CHAPTER 2

A multiple means-of-payment model

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In this chapter we study an economy in which there are two technologies for making payments. The first is currency; the second, bank drafts drawn on interest-bearing demand deposits. The interest-bearing asset does not dominate the noninterest-bearing currency because there is a fixed recordkeeping cost incurred whenever a bank draft is used as the means of payment. The steady-state equilibrium is characterized. It is found that the value of the good or (more precisely) package of goods purchased at a given location determines which means of payment is used. Bank drafts are used for large purchases and currency for small purchases.

In the environment studied, the highly centralized Arrow–Debreu competitive equilibrium is impractical, because the number of date-, event-, and location-contingent commodities is so large that the resources required for information collection and processing would be prohibitive. In this sense we follow Brunner and Meltzer (1971) and consider as the chief role of money economizing on costly information collection and processing.

The approach is close in spirit to that of Townsend (1980), who views the payment system as a communication system. It differs in that no effort is made to find the best arrangement. The arrangement studied, however, is sufficiently explicit that one can calibrate the model and then examine the costs and benefits associated with modifying the scheme—say, by imposing reserve requirements or interest-rate ceilings. Upper bounds for the gains that can be realized from alternative systems can

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Other related models are those of Lucas (1980) and Gale and Hellwig (1984).

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be computed. A system that is simple and implementable and nearly optimal-independent of the exact specification of the environment: This is the most that economic theory can provide.

The scheme studied requires collective actions, which are virtually necessary for any payment system which uses fiat money. The question of what would develop absent any collective action besides enforcement of contracts is not addressed. In this environment there are no gains from credit arrangement, as the interest-bearing debit account dominates. In fact, credit is often used, particularly when there are ongoing relations. An important extension of this research would be to introduce some feature into the environment that would give rise to the use of credit as well as the use of currency and bank drafts.

The model is close to the growth model, a structure that has proven useful in public finance and macroeconomics. This, I think, is desirable; the closer a specification is to those used in other economic applications, the greater the prior knowledge that can be used to restrict the model's parameters. A second desirable feature of the construct is that the record keeping costs associated with using a bank draft as the means of payment can and have been measured. The number obtained is not small -- nearly a half dollar per draft -- and would be larger if the value of the transactor's time associated with writing a check and verifying the payee's identity were taken into account. The number of transactions also can be measured and then used to restrict the theory. To summarize, the hope is that this line of research will lead to the development of a theory that can be used to quantitatively evaluate alternative payment systems and aid in the design of better payment arrangements.

The chapter is organized as follows. In Section 1 the environment is specified. The means-of-payment decisions of an agent, given the market interest rate and that agent's total expenditures, is solved in Section 2. In spite of the fixed cost per bank draft, standard convex analysis is applicable. This is possible because each purchase is a negligible fraction of an individual's total expenditures. Others have previously assumed a finite number of transactions, and the resulting nonconvexity held back the development of a transaction-based general equilibrium theory of money. In Section 3 the saving-consumption expenditure decision is considered. It is found that the resulting behavior is essentially the same as for the neoclassical growth model. The steady-state capital stock equates the marginal product of capital to the sum of the depreciation rate of capital and the subjective time discount rate. This steady-state capital

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2 See King (1983) for an insightful discussion of the economics of the private provision of money.
stock is invariant to the inflation rate for this model. The steady-state equilibrium is determined in Section 4, and Section 5 contains some illustrative uses of the construct.

1 The economy

There is a continuum of agents and products. Time is discrete, with half the agents allocating their time to production in odd periods and to shopping-consumption in even periods. The other half of the population produce goods in even periods and shop in odd periods. The shoppers purchase many different goods at different locations — indeed, a continuum of them. For an individual, the goods can be categorized into a finite number of equivalence classes, which are indexed by \( \theta \) belonging to a finite set of positive reals \( \Theta = \{ \theta_1, \ldots, \theta_n \} \). The classes of goods are ordered so that \( \theta_i < \theta_{i+1} \). Element \( \pi_\theta \) is the fraction of goods of type \( \theta \). Within an equivalence class of goods, all goods enter the individual’s utility function symmetrically. Letting \( e \) name the goods within an equivalence class, the utility function used has the form

\[
\sum_{i, \theta} \beta^t \left[ \int_0^{r_\theta} u(\theta x_{\theta t}(e)) \right] \, de,
\]

(1.1)

where \( x_{\theta t}(e) \) is the quantity of good \( e \) in class \( \theta \) that is consumed in period \( t \), and \( 0 < \beta < 1 \) is the subjective time discount factor. The function \( u \) is continuously differentiable, strictly increasing and strictly concave, and is defined on the nonnegative reals. Furthermore, \( u(0) = 0 \).

Rather than keeping track of the quantities of every good consumed by every person, I will (and need only) monitor the distribution of quantities of goods consumed in the various goods’ equivalence classes. The quantity of a good consumed is indexed by \( x \) belonging to some finite set of positive real numbers. Indirect utility functions will be defined over arrays \( z_{\theta x t} \) for \( \theta \in \Theta, x \in X, \) and \( t \in \{0, 1, 2, \ldots \} \). Element \( z_{\theta x t} \) is the number (or more precisely the measure) of goods of type \( \theta \) consumed in quantity \( x \) in period \( t \). The utility function defined on \( z \) has the form

\[
U(z) = \sum_{i, \theta, x} \beta^t u(\theta x) z_{\theta x t}.
\]

The sets \( \Theta \) and \( X \) are assumed (for expository — not technical — reasons) to have a finite number of elements. In the following arguments, the only property needed is that the cross product of \( \Theta \) and \( X \) be a compact separable metric space. Since \( \Theta \) and \( X \) are subsets of the real line, their cross product is compact if the sets are closed and bounded. Sets of measures that are defined on the Borel sigma algebra of a separable compact metric space and that are closed and bounded are compact with
respect to the weak* topology. At points in the analysis we will proceed as if \( X \) were a continuum and differentiate with respect to \( x \). Implicitly we are assuming the points in \( X \) are so closely spaced that the derivative and the finite difference are the same for all practical purposes.

Producers are located at spatially distinct points or islands and shoppers visit a random sample of islands. At each location there is precisely one type (not class) of goods sold by a number of producer-sellers and as a result prices are determined competitively. The sample of purchasing opportunities is large. Consequently, a shopper receives a representative sample of purchase opportunities. Letting \( \lambda \) denote the number (measure) of purchase opportunities, \( \pi_\theta \lambda \) is the number of opportunities for the purchase of different goods in class \( \theta \). A given good is of type \( \theta \) for fraction \( \pi_\theta \) of the population. Thus some goods are more highly valued by a given individual (i.e., have higher \( \theta \)), but the fraction of the population which values a given good at a given level is the same for all goods. This introduces symmetry in both the goods and agent space, which simplifies the subsequent analysis. Because of this symmetry, in equilibrium all goods will have the same price and the same distribution of purchase quantities.

There is the additional restriction that a given good, if purchased, is purchased only once, using either currency or a draft. This constraint greatly simplifies the formulation and is nonbinding. Using currency, there is no gain from making five one-dollar purchases of a given good rather than one five-dollar purchase. If drafts are used, it is wasteful to make multiple purchases of the same good because the fixed cost would be unnecessarily incurred more than once.

2 Means of payment and quantity purchased

The decisions first considered are the means of payment and the quantity purchased. By symmetry, the price of all goods must be the same in equilibrium. Units are selected so that this price is one unit of currency. Let \( M \) denote currency holdings and \( B \) bank deposits at the beginning of a shopping period. Letting \( m = (m_{\theta x}) \) be the measure of cash purchases and \( d = (d_{\theta x}) \) the measure of debit purchases, wealth at the end of the period is

\[
W' = (1+r)B + M - \sum_{\theta, x} x m_{\theta x} - \sum_{\theta, x} x d_{\theta x} - \gamma \sum_{\theta, x} d_{\theta x}.
\]

\( 3\) For the development of general equilibrium theory with signed measures used as the commodity point, see Mas-Colell (1975) and Jones (1984). They exploit them for the case of a continuum of differentiated products.
The first summation is the value of all cash purchases, the second is the value of all credit transactions, and the third is sum of the fixed costs of credit transaction (γ is the cost per transaction and \( \sum d_{\theta x} \) is the number of transactions).

Let
\[
M = \sum_{\theta, x} x m_{\theta x}, \tag{2.2}
\]
\[
D = \sum_{\theta, x} x d_{\theta x}, \tag{2.3}
\]
\[
S = \gamma \sum_{\theta, x} d_{\theta x}. \tag{2.4}
\]

As \( W = M + B \) it follows that
\[
W' = (1 + r) W - Y,
\]
where
\[
Y = (1 + r) M + D + S. \tag{2.5}
\]

Variable \( Y \) is total expenditures on goods and bank services plus forgone interest earnings on currency holdings.

The program facing the individual with these definitions is
\[
U(Y, r) = \max_{m, d \geq 0} \sum_{\theta, x} u(\theta x)(m_{\theta x} + d_{\theta x}), \tag{2.6}
\]
subject to
\[
\sum_{\theta, x} (m_{\theta x} + d_{\theta x}) \leq \lambda \pi_\theta \quad \text{for all } \theta \tag{2.7}
\]
(purchases of a given class of good are constrained by the number of goods in that class found while searching), and subject also to
\[
(1 + r) \sum_{\theta, x} x m_{\theta x} + \sum_{\theta, x} x d_{\theta x} + \gamma \sum_{\theta, x} d_{\theta x} \leq Y. \tag{2.8}
\]

This is a linear program. The constraint set is closed and bounded and is nonempty. Consequently an optimum exists.

The first-order conditions are
\[
u(\theta x) \leq \mu_{\theta} + (1 + r) x \phi \quad \text{with equality if } m_{\theta x} > 0 \tag{2.9}
\]
and
\[
u(\theta x) \leq \mu_{\theta} + (\gamma + x) \phi \quad \text{with equality if } d_{\theta x} > 0, \tag{2.10}
\]
where \( \mu_{\theta} \) are the Lagrange multipliers associated with constraints (2.7) and \( \phi \) is the multiplier associated with constraint (2.8).

If purchases are made using currency, the quantity purchased satisfies
\[
\theta u'((\theta x)) = (1 + r) \phi, \tag{2.11}
\]
whereas, if a draft is the means of payment, the quantity purchased satisfies

$$\theta u'(\theta x) = \phi. \quad (2.12)$$

Let $x_m(\theta, \phi)$ and $x_d(\theta, \phi)$ be the solutions to (2.11) and (2.12), respectively. Currency will be used if

$$u[\theta x_m(\theta, \phi)] - (1 + r)\phi x_m(\theta, \phi) > u[\theta x_d(\theta, \phi)] - \theta x_d(\theta, \phi) - \gamma \phi; \quad (2.13)$$

drafts will be used if the inequality is in the opposite direction. With equality, it is optimal to use either means of payment for purchase of goods of that marginal type. The fraction of purchases of that type good using the two alternative means of payment is, however, determined, so there is a unique solution to the program. For $\theta$ less than some critical value $\theta(Y, r)$, currency is the means of payment. For $\theta$ greater than $\theta(Y, r)$ bank drafts are used.

The larger is $Y$, the smaller is the marginal utility $\phi$ of additional expenditures. The smaller is $\phi$, the larger are the purchase quantities $x_m(\theta, \phi)$ and $x_d(\theta, \phi)$. And as $\theta$ increases, drafts are used for the purchases of more goods; that is, $\theta(Y, r)$ is decreasing in $Y$. Consequently the value of purchases using drafts increases as $Y$ increases. Because

$$M = (Y - D - S)/(1 + r), \quad (2.14)$$

a one-unit increase in $Y$ results in a change in $M$ that is bounded from above by $(1 + r)^{-1}$. Thus the optimal currency holding $M(Y, r)$ has slope less than 1 with respect to $Y$.

It is readily verifiable that increases in $r$ decrease the use of currency and therefore increase the use of bank drafts.

3 The dynamic problem

The production function of an individual is

$$f(k), \quad (3.1)$$

where $f$ is increasing, strictly concave, and continuously differentiable, with $f'(0) = \infty$ and $f''(K) = 0$. Because all goods enter symmetrically, all goods have the same equilibrium price, which we normalize to be 1. Then, if $k$ units of capital are rented and $W$ is wealth at the beginning of the production period, end-of-period wealth is

$$W' = (1 + r)W + f(k) - rk \quad (3.2)$$

as sales, $f(k)$, are realized and capital rental payments, $rk$, made at the end of the period.
Letting $v_1(W)$ and $v_2(W)$ be the dynamic programming value functions for (respectively) beginning-of-production and purchase periods, one optimality condition is

$$v_1(W) = \max_k \{ \beta v_2([(1+r)W+f(k)-r]) \}. \quad (3.3)$$

Given that $v_2$ is increasing, the first-order condition is

$$f'(k) = r, \quad (3.4)$$

which is the usual steady-state condition for the optimal growth model.

Another result is that if $v_2$ is concave then $v_1$ is concave, given that $f$ is concave.

Using the indirect utility function derived in Section 2, the optimality condition requires that

$$v_2(W) = \max_Y \{ U(Y, r) + \beta v_1([(1+r)W-Y]) \}. \quad (3.5)$$

As $U$ is concave in $Y$, $v_2$ is concave if $v_1$ is concave. By standard discounted dynamic programming results, functional equations (3.3) and (3.5) have unique solutions which are concave and continuous. These solutions are the optimal value functions for this discounted dynamic program.

4 Steady-state equilibrium

In order to determine the steady-state equilibrium, the interest rate $r$ must be determined. An individual shopper at date $t$ can transform $Y_t$ into $Y_{t+2}$ via borrowing or lending at rate $(1+r)^2$. The marginal rate of substitution between $Y_t$ and $Y_{t+2}$ relative to the indirect utility function $U$ is $1/\beta^2$. Consequently, the steady-state interest rate is

$$r = \beta^{-1} - 1. \quad (4.1)$$

Given $r$, steady-state $k$ is determined by condition (3.4).

A unique steady-state wealth for an individual is determined not by the interest rate $r$, but rather by equilibrium in the goods market. In particular, equilibrium in the goods and service markets requires that

$$f(k) = M(Y, r) + D(Y, r) + S(Y, r). \quad (4.2)$$

Because purchases of goods and services are increasing in $Y$, equation (4.2) can be solved for $Y$ given $r$ and $k$.

Letting $W_1$ and $W_2$ denote (respectively) beginning-of-production and shopping-period wealth,

$$k = W_1 + (W_2 - M). \quad (4.3)$$

Further, from (3.2),
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\[ W_2 = (1 + r)W_1 + f(k) - rk. \]  \hspace{1cm} (4.4)

Given \( M, k, r, \) these equations, linear in \( W_1 \) and \( W_2, \) determine \( W_1 \) and \( W_2. \)

The one remaining variable to be determined is the price level. The steady-state price level \( P \) satisfies

\[ \frac{M(Y, r)}{2} = \frac{M^e}{P}, \]  \hspace{1cm} (4.5)

where \( M^e \) is the per capita money supply and \( M(Y, r)/2 \) is the average real cash balance of agents. Thus, the quantity theory holds for this economy.

5 Illustrated uses of the construct

The purpose of this discussion is to indicate the type of questions that can be addressed within models of this type. Before such a model is confronted with data it will be necessary to incorporate many additional features. For example, one reason for using drafts as a means of payment is that they provide a record of payment necessary for tax purposes. Similarly, currency may be used so that there is no record of the payment – in order to facilitate illegitimate economic activities or to avoid taxation – or not be used as much as it might otherwise be, in order to reduce the risk of loss by theft or fire. Still another possibly important feature is that income allocated to bank services is not taxable when it is financed by lower interest payments on deposits. A final caveat before discussing applications is that the model is a steady-state model. Such models are not designed for studying fluctuations in interest rates and various monetary aggregates. It is suited only for the study of smoothed data series for a given economy or for cross-country, time-averaged data. Given these caveats, the illustrative uses are as follows.

Suppose there are two economies alike in every way except that, for one, the marginal product of capital is uniformly higher. Steady-state capital and output are greater for the more productive economy. Checks will be used for more purchases and these purchases will be larger in the rich country. One implication is that more banking services are used in the rich country. Predictions with respect to the use of currency are ambiguous, as they depend upon the distribution of goods by types. In the high-income country, goods using currency as the means of payment are purchased in greater quantities, leading to a greater use of currency. This effect is offset by the use of checks for a greater fraction of the purchases.

A second application is the question of the optimal growth rate of money. Suppose injections of money are in the form of lump-sum transfers of money to agents at the time they are selling their goods. Withdrawals of money are accomplished by lump-sum taxes, also at the times
when households–firms sell their products. Assuming the money supply grows (declines) at a constant rate \( \phi \), prices will grow at rate \( \phi \) and the nominal interest rate paid on demand deposits will be \( i = \phi + r \). The larger is \( \phi \), the greater is the use of checks and the smaller is the steady-state consumption (output less banking services). For \( \phi = -r \), the nominal interest rate \( i \) is zero. This minimizes the amount of resources allocated to banking services and maximizes steady-state consumption. In this sense the model supports the view of Freidman and Samuelson: because currency is costless to produce, it is optimal to deflate at the real interest rate. This, of course, assumes lump-sum taxes and no private information, features which are needed for taxation to have no deadweight loss. Optimal taxation implies a zero tax on liquidity only if other taxes are distortion-free. However, having a zero nominal interest rate does eliminate incentives to economize upon currency holdings for this economy.

The economy considered has zero reserve requirements, but they are easily introduced. Suppose then there is a reserve requirement, that interest is not paid on reserves, and that currency is supplied perfectly elastically. This is closer to the U.S. payment system than is the model. The quantity theory would still hold, but for currency plus reserves rather than (as in the model considered) for currency alone. If \( \rho \) is the reserve requirement, the interest on demand deposits would be \( (1 - \rho) r \) rather than \( r \). Deposits would be just large enough to insure zero deposits after payments were made. The banks would finance only part of capital investment and would charge interest rate \( r \). In summary, the steady-state behavior of this economy is very much like the static, textbook models of money and banking.

One final use is to consider what happens as the cost of record keeping approaches zero. In the limit, currency is not used and there is no numeraire; consequently, the price level is indeterminate. In such an environment, a reserve requirement along with a fixed supply of reserves is an arrangement for which the price level is determined.

REFERENCES