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MONEY, MULTIPLIER ACCELERATOR INTERACTION, AND THE BUSINESS CYCLE*

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I. INTRODUCTION

It has been argued that the fact that the money supply moves cyclically, expanding in periods of prosperity and contracting in recession, constitutes prima facie evidence that the best efforts of the Federal Reserve System have had the perverse effect of contributing to economic instability. Our analysis reveals that it is quite possible that the American economy would be subject to even greater instability if discretionary monetary policy were replaced by the simple rule of having the money supply expand at a constant rate per annum in both boom and recession. We first specify the behavioral equations of a model constituting a synthesis of the real theory of the business cycle provided by the multiplier-accelerator model of Samuelson [1939] with the Hicks' [1937] static IS-LM apparatus for the analysis of the role of money in the determination of aggregate income. Then we analyze the effects of alternative stabilization strategies that might be pursued by the monetary authorities. Stochastic disturbances are shown to create complications in the formulation of appropriate stabilization policy. In the concluding section we consider the implications of our analysis with regard to the debate on rules vs. discretion in the execution of monetary policy.

II. BEHAVIORAL ASSUMPTIONS

Any effort at incorporating monetary variables into a model of the business cycle inevitably involves a number of issues that the best of empirical studies have not successfully resolved. Milton Friedman, the strongest advocate of the importance of money in the business cycle, insists [1964a, p. 21-22]:

We are still a long way from having a detailed and tested theory of the mechanism that links money with other economic magnitudes. . . . Identification of the channels through which short-run monetary changes work their effects, and specification in quantitative terms of the characteristics of the channels and of the effects exerted through them, remain major tasks for future research.

Since empirical investigations have not yet served to specify the precise framework to be employed in the analysis of the role of monetary policy in the business cycle, let us proceed with a model of minimal complexity.¹

We shall first spell out precisely the behavioral equations of our system. Time is treated as a discrete variable in order to facilitate comparison of our system with models of the real cycle developed by Samuelson [1939], Metzler [1941], and Hicks [1950]. All symbols refer to variables meas-

¹ Certain complications of the basic model will be discussed in the Appendix.

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ured in real magnitudes. Unless a subscript indicates otherwise, all variables refer to the current time period.

Investment spending is assumed to be determined by a capital stock adjustment process. The desired capital stock is

\[ K^d = a_1 Y_{-1} - a_2 r, \]

where \( Y_{-1} \) is last period's net national product and \( r \) the rate of interest. How fast business firms attempt to adjust their capital stock towards this desired level, given the costs involved, determines the volume of investment. In many empirical studies of investment behavior it is assumed that some definite fraction \( \delta \) of the discrepancy between the desired capital stock and that inherited from the preceding period is eliminated; i.e., net investment is

\[ I = \delta (K^d - K_{-1}), \quad 1 \geq \delta > 0 \quad (2.2) \]

where \( K_{-1} \) is the inherited capital stock.\(^2\)

Substituting 2.1 into 2.2 yields

\[ I = \delta a_1 Y_{-1} - \delta a_2 r - \delta K_{-1}; \quad (2.3) \]

investment thus depends upon last period's income, the inherited capital stock, and the interest rate. In the special case of \( \delta = 1 \) we have from 2.1 and 2.2 that \( K_{-1} = K^d_{-1} = a_1 Y_{-2} - a_2 r_{-1}; \) consequently, the volume of investment spending with instantaneous adjustment of the capital stock is simply

\[ I = K - K_{-1} = a_1 (Y_{-1} - Y_{-2}) \]

\[ - a_2 (r - r_{-1}), \quad (2.3a) \]

investment depending upon changes in both income and the interest rate. Note that if the desired capital stock is insensitive to the cost of capital (\( a_2 = 0 \)), equation 2.3a reduces to the simple accelerator. On the other hand, equation 2.3 reduces to the standard marginal efficiency of capital schedule, complicated by the inclusion of the lagged capital stock as an initial condition, if \( a_1 \) is zero.

The other equations of our model are conventional. Specifically, we adopt a consumption function involving the Robertsonian lag:

\[ C = c_0 + c Y_{-1} . \quad (2.4) \]

In addition, output is the sum of consumption, investment, and government spending:

\[ Y = C + I + G . \quad (2.5) \]

Finally, the demand for money is determined in accordance with liquidity preference theory by income and the interest rate, either in accordance with the linear equation

\[ M^d = m_0 + m_1 Y_{-1} - m_2 r; \quad (2.6) \]

or alternatively,

\[ M^d = k(r) Y_{-1} . \quad (2.6a) \]

The rate of interest adjusts so as to insure equilibrium in the money market:

\[ M^d = M^s \quad (2.7) \]

Our model thus involves a total of nine variables and seven equations. Given the level of government spending, the real money supply, and historically determined values of the lagged variables, the seven equations can be solved to determine the remaining variables of the system.

The short run implications of these behavioral equations may be determined by substituting equations 2.3 and 2.4 into 2.5. This yields

\[ Y = c_0 + (c + \delta a_1) Y_{-1} \]

\[ - \delta a_2 r - \delta K_{-1} + G . \quad (2.8) \]

In essence, this is the IS schedule developed by Professor Hicks [1937] and popularized in
standard macroeconomic textbooks, but complicated by the inclusion of the inherited capital stock and lagged income as initial conditions. Given the historically determined values of these predetermined variables, this IS relation may be employed to solve, with the aid of the liquidity preference schedule (equation 2.6 or 2.6a), for the level of current income and the rate of interest as functions of government spending and the real money supply. But unless the resulting value of net investment happens to be zero, the capital stock changes with the passage of time and the IS curve shifts. We shall take this capital stock adjustment process into account in analyzing the implications of alternative stabilization strategies that may be pursued by the monetary authorities.

III. AN ENDOGENOUS MONETARY POLICY

Those who argue that discretionary monetary policy should be abandoned base their case upon the proposition that given the limited state of current knowledge, the best countercyclical efforts of the Federal Reserve System have actually been destabilizing. In order to analyze this issue, it is necessary for us to recognize that under present institutional arrangements neither the interest rate nor the money supply is uninfluenced by fluctuations in economic activity. We shall incorporate into our model a feedback mechanism capable of reflecting a variety of possible monetary response patterns to changing economic conditions.3

Interest rates are frequently regarded as an index of the degree of ease or tightness of credit conditions. Suppose that the central bank concentrates its attention on interest rate movements in attempting to moderate fluctuations in economic activity. Specifically, let us suppose that the money supply is adjusted in such a way as to insure that the interest rate departs from some normal or "natural" value \( \bar{r} \) whenever income departs from a desired level, \( \bar{Y} \), in accordance with the rule:

\[
\tau = \bar{r} + \lambda(Y_{-1} - \bar{Y}), \quad \lambda > 0. \quad (3.1)
\]

First differencing of this equation reveals that such a policy is equivalent to raising the rate of interest when income is increasing, and conversely:

\[
\tau - \tau_{-1} = \lambda(Y_{-1} - Y_{-2}). \quad (3.2)
\]

If it may be assumed that the demand for money is correctly described by equation 2.6, with \( m_2 > 0 \), the execution of this policy will require that the real money supply be adjusted so as to achieve

\[
M^* = m_0 + (m_1 - m_2\lambda)Y_{-1} + m_2(\lambda\bar{Y} - \bar{r}); \quad (3.3)
\]

hence

\[
M^* - M_{L-1} = (m_1 - m_2\lambda)(Y_{-1} - Y_{-2}). \quad (3.4)
\]

How the money supply responds to guided by a monetary "superman" equipped with precise knowledge of the economy's structure. Our approach is more analogous to that of Baumol [1961] in his analysis of the effects upon the stability of the economy of certain quite simple behavioral rules for manipulating the level of government spending. But in order to focus our attention more fully upon issues of monetary policy, we assume that government spending is exogenous rather than expanding the discussion to incorporate the effects of fiscal stabilization activity.4 But if the demand for money were determined in accordance with equation 2.6a, the response of the money supply to changes in income would have to be:

\[
\frac{\partial M^*}{\partial Y_{-1}} = k + \frac{\partial k}{\partial \tau} \frac{\partial \tau}{\partial Y_{-1}} = k + \lambda \frac{\partial k}{\partial \tau} Y_{-1},
\]
fluctuations in economic activity depends upon the magnitude of the policy coefficient \( \lambda \). The money supply will move cyclically rather than countercyclically if the policy coefficient is sufficiently small, as when the central bank attempts only to lean against the wind in adjusting the cost of credit. 

Under the Real Bills Doctrine we had \( \lambda < \frac{m_1}{m_2} \), for then the money supply expanded with income in response to the increased supply of short-term, self-liquidating agricultural, industrial and commercial paper occasioned by rising economic activity. Historically, both interest rates and the real money supply have moved cyclically, the central bank attempting only moderate adjustments in interest rates as output fluctuated. If Federal Reserve policy can be roughly characterized by equation 3.1, this means that the policy coefficient has, in practice, been subject to the following restriction:6

\[
0 < \lambda < \frac{m_1}{m_2}. \tag{3.4}
\]

In contrast, many outspoken critics of Federal Reserve policy argue that the nominal money supply should be completely insensitive to fluctuations in income. If prices were rigid, such a policy would involve setting \( \lambda = \frac{m_1}{m_2} \), so that the real money supply would be insensitive to movements in income.6 In practice, of course, prices tend to rise in booms and fall in recessions;7 since the rule is stated in nominal terms, adherence to it would lead to a contraction of the real money supply in booms and its expansion in recession. This "automatic stabilizer" effect involves, in terms of equation 3.1, a value of \( \lambda \) greater than the ratio \( \frac{m_1}{m_2} \); how much larger depends upon the degree of price flexibility. The question of whether it is possible to state categorically that \( \lambda > \frac{m_1}{m_2} \) would be more stabilizing than discretionary Federal Reserve policy will be explored in subsequent sections of this paper.

IV. STABILIZATION POLICY

The implications for the stability of the economy of alternative values for the policy parameter \( \lambda \) will now be explored. Our behavioral assumptions yield a second order linear difference equation explaining the time path of output:

\[
Y + \beta Y_{-1} + \gamma Y_{-2} = \alpha \tag{4.1}
\]

where

\[
\beta = \delta(1 + a_0 \lambda - a_1) - 1 - c,
\gamma = c + \delta(a_1 - c - a_0 \lambda)
\]

and

\[
\alpha = G - (1 - \delta)G_{-1} + \delta \epsilon_0.
\]

This fundamental difference equation is obtained by adding \( Y_{-1} \) to both sides of the first differenced version of 2.8, after eliminating \( r - r_{-1} \) with 3.2 and \( K_{1} - K_{-2} \) by noting from 2.4 and 2.5 that it equals \( I_{-1} = Y_{-1} - c_0 - cY_{-2} - G_{-1} \).

Just as with the multiplier-accelerator model, difference equation 4.1 may generate a number of alternative modes of behavior when disturbed from equilibrium. For certain values of \( \beta \) and \( \gamma \) the system will converge monotonically towards equilibrium; otherwise the system will be unstable. Certain values of the coefficients generate cyclical movements.8 But in contrast to the system where

\[
\frac{\partial k}{\partial r} < 0.
\]

A negative \( \lambda \) would imply that interest rates rise in recessions and fall in periods of boom.

6 Modifications for an economy with a non-zero growth rate are relegated to the appendix, for the complications of an expanding economy do not affect the basic conclusions of this paper.

7 If money wages are rigid, the price level may fluctuate in order to continuously equate the real wage with the marginal productivity of labor.
considered by Samuelson, the type of path is affected by the decision of the monetary authorities as to how to respond to fluctuations in income—by the magnitude of the policy parameter $\lambda$.

As a prelude to analyzing how events will unfold with the passage of time it is necessary to analyze the steady-state solution of our system of behavioral equations. From 4.1 we see that if $G$ is fixed and income is at some unchanging level $Y^*$ we must have

$$Y^* = \frac{1}{1 + \beta + \gamma} \delta (G + c_0)$$

This equilibrium concept is determined by the multiplier and the level of government spending, but it is unaffected by the policy parameter $\lambda$ and the coefficients of the demand for money equation. Because it is characterized by zero net investment, this equilibrium is identical to that obtained with the traditional multiplier-accelerator model. In contrast to the Keynesian equilibrium concept, however, the analysis is not restricted to a period so short as to permit changes in the capital stock to be ignored; on the other hand, the period of adjustment is assumed to be short enough to allow us to suppress relative price movements, as is customary in both multiplier-accelerator analysis of the real cycle and Hicksian IS-LM analysis of the effects of changes in government spending and the real money supply upon the level of income.

How output fluctuates when disturbed from equilibrium is easily analyzed with a simple graphical device explained by William Baumol [1959, pp. 221-2]. The value of $\beta$ is plotted on the ordinate and $\gamma$ on the abscissa of Figure 1. As Baumol explains the system will be stable, necessarily converging to $Y^*$ regardless of the current level of income, if and only if the point $(\gamma, \beta)$ falls within the large stability triangle on the graph. The system will be subject to cyclical

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8 The quantity of money that the monetary authorities would have to keep in circulation in order to maintain the interest rate at the level $\overline{r}$ implied by policy equation 3.1 may be determined by substituting $Y^*$ and $\overline{r}$ into 3.3.

10 For the purpose of analyzing such long run issues as "secular stagnation," rather than the business cycle, it might be useful to consider an alternative concept of equilibrium involving flexible money wages and full-employment; it should be noted, however, that secular issues are usually analyzed in abstraction from the short-run dynamic complications that constitute the essence of cyclical phenomena; indeed, net investment is sometimes assumed to be a function only of the rate of interest rather than being sensitive to either the level of output or the magnitude of the capital stock! It may be shown that precisely the same equation for $Y^*$ is obtained with the quantity theory version of the demand for money, equation 2.6a. Also, the assumption that the dependence of the desired capital stock upon output and interest rates is of linear form is not essential. However, the direct inclusion in the consumption function of wealth as measured by the capital stock would require a modification of 4.2.
oscillations if the point \((y, \beta)\) falls in the shaded area to the right of the parabola.\(^{11}\)

As an illustration of Baumol's procedure, note that equation 4.1 reduces to the elementary version of the multiplier-accelerator interaction mechanism if \(\delta = 1\) and either investment is insensitive to the interest rate or interest rates are constant. Specifically, of \(\delta = 1\) and \(\lambda a_{2} = 0\) we have

\[
Y = (c + a_{1})Y_{-1} - a_{1}Y_{-2} + G + c_{0}. \quad (4.1')
\]

On the basis of an empirical study of pre-World War II data, G. H. Fisher [1952] has suggested that it is reasonable, at least for illustrative purposes, to regard the accelerator coefficient \(a_{1}\) as being of the order of 0.5. Fisher, utilizing a marginal propensity to consume of 0.7, calculated \(\beta = -1.2\) and \(\gamma = .5\), which is point \(S\) on Professor Baumol's graph. Since this point is to the right of the parabola but inside the triangle, the path followed by income will converge to equilibrium with oscillations.\(^{12}\)

Now we are supposing, in contrast to traditional multiplier-accelerator theory, that investment is indeed sensitive to the rate of interest, that \(a_{2} \neq 0\). While both \(\beta\) and \(\gamma\) will now be sensitive to the value of the policy parameter \(\lambda\), the two coefficients always sum to the constant \(\delta - 1 - \delta c\). This convenient property means that the point \((\gamma, \beta)\) must necessarily fall somewhere on a line with slope minus one and intercept \(\delta - 1 - \delta c\). Such a control ray is plotted on the graph for \(\delta = 1\) and Fisher's estimate of \(c = 0.7\). Precisely where the point \((\gamma, \beta)\) falls on the control ray depends upon the magnitude of the policy coefficient \(\lambda\) and the parameters of the investment equation. Point \(S\) on the graph applies for Fisher's estimates if \(X = 0\). If the policy parameter is positive, so that the interest rate moves in conformity with the business cycle, we shall find ourselves at some point on the control ray northwest of \(S\). Conversely, if interest rates fall in periods of prosperity and rise in recession, the point \((\gamma, \beta)\) will be at some point on the control ray southeast of \(S\).

The system will be stable, necessarily converging to equilibrium income \(Y^{*}\), if and only if the point on the control ray is inside the stability triangle. This requires \(\beta - \gamma < 1\) and \(\gamma < 1\). Substituting from equation 4.1 reveals that the policy parameter must fall within the range

\[
\frac{(1 - \delta)c + \delta a_{1} - 1}{\delta a_{2}} < 1 + c + \delta a_{1} - .5\delta(1 + c_{1}) \quad (4.3)
\]

if the system is to be stable. But which value of \(\lambda\) within the stability range demarcated by 4.3 is to be preferred?

One possible criterion concerns the speed of convergence towards equilibrium.\(^{13}\) Convergence is much more rapid at point \(A\) than

\[\text{if the system is to be stable. But which value of } \lambda \text{ within the stability range demarcated by 4.3 is to be preferred?}\]

\[\text{One possible criterion concerns the speed of convergence towards equilibrium.}^{13}\]

\[\text{Convergence is much more rapid at point } A \text{ than}\]

\[\text{In other words, the policy parameter } \lambda \text{ may be selected so as to minimize the absolute value of the dominant characteristic root of 4.1. This assures most rapid convergence except for the coincidental occurrence of initial conditions such that only the minimal root enters into the solution of the difference equation with nonzero coefficient.}\]
This fact may be appreciated by an inspection of Figure 2 where a number of Baumol’s “isostability triangles” are plotted. As Baumol explains, each point on any given isostability triangle yields the same rate of convergence towards equilibrium, and the smaller the triangle the more rapid the convergence. Since A is on a smaller isostability triangle than S, convergence must be more rapid. Indeed, point A yields the most rapid convergence towards equilibrium of any point on the control ray for it is on the smallest isostability triangle touched by the control ray. That is, for the hypothesized values of the parameters of the consumption function and the investment equation, the optimal feasible point is A if the objective is to maximize the speed of convergence towards equilibrium. Furthermore, convergence is necessarily monotonic for the point A lies on the parabola rather than inside the region that leads to the generation of cyclical movement. In general, the value of the policy parameter maximizing the speed of convergence towards equilibrium may be shown to be

\[
\lambda^0 = \left[ c + \delta(a_1 - 1) - 1 + 2 \sqrt{\delta(1 - c)/a_0} \right]/a_2. \tag{4.4}
\]

Since \( \lambda^0 \) may be either positive or negative, it is apparent that even the sign of the appropriate response of interest rates to changes in income is ambiguous. Substituting \( \lambda^0 \) into 3.3 reveals that the optimal response of the money supply to change in income is

\[
M - M_{-1} = (m_1 - m_2 \lambda^0)(Y_{-1} - Y_{-2}). \tag{4.5}
\]

The proposition that a money supply moving counter cyclically would best contribute to economic stability could be validated only by empirical evidence demonstrating that \( m_1 - m_2 \lambda^0 < 0 \).

Can categorical statements concerning the appropriate way in which monetary authorities should respond to fluctuations in income be made without precise knowledge as to the magnitudes of the parameters of the system? It is tempting to argue that since our economy is in fact subject to fluctuation, the policy pursued historically by the monetary authorities has placed us to the right of the parabola in the region yielding oscillatory fluctuations of income. This would imply that the value of \( \lambda \) has been too low, that interest rates should have been more responsive to movements in income, and that it is indeed conceivable that the economy would have been more stable if the money supply had been insensitive to

\[\lambda^0 = \left[ c + \delta(a_1 - 1) - 1 + 2 \sqrt{\delta(1 - c)/a_0} \right]/a_2. \tag{4.4}\]

Since it is apparent from the procedure for constructing iso-stability triangles that the optimal point A is at the intersection of the parabola and the control ray, its coordinates must satisfy both \( \beta^2 = 4 \gamma \) and \( \gamma = \delta(1 - c) - 1 - \beta \). Solving these equations for \( \gamma \) yields the optimal value \( \gamma^0 \). The value of the control parameter which sets \( \gamma \) equal to \( \gamma^0 \) is then obtained from 5.1.

The interested reader can verify that the absolute value of the dominant characteristic root of the difference equation is reduced from 0.71 to 0.45 as a result of the change in \( \lambda \) from zero to 3.
fluctuations in income. Against this argument it can be objected that the fluctuations observed in income may not in actuality be generated by a deterministic oscillatory process. The business cycle may be the consequence of erratic shocks. After all, a simulation study by Irma Adleman [1960] has revealed that although the Klein-Goldberger econometric model generates non-oscillatory behavior when it is regarded as deterministic, it produces fluctuations with a remarkable resemblance to historical movements once the system is subjected to random shocks. Thus, the fact that our economy has been subject to fluctuations does not suffice to establish that a more vigorous monetary policy characterized by a larger value of X would necessarily have contributed to stability.

V. STOCHASTIC COMPLICATIONS

The impact of stochastic disturbances must be considered in selecting the appropriate value of the policy parameter X. It has long been recognized, thanks in part to the contributions of Ragnar Frisch [1933] and Haalvelmo [1940], that the time series generated by a stochastic difference equation model differs markedly from that traced by the deterministic scheme. Suppose that equation 4.1 is modified to read

\[ Y + \beta Y_{-1} + \gamma Y_{-2} = \alpha + \epsilon, \]  

(5.1)

where \( \beta, \gamma \) and \( \alpha \) are defined as before, and \( \epsilon \) is a temporally independently distributed random disturbance with constant variance and zero mean. Income will be subject to continued fluctuations as a result of the cumulative effects of the erratic shocks.

Minimizing the variance of income constitutes an appropriate objective for monetary policy.\(^{16}\) Provided that the coefficients \( \beta \) and \( \gamma \) constitute a point inside Baumol's stability triangle of Figure 1, the variance of income will approach\(^{17}\)

\[
\sigma^2_y = \frac{1}{1 - \beta^2 - \rho^2(1 - \gamma)(1 + \gamma)} \sigma^2_\epsilon. \tag{5.2}
\]

From this equation it is clear that the long-run variance of income is proportional to the variance of the stochastic disturbance \( \epsilon \), where the factor of proportionality depends upon \( \beta \) and \( \gamma \), and is consequently subject to manipulation by adjusting the policy parameter \( \lambda \).

As an aid in determining how the variance of income is affected by the magnitude of the

\(^{16}\) Although such a measure is somewhat arbitrary, such writers as Milton Friedman [1953] and Daniel Orr [1963] have employed the concept in the analysis of macroeconomic models. If the loss function happens to be quadratic, minimizing variance does minimize expected loss; loss functions of this form have been employed by H. Theil [1964] and C. Holt [1962].

\(^{17}\) As Mann and Wald explain [1943], this autoregressive scheme generates a stationary time series if the characteristic roots of 5.3 are less than one in absolute value; otherwise the variance will increase without bound. Equation 5.2 may be derived under the assumption of stationarity if \( \epsilon \) is free of autocorrelation by first taking expected values of the four equations obtained by multiplying 5.1 by \( \epsilon, Y_{-2}, Y_{-1} \) and \( Y \), all variables being measured as deviations from their expected value. Successive substitution of the first three of the equations obtained in this way into the fourth yields 5.2.

\(^{18}\) It is also of some interest to consider the path of expected future levels of \( Y \) conditional upon its current value. It can be shown that \( E[Y_t/Y_{10671}] \) satisfies the same difference equation, 4.1, that generated \( Y_t \) in the deterministic case. Thus the value of the control parameter regarded as optimal for the deterministic case, \( X^0 \), assures that the expected level of future income will converge most rapidly to the equilibrium level; \( X^0 \) would be appropriate for the stochastic case if this were the relevant criterion. The discussion of the deterministic case is also of interest in evaluating the stochastic scheme from a second point of view. Consider the correlogram of income, the sequence of correlations of \( Y_{t+r} \) on \( Y_t \), for positive integers \( r \). It can be shown that this sequence of correlation coefficients also satisfies the homogeneous form of equation 4.1. Consequently, the value of \( X^0 \) derived for the discussion of the deterministic model serves also to minimize the correlation of \( Y_t \) with future levels of income. These properties of stochastic difference equations are not entirely irrelevant to the question of determining the optimal value of the policy parameter; but the variance of income about its expected level will be focused upon for it seems to us to be the most important factor for the monetary authorities to consider, particularly in the long run.
policy parameter $\lambda$, a number of iso-variance curves have been plotted on Figure 3. All points on a given iso-variance curve yield the same long run variance of income, given $\sigma_*^2$. Note that in contrast to the iso-stability curves considered in the deterministic case, the iso-variance curves are not triangular. The control ray RR' is the same one considered initially in the analysis of the deterministic case.\textsuperscript{19} At the constant interest rate point, S, the variance of income is 3.7 times the variance of the error term. If the value of $X$ is increased so that we move northwest along the control ray RR' to point A, where the parabola and the control ray intersect, the variance will be $2.2\sigma_*^2$, smaller than before. This point, which does maximize the speed of convergence for the deterministic case, does not minimize the variance. Increasing $\lambda$ still further, until we reach C, reduces the variance of net national product to only $1.6\sigma_*^2$. As a matter of fact, this point of tangency is the minimum attainable variance point on RR'. The equation for the optimal level of the policy parameter is\textsuperscript{20}

$$
\lambda_*^0 = \frac{1 + \delta a_t + (1 - \delta)c}{\delta a_2} \frac{- \sqrt{\delta(1 - c)}}{\delta a_2}. \quad (5.3)
$$

The optimum value of the policy parameter $\lambda$ is necessarily larger for the stochastic case than for the deterministic situation considered earlier, as may be seen from a comparison of equation 5.3 with 4.4. The monetary authorities should be more responsive to changes in income than the analysis of the deterministic case suggested. It is possible in the deterministic case that the value of $\lambda$ maximizing the speed of convergence to equilibrium involves an interest rate that falls during periods of prosperity and rises in recession; this would occur if the accelerator coefficient were so weak as to place the constant interest rate point S to the northwest of the intersection of the control ray with the parabola. In contrast, the point of tangency between the control ray and the iso-variance curve must be northwest of S. Consequently, $\lambda_*^0$ is always greater than zero; the variance minimizing monetary policy necessarily involves a cyclically conforming pattern of interest rates.

VI. RULES VERSUS DISCRETIONARY MONETARY POLICY

Given the limited state of current knowledge, the concept of a neutral monetary policy has considerable intuitive appeal. Professor Milton Friedman argued before the House Committee on Banking and Currency [1964b]:

\textit{...in the present state of our knowledge—my}

\textsuperscript{19} The control ray is drawn for a marginal propensity to consume of 0.7.

\textsuperscript{20} The optimum value of the policy parameter is found by minimizing the expression for the variance given by 5.2 subject to the restriction of 4.1 that $\gamma = \delta(1 - c) - \beta - 1$. Now 5.3 is not valid if $\epsilon$ is autocorrelated, but we found that the optimal value of $\lambda$ is only moderately sensitive to this complication.
knowledge, your knowledge, the knowledge that economists in general have—we simply do not know enough to be able to know . . . how to use monetary policy as a balance wheel. . . . We ought to convert monetary policy from being a destabilizing force into at least being a neutral factor.

If by neutrality is meant a nominal money supply that is insensitive to current economic developments, this would require that the parameter $\lambda$ of the policy equation $\Delta r = \lambda \Delta Y_{-1}$ discussed in Part III assume a value $\lambda_{\text{neu}} > m_1/m_2$ (here $m_1$ and $m_2$ are parameters of the demand for money equation 2.6). An alternative concept of neutrality, the elimination of fluctuations in the cost of holding money by keeping interest rates constant, requires that $\lambda = 0$. Since historically both interest rates and the money supply have risen in booms and fallen in recession, the exercise of discretionary monetary policy by the Federal Reserve has constituted a compromise between these two alternative definitions of neutral monetary policy and can be roughly characterized by $0 < \lambda_{\text{FED}} < m_1/m_2$.

A review of the analysis presented in this paper reveals that if we do not know enough about the economy's structure to effectively utilize monetary policy as a balance wheel, then we also do not know enough to specify whether either of these two alternative concepts of neutrality is superior to $\lambda_{\text{FED}}$. The range in which the policy parameter must fall in order to achieve convergence is specified by 4.3. It is clear that a constant interest rate policy could lead to instability; this would occur if the accelerator coefficient ($a_1$) were greater than unity and firms attempt an immediate adjustment of the capital stock to the desired level ($\delta = 1$).

But a policy of eliminating fluctuations in the nominal money supply ($\lambda_{\text{neu}}$) would lead to instability. For example, the second stability condition of 4.3 will certainly be violated if $m_1/m_2 > (1 + a_1)/a_2$ and $\delta = 1$; and this will happen, other things being equal, if the demand for money is rather insensitive to the interest rate, for then a small fluctuation in income may generate excessive movements in the rate of interest and consequent destabilizing fluctuations in investment spending.

Two alternative criteria for specifying the optimal value of the policy parameter have been considered. Equation 4.4 specified the value of $\lambda^0$ leading to most rapid convergence to equilibrium, the criterion considered by such writers as Baumol [1961] in analyzing the effectiveness of automatic stabilizers. Equation 5.3 specified the value $\lambda^0$ that serves to minimize the long run variance of income for the stochastic version of our model, the criterion considered by Friedman [1953] and Orr [1963]. Inspection of these equations revealed that $\lambda^0 < \lambda^s$ and $\lambda^s > 0$. The first inequality reveals that minimizing the variance of income requires an interest rate that is more responsive to cyclical movements than an analysis in terms of the speed of convergence would suggest. The second inequality implies that a constant interest rate policy ($\lambda = 0$) can not serve to minimize the variance of income. None the less, inspection of equations 5.2 and 4.1 reveals that the variance might well be smaller for $\lambda = 0$ than for $\lambda = \lambda_{\text{neu}}$. All this means that while the optimal value of the policy parameter, in terms of either criteria, cannot be calculated without precise knowledge of the system's parameters, it may well be that $\lambda_{\text{FED}}$ is superior to either $\lambda = 0$ or $\lambda_{\text{neu}}$. This is not to say that the monetary authorities have been perfect. But it does suggest that pending the accumulation of sufficient knowledge, our economy may well be more stable if in future years we continue to rely upon the best judgment of the
monetary authorities rather than take the radical step of abandoning discretionary monetary policy in favor of an extreme concept of neutral money.

APPENDIX

A host of simplifying assumptions were invoked in the text in order that the task of analyzing our synthesis of real and monetary theories of the business cycle might be kept within reasonable bounds. In a quest for greater realism we have worked out the implications of a number of generalizations of our simple model. The real cash balance effect and growth consequences will be briefly explored in this appendix.

A. Real Cash Balance Effect

If real cash balances have a direct effect upon consumption spending, the consumption function, equation 2.4, becomes:

\[ C = c_0 + c_1 Y_{-1} + c_2 M. \]  (A.1)

Consumption functions of this form have been employed in a number of econometric studies. It should be noted that incorporating the money stock as a direct determinant of consumption is subject to the objection that a more inclusive wealth variable, not directly amenable to manipulation by the monetary authorities, is required. For a critical discussion of empirical studies of the real balance effect see Patinkin [1965, Appendix M]. An equation of this same form may also be obtained by assuming that it is the rate of interest and income that determine consumption, once equation 2.6 is utilized to eliminate \( r \).

The effects of this complication will be spelt out only for the case of immediate adjustments, \( \delta = 1 \). Equation 4.1 becomes:

\[ Y + \beta Y_{-1} + \gamma Y_{-2} = \alpha \]  (A.1)

where

\[ \beta = - a_1 - c_1 - c_2 m_1 + \lambda (a_2 + c_2 m_2) \]

and

\[ \gamma = a_1 - \lambda a_2. \]

Furthermore, equilibrium income is now

\[ Y^* = \frac{c_0 + G + c_2 m_0 + c_2 m_4 (\lambda \bar{Y} - \bar{r})}{1 - c_1 - (m_1 - m_2 \lambda) c_2}. \]  (A.2)

If for simplicity we assume that \( a_2 = 0 \), so that monetary policy influences the stability of the system only via the real balance effect, only \( \beta \) will be sensitive to \( \lambda \) and the control ray will be a vertical line. For the deterministic analysis the fact that the right-hand sides of Baumol's iso-stability triangles are vertical implies that the speed of convergence will be completely insensitive to the magnitude of \( \lambda \), provided that it is not so large as to place us outside the region of oscillations demarcated by the parabola. For the stochastic case a review of Figure 3 reveals that with a vertical control ray the best iso-variance curve is obtained when \( \lambda = 0 \), and we have from A.1 that the value of the control parameter that minimizes the long run variance of income is

\[ \lambda^* = \frac{a_1 + c_1 + c_2 m_1}{a_2 + c_2 m_2}. \]  (A.3)

The execution of the variance minimizing policy requires that the money supply should move countercyclically, in contrast to the experience of the American economy under the Federal Reserve System—this may be seen by substituting equation A.3 into 3.3. This is a strong conclusion, but it requires the assumption that interest rates have no

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1 Space limitations prevent us from presenting our analysis of the effects of changes in the lag structure that would result if forecasting procedures utilized by the monetary authority permitted changes in the money supply to be related to current rather than lagged income. A mimeographed draft of this material and an analysis of the role of money in the inventory cycle is available from the authors on request.

2 Michio Morishima and Mitsuo Saito [1964] estimate \( c_2 = .228 \); they summarize the results of a number of empirical studies in which the magnitude of \( c_2 \) ranges from 0.024 (Klein and Goldberger) to 0.384 (Goldsmith).
direct effect upon investment spending. The interested reader who works through the more complicated case in which this assumption is relaxed will find that it is no longer possible to demonstrate, without precise knowledge of the parameters of the model, that the money supply should not expand in prosperity and contract in recession.

B. Monetary Policy in a Growth Context

In order to demonstrate how the analysis of the effects of monetary policy upon the business cycle generalizes to encompass economic growth, we shall augment Hick's [1950] Trade Cycle model to explicitly encompass the effects of interest rate fluctuations upon investment spending. Following Professor Hicks, let us assume that the sum of autonomous investment plus government spending, denoted \( G_t \), grows at some constant rate through time; i.e.,

\[
G_t = G_0(1 + g)^t. \quad (B.1)
\]

We shall find a unique equilibrium characterized by a constant rate of interest \( r^* \), and a constantly expanding level of net national product:

\[
Y^*_t = (1 + g')^t Y^*_t. \quad (B.2)
\]

We shall show that the equilibrium growth path is unique, that it grows at the same rate as autonomous spending (i.e., \( g' = g \)), and that the stability conditions are identical to those of the no growth situation (\( g = 0 \)) already analyzed in the text.

It is necessary, in a growth context, to make minor modifications in two of the equations employed in the earlier analysis. First, it is appropriate to modify equation 3.1 in order that the response of monetary authorities to changes in income is proportional to the departure of income from its equilibrium level:

\[
r - \bar{r} = \lambda \frac{Y_{t-1} - Y^*_{t-1}}{Y^*_t}. \quad (B.3)
\]

Second, since the investment equation is best regarded as representing an aggregate relationship obtained by summing over micro units, we would expect the parameter \( a_2 \) to increase in magnitude as the number of economic units increases; otherwise, the effect of a 1% change in the rate of interest upon investment spending would become insignificant with the passage of time. The analysis is simplified if this complication is taken into account by assuming:

\[
K^d = a_1 Y_{t-1} - a_2 r,
\]

where

\[
a_2 = o a_2(1 + g')^t = o a_2(Y_t^*/Y^*_o). \quad (B.4)
\]

Appropriate substitutions into equation 2.8 now yield the fundamental second order difference equation of the growth model:

\[
Y_t + \beta Y_{t-1} + \gamma Y_{t-2} = G_0(1 + g)^t + \alpha(1 + g')^t, \quad (B.5)
\]

where:

\[
\alpha = (\lambda - \bar{r}) o a_2 g'/(1 + g'), \quad \beta = \delta_2 \lambda - \alpha_2 - c, \quad \gamma = \alpha_1 - \delta_2 \lambda, \quad \delta_2 = -\delta_2 a_2/Y^*_{t-2} = (1 + g')a_2/Y^*_t.
\]

Since equilibrium net national product must satisfy this last equation, as well as B.2, we find eventually that \( g = g' \) and

\[
M_s = k_t Y_{t-1}/(\bar{r} - \lambda + \bar{Y}_{t-1}/Y^*_t + k_3).
\]

As a special case, if \( \bar{r} - \lambda = k_3 , M_s = (k_1/\lambda) Y_{t-1} = (k_1/\lambda) Y^*_t (1 + g')^{t-1} \), where the last equality follows from equation B.2, the money supply would grow at the equilibrium rate in accordance with the Friedman-Shaw rule regardless of the behavior of actual net national product.

4 This is a clumsy procedure. The seemingly neater approach of employing a desired capital stock equation of the form \( K^d = a(r) Y_{t-1} \), would complicate the analysis of stability considerably, unless our attention were restricted to infinitesimal departures from equilibrium. We also assume \( \alpha_2 = 0 \).

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3 Precisely how the money supply must be adjusted in order for the rate of interest to move in accordance with this policy rule depends upon the form of the demand for money equation. For example, if \( M_s = k(r) Y_{t-1} \), with \( k(r) = k_1/(r + k_3) \), the money supply must behave as

\[
r - \bar{r} = \lambda \frac{Y_{t-1} - Y^*_{t-1}}{Y^*_t}. \quad (B.3)
\]
The first expression in brackets is the "supermultiplier concept" of Professor Hicks.\(^5\) As with Hicks' **Trade Cycle** model of the real cycle, the rate of growth is determined by the rate of growth of autonomous expenditure and is completely insensitive to the other parameters of the model. It is interesting to note, however, that when \(g \neq 0\) the magnitude of equilibrium net national product at each point of time is influenced by \(\bar{r}\), the rate of interest set by the monetary authorities. When the economy is growing along the equilibrium path, the rate of interest is \(\bar{r}\). From equation 2.6a, it is clear that in equilibrium the income velocity of money, \(Y/M\), will be constant.\(^6\)

For purposes of stability analysis, it is necessary only to note from B.5 that the homogeneous difference equation explaining deviations of net national product from equilibrium depends upon the behavioral parameters in precisely the same way as when we were abstracting from growth complications in the earlier analysis. This means that the earlier analysis of stability generalizes immediately. The value of the policy parameter \(\lambda\) insuring most rapid convergence in the absence of stochastic disturbances may be obtained by substituting \(a_1\), \(c\) and \(a_2\) into 4.4. For the stochastic case, equation 5.3 yields the magnitude of the policy parameter that minimizes the variance of net national product about its equilibrium path.

\(^5\) If the rate of growth of autonomous expenditure is zero, this equation reduces to 3.1.

\(^6\) This means that if we observed a Keynesian economy in which the monetary authorities adjusted interest rates in accordance with equation B.3, the velocity of money would approach a constant if the economy were stable. If the system were subject to oscillations, the velocity of income would average to roughly the same figure from cycle to cycle.

**REFERENCES**


