SHOULD CONTROL THEORY BE USED
FOR ECONOMIC STABILIZATION?*

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In a genetic-historical view of the fundamental revolution in outlook which represents the real beginning of modern natural science was the discovery that the inert objects of nature are not like men, i.e., subject to persuasion, exhortation, coercion, deception, etc., but are "inexorable." The position which we have to combat seems to rest upon an inference, characteristically drawn by the "best minds" of our race, that since natural objects are not like men, men must be like natural objects (Knight, 1941, p. 121).

I. INTRODUCTION

Modern control theory has become a useful tool in both engineering and management science. It made possible manned landings on the moon by finding trajectories with dramatically reduced fuel requirements. It has made possible significant efficiency gains in production scheduling and inventory control. It has become the basic tool used to describe individual and firm behavior when economic activity takes place over time. The question that naturally arises is, can control theory be used to improve the performance of a market economy? Many if not most in the profession believe the answer to this question is yes. There have been numerous studies which have demonstrated convincingly that econometric models can be more effectively controlled through its use.\(^1\) Currently, Kalchbrenner and Tinsley (1975), staff members of the Board of Governors of the Federal Reserve System, are applying optimal control techniques to the design of macro stabilization policy. They are using a simplified version of the Fed-MIT model, which reflects the econometric tradition at its best and was designed specifically for policy evaluation.

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1See Kendrick (1976) for a survey of over ninety such applications.
In fact, for some time, control theory has been used in an informal way in actual policy selection. Policymakers have looked at the current state of the economy and used an implicit law of motion to evaluate the consequences of alternative decisions upon output, employment, and prices. Given that control theory is being used for policy selection, it is surely better that it be applied in a formal way using what is generally considered to be the best econometric model for policy evaluation, as Kalchbrenner and Tinsley advocate, than in the informal way it has and is being used.

Friedman (1953) has sounded a note of caution. He has argued that econometricians are a long way from having a detailed and tested theory that predicts the timing and magnitude of the effects of monetary policy and that, until they do, fine tuning should not be attempted. He argues that the best efforts of the Federal Reserve System have had the perverse effect of contributing to economic instability, and that a neutral policy of a constant growth in the money supply rules is preferable to the use of macro models and control theory methods.

A more fundamental attack upon the use of macro models and control theory to stabilize the economy is due to Lucas (1976). He argues that the structure of the econometric model describing the motion of the economy is not invariant to the policy rule, and that policy simulations using the major macro models are worthless in assessing alternative policy rules. He advocates the use of economic theory to predict the operating characteristics of the economy under alternative policy rules and the selection of a rule which yields the best operating characteristics.

In commenting on Lucas’s essay, Gordon (1976) has suggested that if changes in policy change the law of motion (i.e., structure of the econometric model) in a predictable way, then control theory can still be used. One need only take into consideration such changes in designing policy. Kydland and I (1977) attempt to do precisely that and find it logically impossible. Even in dynamic deterministic situations, the optimal policy is inconsistent. Unlike games against nature, the optimal plan in subsequent periods is not the continuation of the first period optimal plan over the remainder of the planning horizon. If one accepts the rational expectations equilibrium paradigm, optimal control is inappropriate for policy selection.

At this point I would like to emphasize that the issue is not whether economic fluctuations can or should be reduced. The question is, rather, can the economy be better stabilized through the use of econometric simulation models and optimal control theory or through the use of dynamic economic theory to evaluate alternative stabilization rules? Given the enormous costs associated with
unsuccessful macro policy, it seems almost ludicrous to consider experimentation to resolve this debate. The so-called "applications and tests" that have been performed are neither application nor test. They all use an econometric model to simulate the economy, thereby assuming a fixed law of motion invariant to the way policy is selected. Neither are my and Kydland's nor Lucas's analyses tests of the alternative scientific paradigms, as in both cases a rational or self-fulfilling expectations world is assumed.

II. A REVIEW OF STOCHASTIC CONTROL THEORY

Control theory presupposes the existence of a law of motion for the system being controlled. In discrete time—which will be assumed here for expository simplicity and because virtually all quantitative macro models are in the discrete time class—next period's position or state \( x_{t+1} \) depends upon the current position \( x_t \), the action of the controller \( u_t \), and an independent shock \( \varepsilon_t \); that is,

\[
(1) \quad x_{t+1} = F(x_t, u_t, \varepsilon_t).
\]

In a macroeconomic application, the function \( F \) would be the reduced form of an econometric model. If the order of lags is greater than one, as is the case for most macro models, the state variable must include these lagged variables and certain obvious identities added to \( F \). This is just the well-known procedure used to translate an \( n \)th order difference equation with a single variable into a first-order difference equation with \( n \) variables.

The father of quantitative aggregate demand management policy is Tinbergen (1956), in whose scheme a set of target variables and an equal number of instruments are selected. These instruments are adjusted so as to keep the target variables on the desired path. If there are more target variables than instruments, or if there are costs of adjusting instruments, there will be trade-offs, and an objective function must be introduced. Simon (1956) and Theil (1957) developed computationally feasible techniques when the law of motion \( F \) was linear, with additive errors, and the objective function quadratic. These methods were first applied (Holt, Modigliani, Muth, and Simon, 1960) to production scheduling and inventory control. Subsequently, Theil (1964) advocated that they be used for macroeconomic planning.
If the law of motion is nonlinear, then, before applying these techniques, Taylor series expansions are used to form linear approximations. Similarly, quadratic functions are used to approximate the objective function over the relevant range of outcomes. This is the basic method used to control stochastic systems optimally whether these arise in management science or engineering.

Friedman (1953) has argued that there is great uncertainty in the timing and magnitude of the effects of the money supply, and that, given this great uncertainty, attempts at active stabilization are likely to have the perverse effect of contributing to economic instability. This argument has a direct analogue in engineering. When controlling physical systems with transients that are ill understood (for example, some continuous steel making processes), best control practice is not to attempt fine tuning. Adjustments are made infrequently and only when the process moves out of some acceptable control range.

Builders of macro econometric models use static economic theory to deduce the form of relationships determining desired or stationary stocks. The nature of the adjustment of the actual to the desired stock is determined empirically by searching a space of distributed lag relationships for the one which provides the best fit. Recent econometric work by Geisel, McGuire, and Silverman (1975) indicates that, for sample sizes used in estimating econometric models, the probability of selecting an incorrect adjustment process is high. Unlike forecasting, the performance of control theory has not been shown to be robust to specification errors of this type. The only robustness study of which I am aware, Chow (1966),\(^2\) found that optimal policy derived from the Michigan model worked poorly if the Wharton model generated the data, suggesting a lack of robustness. On the other hand, the optimal policy derived from the Wharton model performed well when the Michigan model generated the data, suggesting the Wharton model may be more robust for control purposes.

If the uncertainty in the system's law of motion is moderate, control theory can still be effectively applied. The uncertainty in the parameters specifying the law of motion \(F\) is represented by a prior distribution. Using the dynamic programming methods of Bellman (1957) and Blackwell (1965), the decision facing the controller is well defined, the objective being to maximize the expected value of some criterion function. When the state variable is augmented by a set of variables specifying the controller's distribution on the unknown parameters, there is a well-defined law of motion.

\(^2\)John B. Taylor brought this work to my attention.
Practical methods were developed independently in the economics and engineering literatures to find approximately optimal decisions. The methods assume a quadratic objective and a linear law of motion with unknown coefficients and are based on the following observations. If the coefficients are independent draws from some distribution, the dynamic programming value functions are quadratic, and optimal decision rules are linear in the state variable. Thus, as for the case of known coefficients, computing optimal decision rules is feasible even for complex systems with hundreds of state variables. This observation suggests selecting at each stage that decision which would be optimal if the coefficients were independent draws from the controller's current prior on the unknown coefficients. After each observation, the prior is updated before determining the next period's decision. Tests indicate this procedure worked well when uncertainty was moderate.

This decision procedure, however, ignores the expected effect of a decision upon future knowledge of the coefficients. By using an instrument and observing the result, a more precise estimate of its effect is obtained, and more effective control is possible in the future. Some modest progress has been made in dealing with this experimental element in a practical way (e.g., MacRae, 1972). None of the procedures developed, however, perform well when uncertainty is great, and the gains seem modest (see Norman, 1975).

Econometricians have not been very successful in finding relationships stable over time. Even for the Fed-MIT model, Muench, Rolnick, Wallace, and Weiler (1974) found many of the relationships unstable. Provided the changing structure is caused by slowly changing exogenous factors, such as the demographic profile of the population, and not by changes in policy, optimal control is not necessarily inapplicable. Sarris and Athans (1973) have extended adaptive control to structures with varying coefficients. Similarly, Zellner (1971), and others, have incorporated model selection into the control problem.

To summarize, optimal control theory is an increasingly powerful tool for controlling systems subject to a policy invariant law of motion. If one accepts the view that the economy is governed by some policy invariant law of motion, and if one has sufficient knowledge of this law of motion, there is no question that modern control theory should be used for macro planning.

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3In the economics literature, I first used these methods in 1967; I was unable to determine when they first appeared in the engineering literature. For further details on the methods, see Chow (1975, ch. 11).
III. DOES A POLICY INVARIANT LAW OF MOTION EXIST?

Many years ago Knight (1941) argued that it was a fallacy to assume that social problems could be solved by applying to the social sciences the constructs which have produced the celebrated triumphs of the natural sciences. This warning has gone largely unheeded. More recently, Lucas (1976) attacked the assumption that a policy invariant behavioral or structural relationship exists, arguing that it was inconsistent with everything now known about dynamic economic theory. In situations where economic actors care about what happens, and the models, or even the predictions of the models, are public knowledge, is it reasonable, Lucas asks, to model agents as having expectations which are inconsistent with the predictions of the model?

If it is not, then the basic assumption of control theory that there is an underlying law of motion invariant to the way policy is selected must be rejected. Such considerations result in the following reformulation of the stabilization problem. As before, there is a law of motion, but the motion of the state variable depends in addition upon the actions \( d_t \) of the private economic agents:

\[
\begin{align*}
(2) \quad x_{t+1} &= F(x_t, u_t, d_t, \epsilon_t).
\end{align*}
\]

Given a policy rule (here assumed deterministic, but randomized strategies are also possible) which specifies the values of the control as a function of the state variable,

\[
\begin{align*}
(3) \quad u_t &= \pi(x_t),
\end{align*}
\]

and relation (2), the determination of the equilibrium decision rules of the private agents is, like all competitive equilibrium analyses, a fixed point problem. If the structure of preference is recursive, these rules will be time invariant and of the form:

\[
\begin{align*}
(4) \quad d_t &= \delta_{\pi}(x_t).
\end{align*}
\]
The $\delta_{\pi}$ corresponds to the behavioral equations of econometric models, but are indexed by the policy rule. Equations (2), (3), and (4) define the system's law of motion under policy rule $\pi$. The policy design problem is to find a policy $\pi$ which yields desirable operating characteristics for the economy. In his critique of current econometric policy evaluation, Lucas (1976) advocates this approach.

In commenting on this critique, Gordon (1976) suggests that, if changes in the policy rule change the behavioral equations in a predictable way, then control theory can still be used, provided the controller takes into consideration these induced changes. As noted above, Kydland and I (1977) attempted to do just that and found it logically impossible. The best action, given the current situation and a correct evaluation of both current outcomes and the end of period position, is not optimal.

In a dynamic situation, the policymaker (actually, his economic advisers) constructs utility functions which rationalize individual choice and uses them to predict the agent's choice in new environments. If policymakers use utility theory to predict choice, then why not assume that private agents use utility theory to rationalize and predict social choice? There is a wealth of empirical evidence that they do, as indicated by changes in behavior induced only by changes in the public's expectations of future policy.

Most policy analyses are carried out in a static framework, and expectations are not a problem. A policy, say, a set of tax rates, is selected, and agents maximize given the policy. Even for the static optimal taxation problem, Stiglitz (1976) has shown that a randomized tax policy may strictly dominate the best deterministic policy. This is at variance with the fundamental results of control theory which specify that one need consider only non-randomized strategies in searching for an optimal policy. This leads inescapably to the conclusion that policy selection is a game if agents maximize for given policy. With gaming problems, there are many alternative solutions, but in this case the only plausible one is the dominant player solution. In selecting policy, the policymaker takes into consideration the reaction of agents to his decision, while the many agents, being small, behave noncooperatively and maximize for given policy.

The applicability of the hypothesis of rationality of expectations has led to a spirited debate as to its "plausibility." Is the complexity of the forecasting problem facing agents in dynamic uncertain environments so great that empirically determined disequilibrium behavioral rules can better predict behavior than can rational expectations equilibria? How is a rational expectations equilibrium obtained? There are, I think, answers to these
questions. Competitive economic forecasting will likely result in an equilibrium being obtained even if the structure is not understood. Forecasters both introduce an assumption about how future policy will be selected and observe indicators of expectations, such as measures of consumers' confidence and sales anticipations. Given expected policy, plans, and anticipations, the best forecasts are sought. But, the forecasts form the basis for people's expectations of aggregate economic behavior. Unless the rational expectations solution is obtained, there will be a discrepancy between anticipations and forecasts. Such a discrepancy is unlikely to exist in the competitive forecasting business for much the same reason that the stock market is efficient—profitable opportunities would exist.

Consistent policy

A consistent policy is a sequence of rules, one for each time period, which specifies policy action contingent upon the state of the world at that time. The rules have the property that each is optimal given the subsequent elements. For finite period decision problems with a policy invariant law of motion, a consistent policy is optimal, and any optimal policy is consistent. For problems with bounded returns and discounting, Blackwell (1965) has extended this fundamental result of control theory to infinite horizon problems. For dynamic games, Kydland (1975) has demonstrated that a consistent policy is not generally optimal and that optimal policies are not necessarily consistent. Calvo (1976) independently established the same result when he found the optimal monetary policy was inconsistent.

Patent policy example

Given that resources have been allocated to inventive activity which resulted in new products or processes, the efficient policy is not to permit patent protection for existing inventions. No one would seriously consider this optimal control theory solution to be reasonable. Most would agree that the question should be posed in terms of the optimal patent life policy, which takes into consideration both the incentive for inventive activity provided by patent protection and the loss in consumer surplus that results when someone realizes monopoly rent. In other words, economic theory is used to predict the effects of alternative policy rules and a rule with good operating characteristics selected.
Dynamic taxation example

In recent years, a large optimal taxation literature has developed. A tax policy is optimal if it maximizes some social welfare criterion, subject to the constraint that agents supply labor optimally given tax and wage rates. The optimal taxation literature has abstracted from the savings decision and focuses only on labor supply and consumption. Once the time dimension is introduced, optimal taxation policy, whether random or deterministic, is not consistent. The principle of optimality does not hold because current labor supply and consumption decisions depend upon expected future tax rates.

Suppose there is a representative consumer who maximizes utility given the (expected) tax rates on capital and labor incomes. The optimal tax policy is the sequence of tax rates which maximizes the welfare of this consumer, given some stream of government expenditures that must be financed, and the behavior of the consumer, given the tax policy. Assuming expenditures are not too large, only capital income will be taxed in the initial period, as it is supplied inelastically. In subsequent periods, both labor and capital incomes will be taxed. The inconsistency of the optimal solution arises because the optimal tax on labor is zero in the current period and positive in future ones, but eventually future periods become the current one.

Other examples

The inconsistency problem provides the rationale for enforceable contracts and for certain legal principles such as grandfather clauses. In many policy debates, however, the principle is ignored, for example, when, following a crop shortfall, the government controls current price promising not to control the product's future price, or when the government imposes rent controls on existing rental units but not on new units. Similarly, no rational private agent would stockpile oil speculating on another embargo because he would realize that, in such an event, the government would almost surely again control the price of oil.

Use of control theory in management science

Control theory has proved useful in controlling inventory and in scheduling production. The reason it has succeeded is that the law of motion which equates next period's stock to current stocks plus net additions is derived from physical considerations and is invariant to policy selection. If the problem

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4 See Ramsey (1927), Diamond and Mirrlees (1971), and Stiglitz and Dasgupta (1971).
is modified to make price a decision variable, then optimal control is no longer appropriate, to the extent that current demand depends not only on current price but on anticipated future prices. This is probably why firms with some monopoly power rely on a pricing policy and do not use control theory for pricing decisions.

The term "control" is used in accounting, but its use is very different from that in control theory. An accounting control system is a set of rules and monitoring procedures. The effectiveness of a particular system is judged in terms of the resulting operating characteristics of the firm. Accountants clearly recognize that the policy problem is to use equilibrium theory to predict the effect of alternative policy rules.

To summarize, the implication of dynamic equilibrium theory is that control theory should not be used for economic planning. The problem of stabilizing the economy is not analogous to guiding an inert rocket ship, but more nearly analogous to guiding a dual-control rocket ship when the two controllers want to go in different directions.5

IV. LONG-TERM CONTRACTS AND RATIONAL EXPECTATIONS

The suggestion has been made that, even if expectations are rational, activist monetary policy is appropriate if agents enter into long-term nominal wage contracts.6 Activist monetary policy, taken to mean the monetary action which is best given the current nominal wage, is a mistake, as the example below demonstrates. If, on the other hand, the argument is that the operating characteristics of the economy may be superior for some rule other than the constant money supply rule, then there is no disagreement.

Let \( w \) be the log of the nominal wage, and assume it is set efficiently given the policy rule. \( w \) maximizes the expected utility of the representative worker, given he is committed to supply all the labor the firm wants at that wage. The assumed log of labor demand \( n \) is

\[
(5) \quad n = \eta_0 \cdot \eta_1 (w-p) + \epsilon.
\]

5Cyert and DeGroot (1977) consider such a dual-controller problem.

where \( e \) is an aggregate demand or supply shock and has mean zero, and where \( p \) is the log of the price level. Letting \( u(\cdot, \cdot) \) be the appropriate objective function for the individual, and \( \mathbb{E} \) the expectations operator, the \( w \) selected is the one which maximizes

\[
\mathbb{E}_{p,n} [u(w-p, n)],
\]

subject to (5). It is further assumed that the objective function can be approximated by the quadratic function:

\[
u(w-p, n) = \mu_0 (w-p) + (n - \mu_1)^2.
\]

Substituting for \( n \) and taking the expectations yields

\[
\mu_0 (w-p^e) + [\eta_0 - \eta_1 (w-p^e) - \mu_1]^2 + \text{var}(\epsilon) + \eta_1^2 \text{var}(p) + 2\eta_1 \text{cov}(\epsilon, p).
\]

Differentiating with respect to \( w \), setting the resulting expression equal to zero, and solving for \( w \), the nominal wage selected is

\[
w = p^e + (\eta_0 - \mu_1 - \mu_0 / 2 \eta_1) / \eta_1.
\]

Without knowledge of the way in which policy will be selected, there is no basis for forming price expectations.

Control theory or discretionary solution

With this solution, the rate of inflation that will be selected can be said to be best relative to some social objective function \( s(n, p) \), given \( w \), the labor demand function (5), and the value of the realized shock. If there is a representative worker and a democratic society, why should the social objective function and the worker's utility function differ? Phelps (1972) provides some answers, including the observation that labor income is subject to income tax,
which will result in an oversubstitution of home-produced goods and leisure
for market-produced goods. In addition, unemployment insurance drives a
wedge between the private and social product of search, resulting in less than
optimal employment.

Again, using quadratic approximations, the social objective function is

\[ s(n, p) = (n - \sigma_2)^2 + \sigma_0 (p - \sigma_1)^2. \]

Another question to be addressed is, why introduce the price level in the social
objective function? One reason is that the U.S. and many other tax systems
and institutional arrangements are predicated upon stable prices, and that, until
these are changed, price level instability imposes real costs. Another argument
is that, in a democracy, policy choice should reflect the people’s preference;
there is a wealth of evidence that the public perceives inflation as a serious
problem. If this perception is not rational, then it behooves economists to so
convince the people.

Maximizing the social objective function (10), with respect to \( p \), subject
to the labor demand (5), yields

\[ p = (\sigma_0 \sigma_1 - \eta_0 \eta_1 + \eta_1 \sigma_2 - \eta_1 e + \eta_1 w)^2 / (\eta_1^2 + \sigma_0). \]

The expected price prior to observing \( e \) is, therefore,

\[ p^e = (\sigma_0 \sigma_1 - \eta_0 \eta_1 + \eta_1 \sigma_2 + \eta_1^2 w) / (\eta_1^2 + \sigma_0). \]

Substituting this into (9), and then solving for \( w \), yields the rational
expectations nominal wage for the control theory policy.

**Best policy rule**

The control theory policy is not in general best. It is not optimal because the
policymaker fails to take into consideration the effect of his policy rule
upon the selected nominal wage \( w \). There will exist a policy rule of the form
which provides a higher expected value for the social objective function. With such a rule,

\[ p^e = \pi_0, \]

\[ w = \pi_0 + (\eta_0 + \mu_0/2 - \mu_1)/\eta_1, \]

and

\[ n = \eta_0 - \eta_1 (w - \pi_0 - \pi_1 + \epsilon) + \epsilon. \]

The expected value of the social objective function should be maximized with respect to \( \pi_0 \) and \( \pi_1 \), subject to (14) - (16). 7

The assumed model incorporates wage rigidities in the simplest possible way, and the conclusion derived is that such rigidities do not justify the use of control theory for policy selection. The problem is one of policy design, namely, finding a policy rule which yields good or, in this case, best operating characteristics. 8 If it were assumed that wage contracts are overlapping, the analysis would be much more complicated, and the control theory solution non-trivial. These complications only reinforce the argument that optimal control theory is inappropriate if expectations are rational. Another problem that is not addressed here is that the nature of the contract is likely to change if policy changes.

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7 I have not proved here that the best policy lies within the class of linear response functions, nor would I be surprised if it did not. I have established that the control theory solution could be dominated by an element of the class.

8 These conclusions are implicit in Phelps and Taylor (1977, p. 163) when they recognize that monetary authorities should sometimes penalize the economy in the short run for the sake of beneficial effects of the rule upon the economy’s operating characteristics.
V. NONNEUTRALITIES WITH ANTICIPATED POLICY

The equilibrium view of business fluctuations does not necessarily imply that anticipated macro policy is without effect. The following example demonstrates that the way in which government expenditures are financed, whether through issuing debt or taxation, does matter. The example assumes a representative consumer whose preferences depend upon consumption and labor supplied. Letting $c_t$ be consumption in period $t$, and $n_t$ labor supplied, the representative household maximizes the expected value of the function

$$\sum_{t=0}^{\infty} \beta^t (c_t - \mu_1 n_t - \mu_2 n_t^2),$$

where $0<\beta<1$ is the discount rate, and the $\mu_i$ are positive parameters.

There is no capital, and the production relationship is output $y_t$ proportional to labor input; measuring output units appropriately, the proportionality constant may be taken to be one, and the production function is simply

$$y_t = n_t.$$

Government expenditures $g_t$ are determined exogenously and do not affect the relationship which rationalizes the representative household's intertemporal private consumption-leisure choice.

The government finances its expenditures and past debt obligations through a proportional tax on labor income and the issuance of real purchasing power bills which come due the next period. Letting $d_t$ be the debt coming due in period $t$, $\tau_t$ the proportional tax rate, and $\sigma_t$ the price of a real bill coming due next period, the government's budget constraint is

$$\sigma_t d_{t+1} + \tau_t n_t = g_t + d_t.$$

The policy instruments are the tax rate and the amount of the bills issued. The above constraint precludes their independent manipulation, so there is but one instrument, which will be taken to be the tax rate.
For simplicity, it is assumed that the $g_t$ are independent draws from some known distribution with a finite second moment. It is further assumed that financing policy in period $t$ depends only upon debt coming due in that period and on government expenditures; that is, $\tau_t = \tau(g_t, d_t)$, and $d_{t+1} = d(g_t, d_t)$. Given these assumptions, the state variables, or position of the economy, are the pair $(g_t, d_t)$.

The perfect substitutability assumption of $c_t$ for $c_{t+1}$ (assuming no corner solutions) implies that equilibrium $\sigma_t$ must be $\beta$. For the assumed production function, the demand for labor is infinitely elastic at real wage 1. Further, the competitive assumption along with constant returns to scale implies zero profits and dividends, so share ownership need not be considered. The competitive fixed point equilibrium problem is to determine for a given policy rule $\tau_t = \tau(g_t, d_t)$ the equilibrium labor employment function $n_t = n_\tau(g_t, d_t)$. The employment function $n_\tau$ is subscripted by $\tau$ because it will change as a result of changes in the policy function $\tau(g_t, d_t)$.

Equating the ratio of the marginal utility of $c_t$ and the disutility of labor supplied to the ratio of their prices, one obtains

\[
\frac{1}{-\mu_1 - 2\mu_2 n_t} = \frac{1}{-(1-\tau_t)}.
\]

or

\[
n_t = \frac{1}{2\mu_2} \frac{1 - \mu_1}{2\mu_2} \tau_t.
\]

Labor employed in period $t$ is a decreasing linear function of the tax rate in that period. Tax receipts are $\tau_t n_t$, and from (20), (21), and the result that $\sigma_t = \beta$, the equilibrium law of motion is obtained:

\[
d_{t+1} = \frac{1}{\beta} \left[ g_t + d_t - \tau_t \left( \frac{1 - \mu_1}{2\mu_2} - \frac{\tau_t}{2\mu_2} \right) \right].
\]
Because of the very special assumption, expectations of future policies do not affect current labor supply. This is why the equilibrium employment and consumption functions,

\[
(23) \quad c_t = n - g_t = \frac{1-\mu_1}{2\mu_2} - \frac{1}{2\mu_2} \tau_t g_t,
\]

do not depend upon the financing policy rule followed. Nonetheless, policy is relevant, as the expected discounted value of utility flows for the representative household depends upon the policy rule followed.

Let \( v_t(g_t,d_t) \) be the expected discounted utility realized by the household from period \( t \) forward if the policy is \( \tau_t = \tau(g_t,d_t) \). I restrict tax policies to those for which debt remains bounded with certainty; next period’s debt is an increasing function of current debt; and \( \tau(g,d) \) is increasing in \( d \). Result: The function \( v_t(g,d) \) is decreasing in \( d \). Proof: By the recursivity principle,

\[
(24) \quad v_t(g,d) = c - \mu_1 n - \mu_2 n^2 + \beta \int v_t(z,d')dF(z),
\]

where

\[
\begin{align*}
    c &= n - g \\
    \tau &= \tau(g,d) \\
    d' &= \beta^{-1}(d + g - \tau n),
\end{align*}
\]

and \( F(\cdot) \) is the distribution function of \( g \). The right-hand side of (24) and the constraints define a mapping of bounded (measurable) functions of \( (g,d) \) into the same set of functions. The fixed points of this mapping \( M \) are solutions to (24). It is trivial to verify conditions of theorem 5 of Blackwell (1965) to conclude that there is a unique solution, and that \( M^0 v = M(M^{n-1} v) \) converges to this fixed point for all functions \( v(g,d) \). The operator \( M \) has the property that it maps a function decreasing in its second argument into a function also decreasing in that argument. This proves the result.
Discussion

If the representative household owns the government debt and as a taxpayer is liable for its payment, then why does the amount of debt financing matter? The answer is that this is an optimal taxation problem and welfare depends on the efficiency of taxation. By maintaining a relatively constant tax rate, and by running deficits when $g_t$ is large and surpluses when it is small, there is less welfare loss associated with financing a given stream of government expenditures. Having access to capital markets replaces the constraints that government expenditures $g_t$ equal tax receipts $t_t n_t$ for every $t$, with the single constraint that the present value of tax revenues equals the present value of expenditures. It is the same gain realized in static situations when the only constraint is that total receipts equal total expenditures, rather than matching expenditures types to revenue sources and then insisting upon an equality of expenditures and receipts for each match.

An interesting feature of this model is that the representative household's welfare would increase if the government defaulted on its existing debt and then followed a given debt policy promising to honor future debt obligations. Such a policy is implied by optimal control. The inconsistency problem of the optimal solution is that next period the government will again fail to honor existing debt. The control solution, then, is never to honor existing debt and, as a result, the government will not have access to capital markets. The result is inefficient taxation and a dead-weight loss to society.

VI. TESTING THE ALTERNATIVE PARADIGMS

It is unlikely that any single observation will result in the rational expectations hypothesis being accepted or rejected. An observation which results in the ready acceptance of a new theory or paradigm, such as the confirmation of Einstein's prediction from relativity theory that light rays would be bent by the gravitational force of the sun, is a rarity. More typically, a considerable period elapses before a new theory is accepted, and even then many observations appear inconsistent with the new theory. For example, the demise of the mixture theory and the acceptance of Dalton's atomic theory in chemistry required a generation, as described by Kuhn (1970, p. 135):
Chemists could not, therefore, simply accept Dalton’s theory on the evidence, for much of that was still negative. Instead, even after accepting the theory, they had still to beat nature into line, a process which, in the event, took almost another generation. When it was done, even the percentage composition of well-known compounds was different. The data themselves had changed.

Similarly, assuming it is accepted, a considerable period will elapse before the economist beats nature into conformity with rational expectations theory, and even then many puzzles will remain.

The rational expectations paradigm may be considered in the same spirit as the maximizing assumption, once the subject of much debate in economics, but now considered to be fundamental. The rational expectations assumption augmented the maximizing assumption by hypothesizing that agents use their information sets efficiently when maximizing. Like utility, expectations are not observed, and surveys cannot be used to test the rational expectations hypothesis. One can only test if some theory, whether it incorporates rational expectations or, for the matter, irrational expectations, is or is not consistent with observations.

In science, controlled experiments are frequently used in the testing of theories. But, this is hardly the way to test which hypothesis should be used to stabilize the economy—the paradigm which views economic fluctuations as equilibrium phenomena or the paradigm, borrowed from physical science, which views economic fluctuations as generated by an empirically determined policy invariant law of motion. Recent attempts at exploiting a perceived trade-off between employment and inflation are as close to a controlled experiment in aggregate economics as one might hope for. The predictions of the rational expectations theorists that this procedure would fail and that, by contributing to uncertainty, might even increase unemployment were fulfilled. The predictions of the Phillips curve supporters that higher inflation would buy lower unemployment were not confirmed.

There is another notable example of the failure of a generally accepted nonmaximizing macro economic theory: the depression, which economists so confidently predicted for the post-World War II period, and which never occurred. This prediction was proved wrong because the empirically determined consumption function which related current consumption to current disposable income was mistaken. New maximizing theories of consumption were developed.
by Modigliani and Brumberg (1954) and by Friedman (1957) in response to this failure. These theories have forecast well since then, generating, for example, the prediction, subsequently verified, that the temporary tax surcharge of 1968 would have minimal effect.

Many suggested "tests and applications" of the use of control theory for macro stabilization have appeared in the literature. They have all used a set of difference equations, sometimes stochastic, to simulate the economy. For example, Bogaard and Theil's (1959) use of the Klein I model to show that a rapid recovery from the Great Depression would have been possible. Slightly more sophisticated tests (Prescott, 1967) did not assume the model to be known. An econometric model with random shocks was used to generate an historical data set. This data set was then used to draw inferences from the model, and adaptive control methods were used to select policy actions during the control period. One could go one step further and simulate the model selection procedure to obtain more demanding tests. But, even then, such tests are worthless in choosing between the alternative paradigms because, incorporated into the simulated environment, is a policy invariant law of motion. At best, such tests can assess the validity of Friedman's (1953) contention that economists do not have sufficiently precise knowledge of the law of motion to fine tune the economy, and that, until they do, they should rely on a neutral policy.

Kydland and I (1977) evaluated a class of investment tax credit policies within the rational equilibrium framework. Increasing costs associated with rapid adjustment in capacity were assumed. Distributed lags were introduced by assuming that capacity expansion required two periods, with some fraction of the expenditures occurring in the first, and the rest in the second, period. We simulated the use of optimal control in such an environment.

The tests assume that a passive investment tax credit stabilization policy was pursued in the past, and that the function describing investment behavior was equilibrium, given this passive policy. They also assume that econometricians have estimated the investment relationship and that policymakers use control theory to determine which policy rule is optimal, under the incorrect assumption that the equilibrium investment function is invariant to the policy rule used. Subsequent to the implementation of this policy rule, the economy moves to the new equilibrium investment function. Econometricians revise their estimate of the investment function, reasoning that there has been structural change, and the policymaker uses optimal control to determine a new policy rule. The change in policy induces still another change in the investment function, which in turn induces a change in the policy rule once the shift in the investment function is recognized.
The tests found that the iterative process typically converged. For some examples, this simulated application of control theory improved economic performance initially (e.g., early iterations), but then had detrimental effects, eventually converging to a policy which was inferior to the no-investment tax credit policy. For other examples, the iterative process did not converge, and each iteration resulted in a less stable economy. Insofar as this iterative process captures the essence of what is happening, these tests indicate that the use of control theory for macro stabilization is hazardous and can very well increase economic fluctuations. These tests demonstrate that, for equilibrium environments, control theory should not be used for economic planning, but do not indicate whether the economy is best viewed as an equilibrium or disequilibrium process.

These observations suggest that simulation experiments cannot be used to choose between the control and the equilibrium approaches. A fundamental inference problem is that given a policy rule both paradigms predict the existence of a stable law of motion, and that no matter how large the sample size one cannot choose between the theories. Only if there is a change in policy, such as the shift to peacetime inflationary financing as occurred in the late sixties and early seventies, do the predictions of the theories differ.

There are two tests of rational expectations theories which warrant mention. Lucas (1973) uses international evidence to test for an output-inflation trade-off and finds the data consistent with there being none. Sargent (1976) tests a model incorporating rather severe classical hypotheses and turns up "little evidence requiring us to reject the key hypotheses of the model that government monetary and fiscal-policy variables do not cause unemployment or the interest rate" (p. 236). Similarly, one can find laws of motion which are consistent with the data.

VII. CONCLUSION

The primary purpose of this paper is not to enter into the monetarist debate, but to discuss whether optimal control is an appropriate device for economic planning. The use of optimal control is predicated upon the existence of a policy invariant law of motion, which economic theory predicts will not exist if expectations are rational. The inapplicability of control theory is recognized when the situation is well understood, such as for patent policy

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9I proposed the use of simulation experiments with many players assuming the roles of households, firms, banks, forecasters, and policymakers. The near unanimity of opinion among my colleagues that results of such experiments would tell one nothing led me to retract this proposal.
...and management. It is only for aggregate economics that the norm is to borrow from physics the concept of a fixed policy invariant law of motion to be determined empirically. Knight (1941) warned that modeling people as inert objects is unlikely to achieve celebrated triumphs in social science comparable to those obtained by treating physical objects as unlike people.

A secondary purpose of the essay is to correct the fallacy that because some rational expectations theories predict that anticipated monetary policy will have little effect, all theories utilizing this paradigm imply that active policy will either be ineffectual or will contribute to economic instability. Quite the contrary; for some policy rules, the equilibrium operating characteristics of the economy will be preferred to others, which is precisely what Lucas (1976), the foremost advocate of the paradigm, argues. For the long-term wage contracts example, output is perfectly correlated with the unanticipated component of inflations, and uncorrelated with the anticipated, yet the policy rule followed affects the variance of output.

The implication of the equilibrium view of business fluctuations is that until a tested theory of the business cycle is available it is best that active stabilization not be attempted. Reliance on a policy of maintaining a relatively stable currency price and constant tax rates is appropriate. Once a tested theory is available, the implication of the equilibrium view is that economic theory be used to predict the economy's operating characteristics under alternative policy rules, and that one with good characteristics be selected.
REFERENCES


