor incomes, and output.

Until recently, uncertainties in the Solow technological change parameters were thought to be a minor source of aggregate fluctuations. This intuition was based on partial equilibrium reasoning, not on a careful fluctuations accounting exercise in a general equilibrium framework. That sort of exercise was not feasible without two important advances in economic theory.

These advances came in the 1970s. In 1972, William Brock and Leonard Mirman analyzed the optimal growth problem for the stochastic version of the neoclassical growth model. At about the same time, Lucas developed the tools necessary for competitive analysis of recursive economies with uncertainty. The competitive equilibria of such economies are time-invariant Markov processes. Lucas’ idea that an equilibrium is a stochastic process was a radical departure from then-existing approaches.

With these advances in theory and parallel developments in computational tools, the equilibrium stochastic process for stochastic growth models could be computed and its behavior compared with that of actual economies. Such comparisons require decisions about what time series to study and what statistical properties to compare. The neoclassical growth model provides guidance. The key business cycle fact is that, at the higher frequencies, variations in the labor input account for most of the variations in output. The capital stock input, being orthogonal to output at these frequencies, accounts for virtually none of the business cycle fluctuations. (The language I am using here shows the importance of the Solow growth accounting contribution.)

Clearly, to use the stochastic growth model in the study of business cycle fluctuations, the time allocation decision between market and nonmarket activities must be endogenized. This is what Finn Kydland and I did in our 1982 *Econometrica* paper. Our findings challenged the widely held view that variations in the Solow technology parameter are too small to account for a significant amount of business cycle fluctuations. The model can and is being used to estimate the aggregate consequences of public finance and terms of trade shocks as well.

The model has successfully accounted for some observations which had been thought inconsistent with neoclassical theory. For example, some thought aggregate consumption is too smooth, considering the permanent income hypothesis’ prediction for it. This issue is now discussed in the context of the augmented neoclassical stochastic growth

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Some were puzzled by the well-known observation that production is more volatile than final sales, which implies highly volatile inventory investment. Once the model has a multistage production process, neoclassical growth theory predicts the high volatility of inventory investment and also the fact that inventory stocks lag the business cycle.

Another puzzling observation was that most of the adjustment in the labor input is accounted for by variations in the number of people employed rather than in the hours worked per employed person. Gary Hansen (1985) introduced nonconvexities into the mapping between hours allocated to market activities and units of labor service produced, a feature of the economy well-documented by micro observations. Again the neoclassical growth model came through.

Financial Markets

The model has also been used by Sanford Grossman and Robert Shiller (1981) in the study of stock market volatility. Here, though, the model has not yet been so successful. Stock prices are several times as volatile as predicted by the theory. Still, even when the theory fails, it succeeds, in a sense. In failing to account for most of the volatility it nicely defines a scientific puzzle to be solved.

IV. Conclusion

Robert Solow has clearly made a major contribution to economics. His neoclassical growth model is today the organizing structure in macroeconomics. Graduate students at the leading macro research centers focus on this model and spend little, if any, time on the IS–LM mechanism which it replaced. Furthermore, the extension of the neoclassical growth model to the study of fluctuations has resulted in macroeconomics being integrated with the other substantive areas of economics rather than being the one branch of economics using a different paradigm. Macroeconomics has benefited from this integration and even has contributed to these other areas — particularly labor economics and public finance. Assumptions defining macroeconomic models now are restricted by findings in other substantive areas, while their models are restricted by

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macro findings. All areas of economic science, that is, have been influenced by Solow’s neoclassical growth model.

References


