Trade and Labor Market Dynamics*

Lorenzo Caliendo† Maximiliano Dvorkin‡ Fernando Parro§

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Abstract

We develop a dynamic trade model where production and consumption take place in spatially distinct labor markets with varying exposure to domestic and international trade. The model recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, and input-output linkages in determining equilibrium allocations. We show how to solve the equilibrium of the model without estimating productivities, migration frictions, or trade costs, which are usually difficult to identify. We calibrate the model to 38 countries, 50 U.S. states, and 22 sectors and use the rise in China’s import competition to quantify the effects across more than a thousand U.S. labor markets. We find that China’s trade shock resulted in a loss of 0.8 million U.S. manufacturing jobs, about 50 percent of the change in the manufacturing employment share unexplained by a secular trend. We find aggregate welfare gains but, due to trade and migration frictions, the welfare and employment effects vary across U.S. labor markets. Estimated transition costs to the new long-run equilibrium are also heterogeneous and reflect the importance of accounting for labor dynamics.

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† Yale University and NBER. ‡Federal Reserve Bank of St. Louis. §Federal Reserve Board.
1. INTRODUCTION

Aggregate trade shocks can have varying effects across labor markets. One source of variation is the exposure to foreign trade, measured by the degree of import competition across labor markets. Another source of variation is the extent to which trade shocks impact the exchange of goods and the reallocation of labor across and within sectors and locations. Moreover, since labor movement across markets takes time, and mobility frictions depend on local characteristics, labor market outcomes adjust differently across industries, space, and over time to the same aggregate shock. Therefore, the study of the effects of shocks on the economy requires the understanding of the impact of trade on labor market dynamics.

In this paper, we develop a dynamic and spatial trade model to understand and quantify the disaggregate labor market effects resulting from changes in the economic environment. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in shaping the effects of shocks across different labor markets. Hence, our model has intersectoral, interregional, and international trade.

In our economy, production takes place in spatially distinct markets. A market is a sector located in a particular region in a given country.\(^1\) In each market there is a continuum of heterogeneous firms producing intermediate goods \(à la\) Eaton and Kortum (2002, hereafter EK). Firms are competitive, have constant returns to scale technology, and demand labor, local factors, and materials from all other markets in the economy.\(^2\) The supply side of the economy features forward-looking households in the world that, at the beginning of the period, are distributed across labor markets in a given way. Households can be either employed or unemployed. Employed households supply a unit of labor and receive the local competitive market wage; unemployed households obtain consumption in terms of home production.

We model the households’ decision of where to supply labor across markets as a dynamic discrete choice problem building on Artuç, Chaudhuri and McLaren (2010, hereafter ACM). In particular, households decide whether to be employed in the next period and in which labor market to supply labor, conditional on their location, the state of the economy, and an \(i.i.d.\) taste shock. Moving across labor markets is costly, and we allow for an arbitrary distribution of mobility costs. Incorporating these elements delivers a general equilibrium, dynamic discrete choice model with realistic

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\(^1\)Our setup can accommodate an arbitrary number of sectors located over an arbitrary number of regions across countries.

\(^2\)The production structure of the model builds on multicountry international trade models \(à la\) EK. We introduce dynamics, international trade, and labor mobility frictions to the rich spatial model of Caliendo, et al. (2014).
Taking a dynamic trade model with all these features to the data, and performing counterfactual analysis, may seem unfeasible since it requires estimating a large set of fundamentals that are usually difficult to identify, like heterogeneous productivity levels across sectors and regions, bilateral mobility (migration) costs across markets, bilateral international and domestic trade costs, and endowments of immobile local factors. Our methodological contribution is to show that by expressing the equilibrium conditions of the model in relative time differences, we are able to solve the model and perform large-scale counterfactual analysis without requiring us to estimate the fundamentals of the economy. Aside from data for the initial period that directly map into the model’s equilibrium conditions, the only parameters needed to solve the full transition of the dynamic model are the intertemporal discount factor and the trade and migration elasticities.

Our method holds irrespective of the number of markets and relies on conditioning on the observed initial-period allocation. The intuition is that the observed allocation, namely data on production, employment, trade, and migration flows across markets, provides information on the implied levels of the fundamentals of the economy. Therefore, by taking time differences, all fundamentals that are time invariant are differentiated out. Our result parallels those of Dekle, Eaton, and Kortum (2008, hereafter DEK), who have shown this result in the context of a static trade model. We show this result in the context of a dynamic discrete choice model.

Our study is complementary to a large body of empirical research aimed at identifying the disaggregate effects of changes in the economic environment. We contribute by introducing a framework that can be used to perform large-scale quantitative analysis and yet not lose track of the main economic insights that deliver the results. Equally important, our model can speak about effects that are usually difficult to quantify or identify in reduced-form empirical research. For instance, we can study how the levels of aggregate employment for different countries and for specific labor markets respond to a change in economic fundamentals. Furthermore, we contribute...
to this strand of the literature by explaining how additional channels account for the change in
welfare and many other economic outcomes at the aggregate and disaggregate levels and over time.

Our approach relates to a fast-growing strand of the literature that studies the impact of trade
shocks on labor market dynamics.\(^7\) The articles most closely related to ours are those by Artuç
and McLaren (2010); ACM; and Dix-Carneiro (2014). We follow Artuç and McLaren (2010) and
ACM in modeling the migration decisions of agents as a dynamic discrete choice. We depart
from their assumption of a partial equilibrium, small open economy and introduce a multicountry,
multi-region, multi-sector general equilibrium trade model with trade and migration costs. Our study
is also complementary to Dix-Carneiro (2014), whose focus is to measure the frictions that workers
face to move across sectors, and interpret their magnitude through the simulation of hypothetical
trade liberalization episodes. Following Dix-Carneiro (2014), we use our general equilibrium model
to quantify the dynamic effects of a trade shock across markets, but unlike him, we rely on our
solution method to compute these effects at a more granular level.

We apply our model and solution method to study the effects of the rise in China’s import
competition on U.S. labor markets. U.S. imports from China more than doubled from 2000 to
2007. During the same period, manufacturing employment fell considerably while employment in
other sectors, such as construction and services, grew. Several reduced-form studies (e.g. Autor,
Dorn, and Hanson, 2013, hereafter ADH; Acemoglu et al., 2014; Pierce and Schott, 2012) document
that an important part of the employment loss in manufacturing was a consequence of China’s
trade expansion, either as a consequence of technological improvements in the Chinese economy
or reductions in trade costs.\(^8\) In most of these studies, the main reason U.S. labor markets are
differentially exposed to Chinese goods is their different degree of import competition.

We use our model to quantify how additional channels can also explain the employment loss in
the manufacturing sector, and how other sectors of the economy, such as construction and services,
were also exposed to the Chinese shock. More importantly, we use our model to compute welfare
effects across labor markets over time. In summary, we account for the distribution of winners and
losers across sectors and regions of the U.S. economy caused by the increase in Chinese competition.

\(^7\) For instance, see Artuç and McLaren (2010); Artuç Chaudhuri and McLaren (2010); Dix-Carneiro (2014); Dix-
Carneiro and Novak (2015); Cosar (2013); Cosar, Guner, and Tybout (2014); Kondo (2013); Menezes-Filho and
Muendler (2011); and the references therein.

\(^8\) ADH argue that structural reforms in the Chinese economy resulted in large technological improvements in
export-led sectors. As a result, China’s import penetration to the Unites States increased. Handley and Limao
(2014) and Pierce and Schott (2012) argue that the U.S.’ elimination of uncertainty about tariff increases on Chinese
goods was another important reason why U.S. imports from China grew.
We do this by calibrating a 38-country, 50-U.S.-state, and 22-sector version of our model. We take the initial distribution of labor across markets in the U.S. economy and match the initial conditions of our model to those in the year 2000. We rely on the identification restriction suggested by ADH to measure China’s shock; namely, we use the predicted changes in U.S. imports from China using as instrument the change in imports from China by other high-income countries for the period 2000 to 2007. Using our model, we compute the change in sectoral productivities in China between 2000 and 2007 that exactly matches the predicted changes in imports in the model. We term these changes in productivity the “China shock” and refer to them as such in the rest of the paper.

We find that increased Chinese competition reduces the aggregate manufacturing employment share by 0.5 percentage points in the long run, which is equivalent to a loss of 0.8 million manufacturing jobs, or about 50% of the change in the aggregate manufacturing employment share unexplained by a secular trend. We also find that workers reallocate to the services sector as this sector benefits from the access to cheaper intermediate inputs from China.

With our model we can also quantify the relative contribution of different sectors, regions, and labor markets to the decline in manufacturing employment. We find that sectors with a higher exposure to import competition from China lose more manufacturing jobs. The computer and electronics and furniture industries contributed to about half of the decline in manufacturing employment, followed by metal and textiles industries that together contributed to about one-fourth of the total decline. Some sectors, such as food, beverage, and tobacco, gained employment, as they were less exposed to China and benefited from cheaper intermediate goods. The fact that U.S. economic activity is not equally distributed across space plus the differential sectoral exposure to China imply that the impact of China’s import competition varies across regions. We find that U.S. states with a larger concentration of sectors more exposed to China lose more manufacturing jobs. California, which by far accounts for the largest share of employment in computer and electronics (the sector most exposed to China’s import competition), contributed to about 12% of the decline.

Our framework also allows us to quantify the welfare effects of the increased competition from China on the U.S. economy. Our results indicate that the China shock increases U.S. welfare by 0.6%. Therefore, even when U.S. exposure to China decreases employment in the manufacturing sector, the U.S. economy is better off, as it benefits from the access to cheaper goods from China.

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9 It is worth noting that for an application of this dimension not using our solution method will require estimating: $N \times R \times J$ productivity levels, $N^2 \times R^2 \times J$ asymmetric bilateral trade costs, $N^2 \times R^2 \times J^2$ labor mobility costs, and $N \times R \times J$ stocks of local factors. Where $N$, $R$, and $J$ are countries, regions and sectors, respectively.

10 We compute the secular trend for the U.S. manufacturing employment share of total private employment as a linear trend from the year 1967 to 1999, the year before the China shock. The trend predicts a share of 12.83% for the year 2007, while the observed share was 11.85%. More details are provided in Section 4.
We also find a large dispersion in welfare effects across individual labor markets. Larger welfare gains are generally in labor markets that produce nonmanufacturing goods as these industries do not suffer the direct adverse effects of the increased competition from China and at the same time benefit from access to cheaper intermediate manufacturing inputs from China used in production. Similarly, labor markets in states that trade more with the rest of the U.S. economy and purchase materials from sectors where Chinese productivity increases, tend to have larger welfare gains as they benefit from the access to cheaper inputs from China purchased from the rest of the U.S. economy. We also compute the welfare effects in the rest of the world and find that all countries gain from the China shock, with some countries having larger welfare gains and others having smaller welfare gains than the U.S. economy. Since reaching the new steady state after the China shock takes time due to mobility frictions, we compute the transition or adjustment costs to the new steady state and find substantial variation across labor markets.

The paper is organized as follows. In Section 2 we present our dynamic model of costly labor reallocation and trade. Operating over the set of equilibrium conditions, we are able to get a more tractable specification. In Section 3 we show how to solve the model and perform counterfactual analysis. In Section 4 we explain how to take the model to the data, and our identification strategy of the China shock. In Section 5 we use our model to quantify the effects of increased Chinese competition on different U.S. labor markets. Finally, we conclude in Section 6.

**2. A SPATIAL DYNAMIC TRADE MODEL**

We consider a world with \( N \) locations, and \( J \) sectors. We use the indexes \( n \) or \( i \) as unique identifiers of a particular location, namely a region irrespective of the country, and index sectors by \( j \) or \( k \). In each region-sector combination there is a competitive labor market. In each market there is a continuum of perfectly competitive firms producing intermediate goods using a technology with time-invariant heterogeneous idiosyncratic productivity.

We follow EK and assume that productivities are distributed Fréchet with a sector-specific productivity dispersion parameter \( \theta^j \). Firms have a Cobb-Douglas constant returns to scale technology, demand labor, a composite local factor that we refer to as structures, and materials from all sectors.

Time is discrete, and we denote it by \( t = 0, 1, 2, \ldots \). At the beginning of the period there is an initial distribution of labor across markets. Workers face costs to move across markets and experience an idiosyncratic shock that affects their moving decision. The household’s problem is closely related to the sectoral reallocation problem in ACM and to the competitive labor search model of Lucas and
Prescott (1974) and Dvorkin (2014). Agents are forward looking and optimally decide where to move. We first characterize the dynamic problem of a household deciding where to move conditional on a path of real wages across time and across labor markets. We then characterize the static subproblem to solve for prices and wages conditional on the supply of labor in a given market.

2.1 Problem of the Households (Dynamic Problem)

At $t=0$ there is a mass $L_{nj}^0$ of households in each location $n$ and sector $j$. Households can be either employed or unemployed. An employed household in location $n$ and sector $j$ supplies a unit of labor inelastically and receives a competitive market wage $w_{nj}^t$. Given her income she decides how to allocate consumption over local final goods from all sectors with a Cobb-Douglas aggregator. We assume that preferences are over the basket of final local goods, in particular

$$U(C_{nj}^t), \text{ where } C_{nj}^t = \prod_{k=1}^{J} (c_{nj}^{k})^{\alpha^k},$$

where $c_{nj}^{k}$ is the consumption of sector $k$ goods in market $nj$ at time $t$, and $\alpha^k$ is the final consumption share, with $\sum_{k=1}^{J} \alpha^k = 1$. We denote the ideal price index by $P_{nj}^t = \prod_{k=1}^{J} (P_{nk}^t / \alpha^k)^{\alpha^k}$. As in Dvorkin (2014), unemployed households obtain consumption in terms of home production, or an unemployed “benefit,” $b^n > 0$. To simplify the notation, we represent sector zero in each region as unemployment; hence, $C_{nj}^{n0} = b^n$.

Assumption 1 Agents have logarithmic preferences, $U(C_{nj}^t) \equiv \log(C_{nj}^t)$.

Assumption 1 specifies the preference structure of the agents in the economy. The household’s problem is dynamic, and agents are forward looking and discount the future at rate $\beta \geq 0$.

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11 Another related model of labor reallocation is Coen-Pirani (2010). Preference shocks are widely used in the literature on worker reallocation. See, for example, Kennan and Walker (2011), ACM, Dix-Carneiro (2014), Redding (2012), and Monte (2015).

12 Alternatively, one could assume that unemployed households use non-market income to buy market goods. In this case, consumption of unemployed households in region $n$ is given by $b^n / P^n_t$. Under this alternative assumption, a decline in the price index in region $n$ increases the incentive to become unemployed compared with our model. We find this property undesirable since it could lead us to under or overestimate the unemployment effects of changes in the economic environment if non-market income also responds to these changes. Since we do not have a theory on home production in our model, we follow the literature, (see, for instance, Alvarez and Shimer, 2011), and assume it constant. A more ambitious modelling of the unemployment sector requires to incorporate a home production theory in the model, and we leave it for future research.

13 To simplify the notation, we abstract from local amenities, which can vary both by sectors and regions. As it will become clear later, our exercise and results are unaltered to the existence of these amenities under the assumption that they enter the period utility additively and are constant over time. More general types of amenities, including congestion or agglomeration effects, can be handled by the solution method we propose, but we do not model them here.
Assumption 2 Labor reallocation costs $\tau_{nj,ik} \geq 0$ depend on the origin ($nj$) and destination ($ik$), and are (i) time invariant, (ii) additive, and (iii) in terms of utility.

Assumption 2 describes the mobility costs in the model. In addition, agents have additive idiosyncratic preference (or cost) shocks for each choice and we denote them by $\epsilon_{t}^{ik}$.

The timing for the workers’ problem and decisions is as follows. Workers observe the economic conditions in all labor markets and the realizations of their own idiosyncratic shocks. If they begin the period in a labor market, they work and earn the market wage. If they are unemployed in a region, they get home production. Then, both employed and unemployed workers have the option to reallocate. Formally,

$$v_{t}^{nj} = U(C_{t}^{nj}) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{ \beta E \left[ v_{t+1}^{ik} \right] - \tau_{nj,ik} + \nu \epsilon_{t}^{ik} \right\},$$

s.t. $U(C_{t}^{nj}) = \left\{ \begin{array}{ll} \log(b^{n}) & \text{if } j = 0, \\ \log(w_{t}^{nj}/P_{t}^{n}) & \text{otherwise;} \end{array} \right.$

where $v_{t}^{nj}$ is the lifetime utility of a worker in region $n$ and sector $j$ at time $t$ and the expectation is taken over future realizations of the preference shock. The parameter $\nu$ scales the variance of the idiosyncratic taste shocks. Note that workers choose to reallocate to the labor market that delivers the highest utility net of costs.

Assumption 3 The idiosyncratic preference shock (or reallocation cost shock) $\epsilon$ has the following properties: It (i) is i.i.d. over time, (ii) follows a Type-I Extreme Value distribution, and (iii) has zero mean.

Assumption 3 is a standard assumption made in dynamic discrete choice models.\footnote{For a survey on this literature, see Aguirregabiria and Mira (2010).} It allows for a simple aggregation of the idiosyncratic decisions made by households as we now show.

Denote by $V_{t}^{nj} \equiv E[v_{t}^{nj}]$ the expected lifetime utility of a representative agent in labor market $nj$, where the expectation is taken over the preference shocks. Then, given Assumption 3, we obtain (Appendix 1 presents the derivation)

$$V_{t}^{nj} = U(C_{t}^{nj}) + \nu \log \left[ \sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left( \beta V_{t+1}^{ik} - \tau_{nj,ik} \right)^{1/\nu} \right]. \tag{2}$$

Equation (2) reflects the fact that the value of being in a particular labor market depends on
the current-period utility and the option value to move into any other market in the next period.\textsuperscript{15} \(V_t^{n_j}\) can be interpreted as the expected lifetime utility of a worker before the realization of her preference shocks or, alternatively, as the average utility of workers in that market under a pure utilitarian welfare.\textsuperscript{16}

Using Assumption 3 we can also show that the share of labor that transitions across markets has a closed-form analytical expression. In particular, denote by \(\mu_t^{n_j,ik}\) the fraction of workers that reallocate from market \(n_j\) to \(ik\) (with \(\mu_t^{n_j,n_j}\) the fraction who choose to remain in their original location); then (Appendix 1 presents the derivation)

\[
\mu_t^{n_j,ik} = \frac{\exp \left( \beta V_{t+1}^{ik} - \tau^{n_j,ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+1}^{mh} - \tau^{n_j,mh} \right)^{1/\nu}}.
\]

Equation (3), which we refer to as migration shares, has a very intuitive interpretation. Other things equal, it reflects that markets with a higher lifetime utility (net of mobility costs) are the ones that attract more migrants. From this expression we can also see that \(1/\nu\) has the interpretation of a migration elasticity.

Equation (3) is a key equilibrium condition in this model because it conveys all the information needed to determine how the distribution of labor evolves over time. In particular, the dynamics of the distribution of labor over markets are described by

\[
L_{t+1}^{n_j} = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_t^{ik,n_j} L_t^{ik}.
\]

The equilibrium condition (4) characterizes the evolution of the state variable of the economy, that is, the distribution of employment across markets \(L_t = \{L_t^{n_j}\}_{n=1,j=0}^{N,J}\). Note that given our timing assumption, the supply of labor at each \(t\) is fully determined by forward-looking decisions at period \(t-1\). Now, conditional on labor supplied at each market, we can specify a static production structure of the economy that allows us to solve for the equilibrium wages at each time \(t\) such that labor markets clear. The model we develop to solve this static subproblem builds on recent advances in the international trade literature. We now proceed to describe the production side of the economy.

\textsuperscript{15}For an example of a model that delivers a similar expression, refer to Aru\textsuperscript{t} and McLaren (2010), ACM, and Dix-Carneiro (2014). ACM also provide an economic interpretation of the different components of the option value to move across sectors.

\textsuperscript{16}In our case, the measure of this representative agent evolves endogenously with the change in economic conditions. See Dvorkin (2014) for further details.
2.2 Production (Static Subproblem)

Technology follows closely EK, particularly the multisector version in Caliendo and Parro (2015) and the spatial model of Caliendo et al. (2014). Goods are of two types: intermediate, denoted by \( q \), and final, denoted by \( Q \). Firms in each sector and region are able to produce many varieties of intermediate goods and a final good. The technology to produce these intermediate goods requires labor and structures, which are the primary factors of production, and materials, which consist of final goods from all sectors. Total factor productivity (TFP) of an intermediate good is composed of two terms, a sectoral-regional component \( (A^{nj}) \), which is common to all varieties in a region and sector, and a variety-specific component \( (z^{nj}) \). We assume that \( A^{nj} \) is exogenous and deterministic. Since one intermediate variety is identified by \( z^{nj} \), we use it to index a variety.

Intermediate Goods Producers

The technology for intermediate goods is described by

\[
q_{jt}^{nj}(z^{nj}) = z^{nj} A^{nj} \left[ h_{jt}^{nj}(z^{nj}) \xi^n \left[ l_{jt}^{nj}(z^{nj}) \right]^{1-\xi^n} \right] \gamma^{nj} \prod_{k=1}^{J} M_{jt}^{nj, nk}(z^{nj}) \gamma^{nj, nk},
\]

where \( l_{jt}^{nj}(z^{nj}) \) and \( h_{jt}^{nj}(z^{nj}) \) are the demands for labor and structures by firms in sector \( j \) and region \( n \), and \( M_{jt}^{nj, nk}(z^{nj}) \) is the demand for material inputs from sector \( k \) by firms in sector \( j \) and region \( n \). Material inputs are final goods from sector \( k \) produced in the same region \( n \). The parameter \( \gamma^{nj} \geq 0 \) is the share of value added in the production of sector \( j \) and region \( n \), and \( \gamma^{nj, nk} \geq 0 \) is the share of materials from sector \( k \) in the production of sector \( j \) and region \( n \). We assume that the production function exhibits constant returns to scale such that \( \sum_{k=1}^{J} \gamma^{nj, nk} = 1 - \gamma^{nj} \). The parameter \( \xi^n \) is the share of structures in value added. Structures are in fixed supply in each labor market.

We denote by \( P_{jt}^{nj} \) the price of materials, and by \( r_{jt}^{nj} \) the rental price of structures in region \( n \) and sector \( j \). We define the unit price of an input bundle as

\[
x_{jt}^{nj} = B^{nj} \left[ \left( r_{jt}^{nj} \right)^{\xi^n} \left( w_{jt}^{nj} \right)^{1-\xi^n} \right]^{\gamma^{nj}} \prod_{k=1}^{J} P_{jt}^{nk}^{\gamma^{nj, nk}},
\]

where \( B^{nj} \) is a constant. Then, the unit cost of an intermediate good \( z^{nj} \) at time \( t \) is

\[
\frac{x_{jt}^{nj}}{z^{nj} A^{nj} \gamma^{nj}}.
\]
Trade costs are represented by $\kappa^{nj,ij}$ and are of the “iceberg” type. One unit of any variety of intermediate good $j$ shipped from region $i$ to $n$ requires producing $\kappa^{nj,ij} \geq 1$ units in region $i$. If a good is nontradable, then $\kappa = \infty$. Competition implies that the price paid for a particular variety of good $j$ in region $n$ is given by the minimum unit costs across regions, taking into account trade costs. That is,

$$p_t^{nj}(z^j) = \min_i \left\{ \frac{\kappa^{nj,ij} x_t^{ij}}{z_t^{ij} A^{ij}_s^\gamma_{ij}} \right\}.$$  

**Final Goods Producers**

Final goods in region $n$ and sector $j$ are produced by combining intermediate goods from sector $j$ across all other regions. Let $Q_t^{nj}$ be the quantity of final goods in region $n$ and sector $j$ and $\tilde{q}_t^{nj}(z^j)$ the quantity demanded of an intermediate good of a given variety such that, for that variety, the vector of productivity draws received by the different regions is $z^j = (z_1^j, z_2^j, \ldots, z_N^j)$. The production of final goods is given by

$$Q_t^{nj} = \left[ \int [\tilde{q}_t^{nj}(z^j)]^{1-1/\eta^{nj}} \phi^j(z^j) dz^j \right]^{\eta^{nj}/(\eta^{nj}-1)},$$

where $\phi^j(z^j) = \exp \left\{ -\sum_{n=1}^N (z^{nj})^{-\theta^j} \right\}$ is the joint density function over the vector $z^j$, with marginal densities given by $\phi^{nj}(z^{nj}) = \exp \left\{ -(z^{nj})^{-\theta^j} \right\}$ and the integral is over $R_+^N$. For nontradable sectors the only relevant density is $\phi^{nj}(z^{nj})$ since final good producers use only locally produced goods. There are no fixed costs or barriers to entry and exit in the production of intermediate and final goods. Competitive behavior implies zero profits at all times.

Given the properties of the Fréchet distribution, the price of the final good $j$ in region $n$ at time $t$ is

$$P_t^{nj} = \Gamma \left[ \sum_{i=1}^N \left[ x_t^{ij} \kappa^{nj,ij} A^{ij}_s^\gamma_{ij} \right]^{-\theta^j} \right]^{-1/\theta^j}, \quad (6)$$

where $\Gamma$ is a constant.\footnote{In particular, the constant $\Gamma$ is the Gamma function evaluated at $1 + (1 - \eta^{nj}/\theta^j)$.} To obtain (6), we assumed that $1 + \theta^j > \eta^{nj}$. Following similar steps as earlier, we can solve for the share of total expenditure in market $(n,j)$ on goods $j$ from market $i$.\footnote{For detailed derivations, please refer to Caliendo et al. (2014).}

In particular,

$$x_t^{nj,ij} = \frac{[x_t^{ij} \kappa^{nj,ij} A^{ij}_s^\gamma_{ij} - \theta^j A^{ij}_s^\gamma_{ij}]}{\sum_{m=1}^N [x_t^{mj} \kappa^{nj,mj} A^{mj}_s^\gamma_{mj} - \theta^j A^{mj}_s^\gamma_{mj}]}.$$

$$\eta^{nj,ij} = \frac{[x_t^{ij} \kappa^{nj,ij} A^{ij}_s^\gamma_{ij} - \theta^j A^{ij}_s^\gamma_{ij}]}{\sum_{m=1}^N [x_t^{mj} \kappa^{nj,mj} A^{mj}_s^\gamma_{mj} - \theta^j A^{mj}_s^\gamma_{mj}]} \quad (7)$$

This equilibrium condition reflects that the more productive market $ij$ is, the cheaper is the cost of production in market $ij$, and therefore, the more region $n$ purchases sector $j$ goods from region...
In addition, the easier it is to ship sector \( j \) goods from region \( i \) to \( n \) (lower \( \kappa_{ij} \)), the more region \( n \) purchases sector \( j \) goods from region \( i \). This equilibrium condition is sometimes referred to as the gravity equation.

**Market Clearing**

With an eye towards our application and to accommodate for observed trade imbalances, we assume there is a mass 1 of rentiers in each region. Rentiers cannot reallocate to other regions. They own the local structures, rent them to local firms, and send all their local rents to a global portfolio. In return, rentiers receive a constant share \( \pi_{nk} \) from the global portfolio, with \( \sum_{n=1}^{N} \pi_{nk} = 1 \). The difference between the remittances and the income rentiers receive will generate imbalances, which change in magnitude as the rental prices change, and are given by \( \sum_{k=1}^{J} r_{ik} H^{ik} - \pi_{nk} \), where \( \chi_{nt} = \sum_{i=1}^{N} \sum_{k=1}^{J} r_{ik} H^{ik} \) are the total revenues in the global portfolio. The rentier uses her income share from the global portfolio to buy goods produced in her own region using equation (1).

Let \( X_{nt}^{nj} \) be the total expenditure on final good \( j \) in region \( n \). Then, regional market clearing in final goods implies

\[
X_{nt}^{nj} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_{nk} X_{nt}^{ik} + \alpha^{j} \left[ \sum_{k=1}^{J} \pi_{nk} \left( w_{nt}^{nk} L_{nt}^{nk} + \pi_{nk} \right) \right],
\]

where \( \sum_{k=1}^{J} (w_{nt}^{nk} L_{nt}^{nk} + \pi_{nk}) \) is the total income in region \( n \). We refer to equilibrium condition (8) as the goods market equilibrium condition. Given prices and wages, workers and rentiers exhaust their income in final goods (the last term in the equation), and producers supply exactly these final goods for consumption plus the materials needed for intermediate goods production.

Labor market clearing in region \( n \) and sector \( j \) is

\[
L_{nt}^{nj} = \frac{\gamma^{nj} (1 - \xi^{nj})}{w_{nt}^{nj}} \sum_{i=1}^{N} \pi_{ij} X_{nt}^{ij},
\]

while the market clearing for structures in region \( n \) and sector \( j \) must satisfy

\[
H^{nj} = \frac{\gamma^{nj} \xi^{nj}}{r_{nt}^{nj}} \sum_{i=1}^{N} \pi_{ij} X_{nt}^{ij}.
\]

We now proceed to formally define an equilibrium of the economy.
2.3 Equilibrium

The state of the economy at any given moment in time is determined by the distribution of labor across all markets $L_t$. The fundamentals of the economy are given by the sectoral-regional productivities $A = \{ A_{nj} \}_{n=1,j=1}^{N,J}$, iceberg transportation costs $K = \{ K_{nj,ij} \}_{n=1,i=1,j=1}^{N,N,J}$, the labor mobility costs $Y = \{ Y_{nj,ik} \}_{n=1,i=0,j=1,k=0}^{N,J,J,N}$, the distribution of structures across markets $H = \{ H_{nj} \}_{n=1,j=1}^{N,J}$, and the distribution of regional home production $b = \{ b_{nj} \}_{n=1}^{N}$. We denote the fundamentals by $\Theta = (A, K, Y, H, b)$. We seek to find equilibrium wages $w_t = \{ w_{nj,t} \}_{n=1,j=1}^{N,J}$, given $(L_t, \Theta)$. We refer to this equilibrium as a temporary equilibrium. Formally,

**Definition 1** Given $(L_t, \Theta)$, a temporary equilibrium is a vector of wages $w(L_t, \Theta)$ that satisfies the equilibrium conditions of the static subproblem, (5) to (10).

The temporary equilibrium of our model is the solution to a static multicountry interregional trade model. Suppose that for any $(L_t, \Theta)$ we can solve the temporary equilibrium. Then the wage rate can be expressed as $w_t = w(L_t, \Theta)$, and given that prices are all functions of wages, we can express real wages as $w_{nj,t}/P_{nt} = \omega_{nj} (L_t, \Theta)$. After defining the temporary equilibrium, we can now define the sequential competitive equilibrium of the model. Let $\mu_t = \{ \mu_{nj,ik} \}_{n=1,i=0,j=1,k=0}^{N,J,J,N}$ and $V_t = \{ V_{nj,t} \}_{n=1,j=0}^{N,J}$ be the migration shares and lifetime utilities, respectively. The definition of a sequential competitive equilibrium is given as follows:

**Definition 2** Given $(L_0, \Theta)$, a sequential competitive equilibrium of the model is a sequence of $(L_t, \mu_t, V_t, w(L_t, \Theta))_{t=0}^{\infty}$ that solves equilibrium conditions (2) to (4) and the temporary equilibrium at each $t$.

Finally, we define a stationary equilibrium of the model.

---

19It is important to emphasize that the temporary equilibrium described in Definition 1 is not specific to a multi-sector EK model, but it can also be the equilibrium of other trade models such as Melitz (2003). In other words, an economy has a temporary equilibrium if one can solve for equilibrium prices given the distribution of employment.

20In Appendix 2 we present a version of our model that maps into Alvarez and Lucas’ (2007) model. Alvarez and Lucas (2007) show existence and uniqueness of the equilibrium. For a proof and characterization of the conditions for existence and uniqueness of a more general static model than that of Alvarez and Lucas (2007), refer to Allen and Arkolakis (2014), and for a proof of existence and uniqueness of a static model more similar to our static sub-problem, see Redding (2012).

21Proposition 8 from Cameron, Chaudhuri, and McLaren (2007) shows the existence and uniqueness of the sequential competitive equilibrium of a simplified version of our model. Using the results from Alvarez and Lucas (2007) together with proposition 8 from Cameron, Chaudhuri, and McLaren (2007), there exists a unique sequential equilibrium of the one sector model in Appendix 2. The proof is available upon request.
3. SOLUTION METHOD

Solving for all the transitional dynamics in a dynamic discrete choice model with this rich spatial structure is difficult, and it also requires pinning the values of a large number of unknown parameters. Note from Definitions 1 to 3 that to solve for an equilibrium of the model it is necessary to condition on $\Theta$; namely, the level of the fundamentals of the economy (productivity, endowments of local structures, labor mobility costs, unemployment benefits, and trade costs). As we increase the dimension of the problem, for example by adding countries, regions, or sectors, the number of fundamentals grows geometrically. We now show that one can solve this problem and compute the equilibrium dynamics of the model and perform counterfactuals without knowing the level of $\Theta$.

3.1 Time Differences

We start by showing that we can solve for the temporary equilibrium given a change in $L_t$ or $\Theta$ without knowing the level of $\Theta$. To do this, we first define $\Omega(L_t, \Theta)$ to be an allocation of the temporary equilibrium consistent with a distribution of employment $L_t$ and fundamentals $\Theta$. In other words, $\Omega(L_t, \Theta)$ is the set of trade (expenditure) shares $\pi_{t}^{n,j,ij}$, value added, and gross output, given $L_t$ and $\Theta$.

**Proposition 1** Consider the temporary equilibrium at state $L_t$, and a temporary equilibrium at state $L_{t+1}$. Denote the change in the temporary equilibrium from one state $L_t$ to $L_{t+1}$ by $\hat{\omega}(\hat{L}_{t+1}, \Theta)$ where $\hat{L}_{t+1} = L_{t+1} / L_t$. Given the allocation at state $L_t$, $\Omega(L_t, \Theta)$, the solution to the change in the temporary equilibrium from $\hat{L}_{t+1}$ does not require information on the level of $\Theta$.

Appendix 4 presents the equilibrium conditions in relative changes that are used to solve for the temporary equilibrium, and the proof to Proposition 1 is presented in Appendix 3.
Proposition 1 shows that given an allocation and a change in the distribution of labor, $\hat{L}_{t+1}$, we can solve for the change in wages without requiring information on the levels of fundamentals of the economy. Given this result, we can represent the change in equilibrium real wages as a function of the change in $\hat{L}_{t+1}$—namely, $\hat{\omega}^{nj}(\hat{L}_{t+1})$. From inspecting the equilibrium conditions in relative changes in Appendix 4, one can see that given $\Omega(L_t, \Theta)$ we can solve for the change in wages without knowing the level of $\Theta$. This result, which was first advanced by DEK in the context of static heterogeneous firm trade models, turns out to be very convenient for taking the model to the data and for performing counterfactuals. We view this method as having two main advantages. First, by conditioning on observed trade and production allocations at a given moment in time, one disciplines the model by making it match all cross-sectional moments in the data. Second, after conditioning on data, one can use the model to solve for counterfactuals without estimating the fundamentals of the economy, which are usually difficult to identify.

We can use the result from Proposition 1 not only to solve for wages given a sequence of $\hat{L}_{t+1}$, but also to characterize how the sequence of allocations of the economy change given a change in the sequence of fundamentals. Changes in fundamentals are referred to as counterfactuals in our study, as described below. We state this last observation without proof with Corollary 1.

**Corollary 1** Consider a temporary equilibrium at state $L_t$ with fundamentals $\Theta$. Conditional on the allocation $\Omega(L_t, \Theta)$, solving for a change in the temporary equilibrium due to a change in fundamentals, $\hat{\Theta} = \Theta'/\Theta$, does not require information on the level of $\Theta$.

Note that Corollary 1 does not impose any restrictions on how $\Theta$ changes. Namely, Corollary 1 says that for any arbitrary change in fundamentals (one by one or jointly) across time and space, one can solve for the change in real wages resulting from the change in $\Theta$. Building on these last results, we can now characterize the solution of the dynamic model. The next proposition shows that, given an initial allocation of labor, $L_0$, the matrix of gross migration flows at $t - 1$, and the allocation $\Omega(L_0, \Theta)$, we can solve the sequential equilibrium of the model in relative time differences without knowing $\Theta$.

**Proposition 2** Let $V_{t+1}^{ik} \equiv \exp\left(V_{t+1}^{ik} - V_{t}^{ik}\right)^{1/\nu}$. Conditional on an initial allocation of the economy, $(L_0, \mu_{-1}, \Omega(L_0, \Theta))$, the solution to the sequential equilibrium in relative time differences does not require information on the level of the fundamentals, $\Theta$, and solves the following system
of equations:

\[
\mu_{t+1}^{nj,ik} = \frac{\mu_t^{nj,ik} \left[ Y_{ik}^{t+2} \right]^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_t^{nj,mh} \left[ Y_{mh}^{t+2} \right]^{\beta}},
\]

\[
Y_{t+1}^{nj} = \left[ \hat{\omega}^{nj}(\hat{L}_{t+1}) \right]^{1/\nu} \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_t^{nj,ik} \left[ Y_{ik}^{t+2} \right]^{\beta},
\]

\[
L_{t+1}^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_t^{ik,nj} L_{t}^{ik},
\]

for all \( j, n, i \) and \( k \) at each \( t \), where \( \{ \hat{\omega}^{nj}(\hat{L}_{t+1}) \}^{N,J}_{n=1,j=1} \) is the solution to the temporary equilibrium given \( \hat{L}_{t+1} \).

Proposition 2 is one of the key results of this paper. This transformation reduces the burden of calibration and allows solving the model using only a few parameters and data for the initial period (i.e., the initial value of the migration shares and the initial distribution of workers across labor markets).

Proposition 2 also shows that by expressing the value function of the households (2) and the migration shares (3) in time differences, one can difference-out all the mobility cost parameters. To gain some intuition about how it works, consider the following example. Take migration shares (3) at time \( t - 1 \). As we can see from (3) given \( \beta \) and \( \nu \), there are infinite combinations of values \( V_{t}^{ik} \) and migration costs \( \tau^{nj,ik} \) that can reconcile a given migration flow. So, in principle, there is no way we can uniquely solve for \( V_{t}^{ik} \) without information on \( \tau^{nj,ik} \). However, consider migration flows for the same market at time \( t \) and take the relative time difference between (3) at time \( t \) and \( t - 1 \); namely,

\[
\frac{\mu_{t}^{nj,ik}}{\mu_{t-1}^{nj,ik}} = \frac{\exp \left( \beta V_{t+1}^{ik} - \tau^{nj,ik} \right)^{1/\nu}}{\exp \left( \beta V_{t}^{ik} - \tau^{nj,ik} \right)^{1/\nu}} \cdot \frac{\exp \left( \beta V_{t+1}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}{\exp \left( \beta V_{t}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}.
\]

Given the properties of the exponential function the numerator simplifies to \( \exp \left( V_{t+1}^{ik} - V_{t}^{ik} \right)^{3/\nu} \).

Now multiply and divide each element of the sum in the denominator by \( \exp \left( \beta V_{t+1}^{mh} - \tau^{nj,mh} \right)^{1/\nu} \) and use migration flows at time \( t - 1 \) to obtain (11).\(^{22}\)

The procedure to derive equation (12) is similar and results from taking relative time differences between equation (2) expressed at time \( t + 1 \) and at time \( t \) (refer to Appendix 3 for the proof).\(^{23}\)

\(^{22}\) Another way to understand our method is by relating it to Hotz and Miller (1993) and Berry (1994). Hotz and Miller (1993) and Berry (1994) show that choice probabilities provide information on payoffs and parameters and that can be used to estimate parameters. We show that by taking time differences on choice probabilities one can solve for the model, and perform counterfactuals, without estimating the parameters.

\(^{23}\) It is worth noting that given Assumption 2, we do not require information on the level of wages and local prices across markets in the initial period to solve the model. If instead we had linear utility, then equation (12) would be
A couple of observations are noteworthy about the system of equilibrium conditions (11), (12), and (13) in time differences. First, at the steady state, \( \{Y_{t}^{i,jk}\}_{i=1,j=0}^{N,J} = 1 \) for all \( t \) regardless of the level of the fundamentals; and second, we can use this system of equations conditioning on observed \( L_0 \) and \( \mu_{-1} \) even if the economy is not initially in a steady state.

To see this, let \( \mu^* \) be the steady-state migration flow and \( L^* \) the steady-state employment distribution. Now suppose that \( \mu_{-1} = \mu^* \) and that \( L_0 = L^* \), and initialize the system at \( \{Y_{1}^{i,jk}\}_{i=1,j=0}^{N,J} = 1 \). From (11) note that since \( Y_{1}^{i,jk} = 1 \), then \( \hat{\omega}^{nj}(1) = 1 \). Finally, given that \( \{Y_{1}^{i,jk}\}_{i=1,j=0}^{N,J} = 1 \), then only \( \{Y_{2}^{i,jk}\}_{i=1,j=0}^{N,J} = 1 \) solves (12). Now condition on observed data \( L_0 \) and \( \mu_{-1} \). If \( L_0 \), and \( \mu_{-1} \) were at the steady state, then initiating the system at \( \{Y_{1}^{i,jk}\}_{i=1,j=0}^{N,J} = 1 \) should solve the system of equations. However, if \( L_0 \) is not the steady-state distribution of labor of the economy, then after applying \( \mu_{-1} \) to \( L_0 \) we will obtain \( \hat{L}_1 \neq 1 \) and as a result \( \hat{\omega}^{nj}(\hat{L}) \neq 1 \) and then \( \{Y_{2}^{i,jk}\}_{i=1,j=0}^{N,J} \neq 1 \) from (12). We use these observations to construct an algorithm that solves for the competitive equilibrium of the economy.

In Appendix 5, Part I, we present the algorithm. Part II of the algorithm is the one we use to solve for counterfactuals — namely, for changes in fundamentals. The next subsection describes in greater detail how this is done.

### 3.2 Solving for Counterfactuals

So far we have shown that we can take our model to the data and solve for the sequential competitive equilibrium of the economy. This might be interesting by itself; however, we also want to be able to use the model to conduct counterfactuals. By counterfactual we refer to the study of how allocations change across space and time, relative to a baseline economy, given a new sequence of fundamentals; which we denote by \( \Theta' = \{\Theta'_t\}_{t=0}^{\infty} \).

From Proposition 2 we can solve for a baseline economy without knowing the level of fundamentals. Given this, we can then study the effects from a change in fundamentals — namely, \( \hat{\Theta} = \{\hat{\Theta}_t\}_{t=1}^{\infty} \) — relative to the baseline economy, without explicitly knowing the level of \( \Theta \). Of course, as in any dynamic model, when solving for the baseline economy, as well as for counterfactuals, we need to make an assumption of how agents anticipate the evolution of the fundamentals of the economy. For example, we can assume that the change in fundamentals is anticipated (or not) given by

\[
Y_{t+1}^{nj} = \left( \hat{\omega}^{nj}(\hat{L}_{t+1}) - 1 \right) \omega^{nj}(L_t) \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{nj,ik} \left[ Y_{t+2}^{ik} \right]^{\beta},
\]

which, as we can see, would require conditioning on the level of real wages \( \omega^{nj}(L_t) \) at all \( t \).
by agents at time 0. Consistent with our perfect foresight assumption, we follow the convention that at the beginning of the period in the baseline economy agents anticipate the entire evolution of fundamentals.\(^{24}\) Then, to compute counterfactuals and with an eye to toward our application later, we assume that agents at \(t = 0\) are not anticipating the change in the path of fundamentals and that at \(t = 1\) agents learn about the entire future sequence of \(\Theta'\). This timing assumption allows us to use information about agents' actions before \(t = 1\) to solve for the sequential equilibrium, under the new fundamentals, in relative time differences.\(^{25}\)

The next proposition, defines how to solve for counterfactuals from unexpected changes in fundamentals. It shows that conditioning on an initial allocation \((L_0, \mu_{-1}, \Omega(L_0, \Theta))\) and on the allocation at time 0 of the baseline economy, we can solve for counterfactuals without information on \(\Theta\). Formally,

**Proposition 3** Consider a sequence of changes in fundamentals, \(\hat{\Theta} = \{\hat{\Theta}_t\}_{t=1}^{\infty}\). Conditional on the initial allocation of the economy, \((L_0, \mu_{-1}, \Omega(L_0, \Theta))\), and the baseline sequential competitive equilibrium in time differences at \(t = 0\), \((Y_1, \mu_0)\), the solution to the counterfactual sequential equilibrium in relative time differences given an unanticipated \(\hat{\Theta}\) does not require information on the level of fundamentals \(\Theta\).

Proposition 3 is another key result of this paper. In particular, it shows that we can calculate counterfactuals from unanticipated changes to the economy’s fundamentals without knowing the levels of fundamentals. As before, the proof of the proposition is presented in Appendix 3.

Finally, it is worth emphasizing that our solution method allows us to study the effects of changes in any element contained in the set \(\Theta\), holding constant the rest, without having to estimate the entire set. In fact, we do not need to impose restrictions on how fundamentals change. There can be many simultaneous changes in parameters and for different periods of time.

We now move to the empirical section of our paper where we apply our model and solution method. We first describe how to take the model to the data. Then, in Section 5, we quantify the effects of the China shock.

\(^{24}\) Note that the sequence of fundamentals that defines the baseline economy does not need to be constant. There can be an arbitrary evolution of fundamentals in the baseline economy. The only requirement for the baseline economy is that the initial allocation will reflect this informational assumption.

\(^{25}\) We can also compute perfect foresight counterfactuals. By this we mean counterfactuals in which agents are aware of all the future changes in fundamentals at time 0. Later, in our empirical section, we perform robustness checks by comparing the results from an unanticipated China shock relative to an anticipated China shock.
4. TAKING THE MODEL TO THE DATA

We take the model to the data in the year 2000. Applying the solution method requires initial values of bilateral trade flows $\pi_0^{nj;ij}$, value added $w_0^{nj} L_0^{nj} + r_0^{nj} H_0^{nj}$, the distribution of employment $L_0$, and the initial period migration flows across regions and sectors, $\mu_{-1}$. We also need to compute the share of value added in gross output $\gamma^{nj}$, the material shares $\gamma^{nj,nk}$, the share of structures in value added $\xi^n$, the final consumption shares $\alpha^j$, and the global portfolio shares $\nu^n$. Finally, we need estimates of the sectoral trade elasticity $\theta^j$, the migration elasticity $1/\mu$, and the discount factor $\beta$. This section provides a summary of the data sources and measurement to calibrate the model, with further details provided in Appendix 6.

Regions, sectors, and labor markets. We calibrate the model to the 50 U.S. states; 37 other countries, including China and a constructed rest of the world; and 22 sectors. The 22 sectors are classified according to the North American Industry Classification System (NAICS), 12 are manufacturing sectors, 8 are service sectors, and we also include construction and wholesale and retail trade. Our definition of a labor market in the U.S. economy is thus a state-sector pair, including unemployment, leading to 1150 markets. In the other countries, there is a single labor market despite having many productive sectors.

Trade and production data. We construct the bilateral trade shares $\pi_0^{nj;ij}$ for the year 2000 for the 38 countries in our sample, including the aggregate United States, from the World Input-Output Database (WIOD). The sectoral bilateral trade flows across the 50 U.S. states were constructed by combining information from the WIOD database and the 2002 Commodity Flow Survey (CFS), which is the closest available year to 2000. From the WIOD database we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. From the 2002 CFS we compute the bilateral expenditure shares across regions and sectors. These two pieces of information allow us to construct the bilateral trade flows matrix for the 50 U.S. states across sectors, where the total U.S. domestic sales match the WIOD data for the year 2000.

Bilateral trade flows between the 50 U.S. states and the rest of the countries in the world were constructed by combining information from the WIOD database and regional employment data from the Bureau of Economic Analysis (BEA). In our paper, local labor markets have different exposures to international trade shocks because there is substantial geographic variation in industry

\[ \text{Agriculture, mining, utilities, and the public sector are excluded from the analysis.} \]
specialization. Regions with a high concentration of production in a given industry should react more to international trade shocks to that industry. Therefore, following ADH, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, we split the bilateral trade flows at the country level computed from WIOD into bilateral trade flows between the U.S. states and other countries by assuming that the share of each state in total U.S. trade with any country in the world in each sector is determined by the regional share of total employment in that industry.

To construct the share of value added in gross output $\gamma^{nj}$, the material input shares $\gamma^{nj,nk}$, and the share of structure in value added $\xi^n$, we use data on gross output, value added, intermediate consumption, and labor compensation across sectors from the BEA for the U.S. states and from the WIOD for all other countries in our sample.

Finally, using the constructed trade and production data, we compute the final consumption shares $\sigma^j$, as described in Appendix 6; and we discipline the portfolio shares $\iota^n$ to match exactly the year 2000 observed trade imbalances.

The initial migration flow matrix and the initial distribution of labor. The initial distribution of workers in the year 2000 by U.S. states and sectors (and unemployment) is obtained from the 5 percent Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. Information on industry is classified according to the NAICS, which we aggregate to our 22 sectors and unemployment.\(^{27}\) We restrict the sample to people between 25 and 65 years of age who are either unemployed or employed in one of the sectors included in the analysis. Our sample contains over 5 million observations.

Our quantitative model will not allow for international migration.\(^{28}\) That is, we impose that $\tau^{nj,ik} = \infty$ for all $j, k$ such that regions $n$ and $i$ belong to different countries. Given this assumption, we need to measure the initial matrix of gross flows only for the U.S. economy. We construct the initial migration flow matrix at the quarterly frequency, decision that it is guided by the fact that our framework incorporates unemployment, and unemployment spells in the United States are generally short-lived (less than a year). To construct the initial matrix of quarterly mobility across our regions and sectors ($\mu_{-1}$), we combine information from the Current Population Survey (CPS) to compute intersectoral mobility and from the PUMS of the American Community Survey (ACS)

\(^{27}\)While unemployment in the Census is defined similarly to the Current Population Survey (CPS), design and methodological differences in the Census tend to overestimate the number of unemployed workers relative to the CPS.

\(^{28}\)This simplification is a consequence of data availability. As we discussed previously, our model can accommodate international migration.
to compute interstate mobility. Table 2 in Appendix 6 shows the information provided by these two datasets in terms of transition probabilities.

Table 1 shows some moments of worker mobility across labor markets computed from our estimated transition matrix for the year 2000. Our numbers are consistent with the estimates by Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2012) for interstate moves and Kambourova and Manovskii (2008) for intersectoral mobility.29

![Table 1: U.S. interstate and intersectoral labor mobility](image)

One important observation from Table 1 is that there is a large amount of heterogeneity in transition probabilities across labor markets, which indicates that workers in some industries and states are more likely to switch to a different labor market than other workers. In particular, the 25th and 75th percentiles of the distribution of sectoral mobility probabilities by labor market are 40% lower and higher than the median, respectively. This dispersion is even larger for interstate moves. We interpret the observed low transition probabilities and their heterogeneity as evidence of substantial and heterogeneous costs of moving across labor markets, both spatially and sectorally.

**Elasticities.** We calibrate the quarterly discount factor $\beta$ to 0.99, implying a yearly interest rate of roughly 4%. The sectoral trade elasticities $\theta^j$ are obtained using Caliendo and Parro (2015). We calibrate the migration elasticity, $1/\nu$, by adapting the method and data used in ACM. From their model, they derive an estimating equation that relates current migration flows to future wages and future migration flows. Then, they estimate the equation by GMM and instrument using past values of flows and wages.30

In order to adapt ACM’s procedure to our model and frequency, we have to deal with two issues. First, in our model agents have log utility while in ACM preferences are linear; and second, ACM

29 Since our period is a quarter, our rates are not directly comparable with the yearly mobility rates for state and industry from these studies. Moreover, our sample selects workers from ages 25 to 65, who tend to have lower mobility rates than younger workers.

30 ACM construct migration flow measures and real wages for 26 years between 1975-2000, using U.S. Census Bureau’s March Current Population Surveys (CPS). We use ACM data in our estimation and do not proceed to disaggregate their data forward. Due to its small sample size, using the March CPS to construct interregional and intersectoral migration flows could bias down the amount of mobility. For further details, see ACM and Appendix 6.
estimate an annual elasticity while we are interested in a quarterly elasticity. Dealing with the first issue is not that difficult since from our model we obtain the analogous estimating equation to ACM’s preferred specification but with log utility, namely,

\[
\log \left( \frac{\mu_{t}^{nj,ik}}{\mu_{t}^{nj,nj}} \right) = \tilde{C} + \frac{\beta}{\nu} \log \left( \frac{w_{t+1}^{ik}}{w_{t+1}^{nj}} \right) + \beta \log \left( \frac{\mu_{t+1}^{nj,ik}}{\mu_{t+1}^{nj,nj}} \right) + \omega_{t+1},
\]

where \( \omega_{t+1} \) is a random term, and \( \tilde{C} \) is a constant. The relevant coefficient \( \beta/\nu \) represents the elasticity of migration flows to changes in income, while in ACM it has the interpretation of a semi-elasticity. As pointed out by ACM, the disturbance term, \( \omega_{t+1} \), will in general be correlated with the regressors; thus, we require instrumental variables. As in ACM, our theory implies that past values of the flows and wages are valid instruments; therefore, we instrumented this regression by the past values of flows and wages.\(^{31}\)

Dealing with the second issue is more involved. As discussed in ACM, Kambourov and Manovskii (2013) point out a difficulty in interpreting flow rates that come out of the March CPS retrospective questions. They conclude that although superficially it appears to be annual, the mobility measured by the March CPS is less than annual. ACM correct for this bias, and conclude that the March CPS measures mobility at a five-month horizon. Then, they annualize the migration flow matrix by assuming that within a year the monthly flow rate matrix is constant. We transform the five-month migration flow matrices in ACM to quarterly matrices using the same procedure ACM use to annualize these matrices.

After dealing with these two issues, we obtain a migration-elasticity of 0.2, which implies a value of \( \nu = 5.34 \). This is our preferred estimate and we use this number in our empirical section below. To the best of our knowledge, there is no benchmark value for this elasticity in the literature. Yet, to put it in perspective, our estimate is consistent with the intuition that this elasticity should be smaller, thus \( \nu \) larger, at higher frequencies. It is also consistent with ACM, who estimate \( \nu = 1.88 \) at an annual frequency, and a larger value of \( \nu = 2.89 \) at a five-month frequency.

\(^{31}\)The exclusion restriction is that the error term, \( \omega_{t+1} \), is not correlated over time. Naturally, depending on the context, this is a strong assumption which in some cases could be violated. For example, if there are unobservable serially correlated characteristics of some labor markets, they are going to be subsumed in the residual. We rely on ACM’s strategy but note that future research should focus on finding a different instrument, or a different estimation strategy, that is not subject to this criticism. See ACM for a discussion on other strengths and weaknesses of this approach.
4.1 Identifying the Trade Shocks

In previous work, ADH and Acemoglu et al. (2014) argue that the increase in U.S. imports from China had asymmetric impacts across regions and sectors. In particular, labor markets with greater exposure to the increase in import competition from China saw a larger decrease in manufacturing employment. Given that not all of the observed changes in U.S. imports from China are necessarily the result of a change in Chinese TFP, we replicate the procedure of ADH to identify the supply-driven components of Chinese imports. To do so, we compute the predicted changes in U.S. imports from China using the change in imports from China by other advanced economies as an instrument. This is related to the first-stage regression of the two-stage least squares estimation in ADH conducted under our definition of labor market, that is, at our regional and sectoral level of disaggregation.\footnote{See Appendix 6 for more details on the data construction and estimation. One might be concerned that with our data and at our level of disaggregation the specification from ADH might not deliver employment effects comparable to ADH. Therefore, in Appendix 6 we also run the second-stage regression in ADH with our data and the results we obtain are largely aligned with those in ADH.}

\textbf{Fig. 1:} Predicted change (2000-2007) change in imports vs. model-based Chinese TFP change
We estimate the following regression

\[ \Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j, \]

where \( j \) is one of our manufacturing sectors. \( \Delta M_{USA,j} \) and \( \Delta M_{other,j} \) are the changes in U.S. imports from China and imports from China by other advanced economies between 2000 and 2007.\(^{33}\)

We then use the predicted changes in U.S. imports according to this regression to calibrate the size of the TFP changes for each of the manufacturing sectors in China that will deliver the same change in imports in the model as in the data. To do so, we first employed a static multicountry, multi-sector version of our model and calibrated the TFP changes to our 12 manufacturing sectors of the Chinese economy \( \{ \Delta^{China,j} \}_{j=1}^{12} \) that match exactly the change in U.S. manufacturing imports from China from 2000 to 2007. Since the change in U.S. imports from China is evenly distributed over this period, we interpolated the estimated TFP changes over 2000-2007 across all quarters and feed in this sequence of TFP shocks into our dynamic model.

Figure 1 shows the predicted change in U.S. manufacturing imports from China computed as in ADH and the implied sectoral productivity changes in China. Computer and electronics is the sector most exposed to import competition from China, accounting for about 40% of the predicted total change in U.S. imports from China, followed by the textiles and furniture industries with about 12% each, and metal and machinery with 10% of the total import penetration growth each. On the other hand, the food, beverage, and tobacco, and the petroleum industries are the ones least exposed, accounting for less than 1.5% of the predicted total change in U.S. imports from China.

Our model estimates that TFP increased in all manufacturing industries in China. While our estimated changes in Chinese TFP are correlated with the changes in U.S. imports from China by sector, this correlation is not perfect.

5. THE EFFECTS OF INCREASED IMPORT COMPETITION FROM CHINA

In this section, we quantify the dynamic effects of China’s import penetration growth on the U.S. economy. We first compute the dynamic model, holding productivities in China constant, which is our baseline economy. We do this using the results from Proposition 2 and assuming that agents

\(^{33}\)In particular, the countries are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland.
foresee constant fundamentals over time. We then use the results from Proposition 3 and solve for the changes in equilibrium allocations due to the China shock. We now present our results where we first discuss the effects on aggregate, sectoral, and regional employment and then analyze the effects on welfare across markets.

5.1 Employment Effects

Starting with sectoral employment, the upper-left panel in Figure 2 presents the dynamic response of the manufacturing share of employment with and without the China shock. As the figure shows, there are transitional dynamics toward a steady-state equilibrium even in the absence of any change in Chinese productivity. These dynamics occur since the economy is not in a steady state in the year 2000. In other words, the observed employment in manufacturing in 2000 is the equilibrium result of a series of shocks and structural changes that hit the economy before that year, and the economy is transitioning to a new steady state as a result. For instance, U.S. manufacturing employment has experienced a secular decline over the past several decades, and in 2000 the economy was still adjusting to this structural change. Thus, we observe a decline in manufacturing employment even in the absence of productivity changes in China. The implication of this observation is that calibrating the model under the assumption that the economy is in steady state would overestimate the impact of the increased import competition from China since part of the observed decline in manufacturing employment is not related to Chinese competition.

Therefore, the upper-left panel in Figure 2 shows the transitional dynamics of manufacturing employment with and without the China shock. The difference between the two is our account of the effect of China’s import penetration growth on U.S. manufacturing employment. The figure shows that import competition from China contributed to a substantial decline in the share of manufacturing employment, a result that is in line with ADH. Our results indicate that increased competition from China reduced the share of manufacturing employment by 0.5 percentage point after 10 years, which is equivalent to about 0.8 million jobs or about 50% of the change in manufacturing employment that is not explained by a secular trend.

We also computed the model under the alternative assumption that agents were expecting the change in fundamentals at $t = 0$. The results are slightly different quantitatively, but overall our conclusions about the employment and welfare effects do not change much with this alternative assumption.

If we were to include long-run trends or structural changes in our model, our economy could fully account for the continuous fall in manufacturing employment.

The difference between the observed share of manufacturing employment in the U.S. economy in 2007 and its predicted value using a simple linear trend on this share between 1965 and 2000 is 1%. In other words, the change in the U.S. manufacturing share that is unexplained by a linear trend is 1%. To compute the implied levels of
As shown in the other three panels of Figure 2, increased import competition from China makes workers reallocate to other sectors; thus, the share of employment in services, wholesale and retail, and construction increases. We also find that Chinese competition reduced the U.S. unemployment rate by 0.03 percentage point in the long run. The role of intermediate inputs and sectoral linkages is crucial to understanding these reallocation effects. Import competition from China leads to decreased production among U.S. manufacturing sectors that compete with China, but it also affords the U.S. economy access to cheaper intermediate goods from China that are used as inputs in the non-manufacturing sectors. Therefore, production and employment increase in the non-manufacturing sectors as a result. Moreover, the increase in employment in these sectors slightly more than offsets the decline in manufacturing employment so that the unemployment rate declines. However, in some states, such as Illinois, Oklahoma, and New York, the unemployment rate increase, due to mobility frictions and that other sectors are not large enough to absorb all workers displaced from the manufacturing sector across different locations. Finally, the employment in construction overshoots a bit in the short run, which is explained, as mentioned earlier, by the fact that the economy was transitioning to a steady state when the change in Chinese pro-

manufacturing employment loss in 2007, we take data on total employment from the BEA for the year 2007 (Table SA25N: Total Full-Time and Part-Time Employment by NAICS Industries). To match the sectors in our model, we subtract employment in farming, mining, utilities, and the public sector, which yields a level of employment of 151.4 million. We multiply by our model’s implied change in manufacturing employment share and get 0.81 million jobs.
ductivity hit the U.S. economy. As a result, in the initial year the relative benefits of working in the construction sector are too low and people move more quickly to other sectors than in the long run.

Our quantitative framework also allows us to further explore the decline in manufacturing employment caused by the China shock by explaining the sources of this decline. To do so, we quantify the relative contribution of different sectors, regions, and local labor markets to the decline in the manufacturing share of employment.

Figure 3 shows the contribution of each manufacturing sector to the total decline in manufacturing employment. The figure shows that employment in some industries was affected more than in others. Specifically, sectors with higher exposure to import competition from China lost more manufacturing employment. The computer and electronics and furniture industries contributed to about half of the decline in manufacturing employment, followed by the metal and textiles industries, which together contributed to about one-fourth of the total decline. Sectors less exposed to import competition from China explain a smaller portion of the decline in manufacturing employment. In fact, these sectors also benefit from the access to cheaper intermediate goods from sectors that experienced a substantial productivity increase in China. In some sectors, such as food, beverage and tobacco, the increased production from the access to cheaper intermediate goods more than offsets the negative effects from increased import competition, and employment increased as a result.

**Fig. 3:** Sectoral contribution to the change in manufacturing employment due to the China shock
The fact that U.S. economic activity is not equally distributed across space, combined with the differential sectoral exposure to China, also implies that the impact of import competition from China on manufacturing employment varies across regions.

Figure 4 presents the regional contribution to the total decline in manufacturing employment. States with a comparative advantage in sectors more exposed to import competition from China lose more employment in manufacturing. For instance, California alone accounted for 20% of all employment in the computer and electronics industry in the year 2000. For comparison, the state with the next-largest share of employment in this sector is Texas with 8%, while all other states had shares of employment in computer and electronics of less than 2%. As a result, California is the state that contributed the most to the overall decline in manufacturing employment (about 12%) followed by Texas. States with a comparative advantage in goods were less affected by import competition from China and that benefited from the access to cheaper intermediate goods had a smaller impact on employment.

Fig. 4: State contribution to the change in manufacturing employment due to the China shock

The contribution of each labor market to the total decline in manufacturing employment varies considerably across regions and sectors. We find that most manufacturing labor markets lost jobs, although employment increased in some of them. Computer and electronics in California was the labor market that contributed the most to the decline in manufacturing employment, accounting for 4.2% of the total decline. Employment increased in labor markets such as food, beverage, and tobacco in Wisconsin, California, and Arkansas; and transportation equipment in New Hamp-
shire, among others. Notice that even when California experienced a decline in manufacturing employment due to import competition from China, some labor markets in California such as food, beverage and tobacco gained in employment, highlighting the importance of taking into account the spatial and sectoral distribution of economic activity.\footnote{ADH show evidence that higher exposure to Chinese imports in a labor market causes a larger increase in unemployment in that market. In our model, unemployment falls due to the China shock, but we constructed a measure of import changes per worker in each U.S. state over the period 2000-2007 and find that states with a lower import penetration experience a larger fall in unemployment. Similarly, in states with higher import penetration unemployment does not fall as much. Therefore, our model also accounts for the positive relation between import penetration and unemployment in a labor market.}

### 5.2 Welfare Effects

We now turn to the aggregate and disaggregate welfare effects of increased import competition from China on the U.S. economy. Let $x(\hat{\Theta})/x$ be the relative change in the variable $x$ due to a change in fundamentals $\hat{\Theta}$. The change in welfare from a change in fundamentals $W_{nj}(\hat{\Theta})$, measured in terms of consumption equivalent variation, can be expressed as

$$W_{nj}(\hat{\Theta}) = (1 - \beta) \sum_{s=t}^{\infty} \beta^{s-1} \log \left( \frac{C_{s}^{nj}(\hat{\Theta}) / C_{s}^{nj}(\hat{\Theta})}{(\mu_{s}^{nj,nj}(\hat{\Theta}) / \mu_{s}^{nj,nj}(\hat{\Theta}))^\beta} \right)$$

(14)

We compute the welfare effect of the China shock using equation (14), where $\hat{\Theta}$ incorporates the changes in TFP in the Chinese manufacturing sectors.\footnote{In a one-sector model with no materials and structures, equation (14) reduces to $W_{nj}(\hat{\Theta}) = (1 - \beta) \sum_{s=t}^{\infty} \beta^{s-1} \log \left( \frac{(\pi_{s}^{n,n}(\hat{\Theta})^{1/\theta})}{(\pi_{s}^{n,n}(\hat{\Theta})^{1/\theta})^\beta} \right)$, which combines the welfare formulas in ACM (2010), and Arkolakis, Costinot, and Rodriguez-Clare (2012).} In Appendix 7 we present the derivation of equation (14) and discuss in more detail the different mechanisms that shape the welfare effects of changes in fundamentals in our model.

We find that U.S. aggregate welfare increases 0.6% due to China’s import penetration growth.\footnote{We aggregate welfare across labor markets using the employment shares at the initial year.} The aggregate change in welfare masks, however, a large heterogeneity in the welfare effects across individual labor markets. Figure 5 presents a histogram with the changes in welfare across 1150 U.S. labor markets. An important takeaway from the figure is that there is a very heterogeneous response to the same aggregate shock across labor markets. Welfare effects are more dispersed across labor markets that produce manufacturing goods than those that produce nonmanufacturing goods, as manufacturing industries have different exposure to import competition from China.\footnote{The histograms showing the welfare effects across labor markets in the manufacturing sectors and across labor markets in the nonmanufacturing sectors are presented in Appendix 8.} Also, all labor markets that produce nonmanufacturing goods gain from the China shock, and welfare tends
to be higher than for labor markets in the manufacturing sectors. Labor markets that produce nonmanufacturing goods do not suffer the direct adverse effects of the increased competition from China and at the same time benefit from access to cheaper intermediate manufacturing inputs from China used in the production in these industries. Similarly, labor markets located in states that trade more with the rest of the U.S. economy and purchase materials from sectors where Chinese productivity increased more tend to have larger welfare gains as they benefit from the access to cheaper inputs from China purchased from the rest of the U.S. economy. For instance, all labor markets located in California gain, even when California is highly exposed to China. The reason is that California benefits more than other states from the access to cheaper goods purchased from the rest of the U.S. economy.

We also compute the welfare effects across countries. Figure 6 shows that all countries gain from the China shock, with some countries gaining more and others gaining less than the United States. Countries that are more open to trade, not only to China but to the world, such as Cyprus and Australia, experience bigger welfare gains, as they benefit from the access to cheaper intermediate goods from China as well as from purchasing cheaper goods from other countries that also benefit from purchasing cheaper intermediate goods from China.

5.2.1 Adjustment Costs

Recent papers have highlighted the importance of the transitional dynamics for welfare evalua-
tion; specifically, the fact that comparisons across steady-state equilibria can significantly overstate or understate welfare measures (i.e., Dix-Carneiro, 2014; Alessandria and Choi, 2014; Burstein and Melitz, 2011).

In order to provide a measure that accounts for the transition costs to the new steady state, we follow Dix-Carneiro (2014)'s measure of adjustment cost. Formally, we use

\[ AC_{nj}(\hat{\Theta}) = \log \left( \frac{1}{1-\beta} \left( V_{SS}(\hat{\Theta}) - V_{SS}^{nj} \right) / \sum_{t=0}^{\infty} \beta^t \left( V_{t+1}^{nj}(\hat{\Theta}) - V_{t+1}^{nj} \right) \right) , \]

to measure the adjustment cost for market \( nj \).

We find that transition costs burn 2.5% of the long-term aggregate welfare gains.\(^{41}\) However, the variation across individual labor markets is substantial. Figure 7 presents a histogram of the adjustment costs across individual labor markets.

The distribution has a long right tail, and several labor markets have adjustment costs substantially larger than the aggregate transition cost. We also find that some labor markets have negative adjustment costs as the welfare gains with transition dynamics overshoot the steady state. Similar to the welfare effects, adjustment costs in labor markets in the manufacturing sectors are

\(^{41}\) As we did before with welfare measures, we use the \( t = 0 \) labor shares as weights to aggregate across labor markets.

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![Figure 6: Welfare effects across countries](image-url)
more dispersed than in the nonmanufacturing sectors, reflecting their varying exposure to import competition from China.\footnote{In Appendix 8 we present the histograms that show the adjustment costs across labor markets in the manufacturing sectors and across labor markets in the nonmanufacturing sectors.}

6. CONCLUSION

In this paper, we build on Artuc, Chaudhuri, and McLaren (2010) and Eaton and Kortum (2002) to develop a dynamic and spatial trade model. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in determining allocations. We calibrate the model to 38 countries, 50 U.S. states, and 22 sectors to quantify the impact of increased import competition from China over the period 2000-2007 on employment and welfare across spatially different labor markets. Our results indicate that although exposure to import competition from China reduces manufacturing employment, aggregate U.S. welfare increases. Disaggregate effects on employment and welfare across regions, sectors, and labor markets and over time are shaped by all the mechanisms and ingredients mentioned previously.

We emphasize that our quantitative framework can be applied to an arbitrary number of sectors, regions, and countries and can be used to address a broader set of questions, generating a promising
research agenda. For instance, with our framework we can study the impact of changes in trade costs, or productivity, in any region in any country in the world. It can also be used to explore the effects of capital mobility across regions; to study the economic effects of different changes in government policies, such as changes in taxes, subsidies or unemployment benefits; or to study policies that reduce mobility frictions.\textsuperscript{43} Other interesting topics to apply this framework are the quantification of the effects of trade agreements and other changes in trade policy on internal labor markets and the impact of migration across countries. In addition, we can study the transmission of regional and sectoral shocks across a production network when trade and factor reallocation is subject to frictions.\textsuperscript{44} The model can also be computed at a more disaggregated level to study migration across metropolitan areas, or commuting zones, although the challenge here is to collect the relevant trade and production data at these levels of disaggregation. Quantitative answers to some of these questions using dynamic models of the type developed here present an exciting avenue for future research.

REFERENCES


\textsuperscript{43} There is a rapid and growing interest to answer these type of questions; see for instance, Fajgelbaum, Morales, Suárez-Serrato, Zidar (2015), Ossa (2015), and Tombe and Zhu (2015).

\textsuperscript{44} We can therefore extend the analysis of Acemoglu et al. (2012) to a frictional economy. Moreover, we could incorporate local natural disaster shocks and quantify their effect, as recently analyzed in Carvalho et al. (2014).


APPENDIX 1: EQUILIBRIUM CONDITIONS

In this appendix, we derive the lifetime expected utility (2) and the gross migration flows described by equation (3). The lifetime utility of a worker in market $nj$ is given by

$$v_{t}^{nj} = U(C_{t}^{nj}) + \max_{\{i,k\}} \left\{ \beta E \left[ v_{t+1}^{ik} \right] - \tau^{nj,ik} + \nu \epsilon_{t}^{ik} \right\}, \tag{15}$$

Denote by $V_{t}^{nj} = E[v_{t}^{nj}]$ the expected lifetime utility of a worker, where the expectation is taken over the preference shocks. We assume that the idiosyncratic preference shock $\epsilon$ is i.i.d. over time and is a realization of a Type-I Extreme Value distribution with zero mean. In particular, $F(\epsilon) = \exp(-\exp(-\epsilon - \gamma))$, and $f(\epsilon) = \partial F/\partial \epsilon$. We seek to solve for

$$\Phi_{t}^{ik} = E \left[ \max_{\{i,k\}} \left\{ \beta E \left[ v_{t+1}^{ik} \right] - \tau^{nj,ik} + \nu \epsilon_{t}^{ik} \right\} \right].$$

Let $\hat{\epsilon}_{t}^{ik,mh} = \frac{\beta(V_{t+1}^{ik} - V_{t+1}^{mh}) - (\tau^{nj,ik} - \tau^{nj,mh})}{\nu}$, note that

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \epsilon_{t}^{ik}) f(\epsilon_{t}^{ik}) \prod_{mh \neq ik} F(\epsilon_{t}^{ik,mh} + \epsilon_{t}^{ik}) d\epsilon_{t}^{ik},$$

Denote $\gamma = \int_{-\infty}^{\infty} x \exp(-x - \exp(-x)) dx$ to Euler’s constant. Then we obtain

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \epsilon_{t}^{ik}) \exp(-\epsilon_{t}^{ik} - \gamma) e^{(-e^{-\epsilon_{t}^{ik} - \gamma})} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{-\epsilon_{t}^{ik,mh}} \sum_{h=0}^{J} d\epsilon_{t}^{ik},$$

Defining $\lambda_{t} = \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\epsilon_{t}^{ik,mh})$ and considering the following change of variables, $\zeta_{t} = \epsilon_{t}^{ik} + \gamma$ we get

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu(\zeta_{t} - \gamma)) \exp(-\zeta_{t} - \exp(-(\zeta_{t} - \lambda_{t}))) d\zeta_{t}.$$ 

Consider an additional change of variables; let $\tilde{y}_{t} = \zeta_{t} - \lambda_{t}$. Hence, we obtain

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(-\lambda_{t}) \left[ (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu(\lambda_{t} - \gamma)) + \nu \int_{-\infty}^{\tilde{y}_{t}} \tilde{y}_{t} \exp(-\tilde{y}_{t} - \exp(-\tilde{y}_{t})) d\tilde{y}_{t},$$

and using the definition of $\gamma$, we get

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(-\lambda)(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \lambda),$$

and replacing the definition of $\lambda$, we get

$$\Phi_{t}^{ik} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(-\log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\epsilon_{t}^{ik,mh})) (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\epsilon_{t}^{ik,mh})).$$
Substituting the definition of $\tilde{\epsilon}_{t}^{ik,mh}$, we get,

$$
\Phi_{t}^{ik} = \nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau_{nj,mh})^{1/\nu} \left[ \frac{\sum_{i=1}^{N} \sum_{k=0}^{J} \exp(\beta V_{t+1}^{ik} - \tau_{nj,ik})^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau_{nj,mh})^{1/\nu}} \right],
$$

which implies

$$
\Phi_{t}^{ik} = \nu \left[ \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau_{nj,mh})^{1/\nu} \right],
$$

and therefore

$$
V_{t}^{nj} = u(C_{t}^{nj}) + \nu \left[ \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{ik} - \tau_{nj,ik})^{1/\nu} \right].
$$

We now derive equation (3). Define $\mu_{t}^{nj,ik}$ as the fraction of workers that reallocate from labor market $nj$ to labor market $ik$. This fraction is equal to the probability that a given worker moves from labor market $nj$ to labor market $ik$ at time $t$; that is, the probability that the expected utility of moving to $ik$ is higher than the expected utility in any other location. Formally,

$$
\mu_{t}^{nj,ik} = \Pr \left( \frac{\beta V_{t+1}^{ik} - \tau_{nj,ik}}{\nu} + \tilde{\epsilon}_{t}^{ik} \geq \max_{mh \neq ik} \left\{ \frac{\beta V_{t+1}^{mh} - \tau_{nj,mh}}{\nu} + \epsilon_{t}^{mh} \right\} \right).
$$

Given our assumptions on the idiosyncratic preference shock,

$$
\mu_{t}^{nj,ik} = \int_{-\infty}^{\infty} f(\epsilon_{t}^{ik}) \prod_{mh \neq ik} F \left( \beta (V_{t+1}^{ik} - V_{t+1}^{mh}) - (\tau_{nj,ik} - \tau_{nj,mh}) + \tilde{\epsilon}_{t}^{ik} \right) d\epsilon_{t}^{ik},
$$

From the above derivations, we know that

$$
\mu_{t}^{nj,ik} = \int_{-\infty}^{\infty} \exp(-\epsilon_{t}^{ik} - \tilde{\gamma}) \exp \left[ - \exp(-\epsilon_{t}^{ik} - \tilde{\gamma}) \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\epsilon_{t}^{ik,mh}) \right] d\epsilon_{t}^{ik}.
$$

Using the definitions from above, we get

$$
\mu_{t}^{nj,ik} = \exp(-\lambda_{t}) \int_{-\infty}^{\infty} \exp (-\tilde{\gamma}_{t} - \exp(-\tilde{\gamma}_{t})) d\tilde{y}_{t},
$$

and solving for this integral we obtain

$$
\mu_{t}^{nj,ik} = \frac{\exp \left( \beta V_{t+1}^{ik} - \tau_{nj,ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+1}^{mh} - \tau_{nj,mh} \right)^{1/\nu}}.
$$
APPENDIX 2: THE ONE-SECTOR DYNAMIC AND SPATIAL TRADE MODEL

In this appendix, we present the one-sector model. To simplify notation, we index labor markets by $\ell$ and assume that there are a total of $N$ labor markets. As in the main text, we abuse notation and let $\ell = 0$ denote the unemployment status.

Households (Dynamic Problem)

The problem of the agent is as follows:

$$v_t^\ell = U(C_t^\ell) + \max_{\ell'} \left\{E \left[v_{t+1}^\ell - \ell, \ell' + \nu c_t^\ell \right] \right\},$$

s.t. $U(C_t^\ell) \equiv \begin{cases} \log(b_t^\ell) & \text{if } \ell = 0, \\ \log(w_t^\ell/P_t^\ell) & \text{otherwise}; \end{cases}$

and after using the properties of the Extreme Value distribution, we obtain that the expected lifetime utility of a worker is given by

$$V_t^\ell = u(C_t^\ell) + \nu \log \left[ \sum_{\ell'=0}^N \exp \left( \beta V_{t+1}^{\ell'} - \tau_{\ell, \ell'} \right)^{1/\nu} \right].$$

Similarly, the transition matrix, or choice probability, is given by

$$\mu_t^\ell, \ell' = \frac{\exp \left( \beta V_{t+1}^{\ell'} - \tau_{\ell, \ell'} \right)^{1/\nu}}{\sum_{\ell''=0}^N \exp \left( \beta V_{t+1}^{\ell''} - \tau_{\ell, \ell''} \right)^{1/\nu}},$$

and the evolution of the distribution of labor across markets is given by

$$L_{t+1}^\ell = \sum_{\ell'=0}^N \mu_t^\ell, \ell' L_t^\ell.'$$

Production (Temporary Equilibrium)

As in the main text, at each $\ell$ there is a continuum of perfectly competitive intermediate good producers with constant returns to scale technology and idiosyncratic productivity $z^\ell \sim \text{Fréchet}(1, \theta)$. In particular, the problem of an intermediate good producer is as follows,

$$\min_{\{w_t^\ell, M_t^\ell\}} w_t^\ell l_t^\ell + P_t^\ell M_t^\ell, \text{ subject to } q_t^\ell(z^\ell) = z^\ell A^\ell [l_t^\ell]^\gamma [M_t^\ell]^{1-\gamma},$$

where $M_t^\ell$ is the demand for material inputs, and $A^\ell$ is fundamental TFP in $\ell$. As is shown shortly, material inputs are produced with intermediates from every other market in the world. Denote by
the price of materials produce in $\ell$. Therefore, the unit price of an input bundle is given by
\[
x^\ell_t = B^\ell \left[ w^\ell_t / \gamma \right] \gamma \left[ P^\ell_t / (1 - \gamma) \right]^{1 - \gamma}.
\] (21)

The unit cost of an intermediate good $z^\ell$ at time $t$ is
\[
\frac{x^\ell_t}{z^\ell A^\ell}.
\]

Competition implies that the price paid for a particular variety is in market $\ell$ is given by
\[
p^\ell_t(z) = \min_{\ell' \in N} \left\{ \frac{x_{\ell,t}^{'\ell'}}{z^\ell A^\ell} \right\}.
\]

Final goods in $\ell$ are produced by aggregating intermediate inputs from all $\ell$. Let $Q^\ell_t$ be the quantity of final goods in $\ell$ and $\tilde{q}^\ell_t(z)$ the quantity demanded of an intermediate variety such that the vector of productivity draws received by the different $\ell$ is $z = (z^1, z^2, \ldots, z^N)$. The production of final goods is given by
\[
Q^\ell_t = \int_{R^N_+} \left[ \tilde{q}^\ell_t(z) \right]^{1 - 1/\eta} \phi(z) dz
\]
where $\phi(z) = \exp \left\{ - \sum_{\ell=1}^N (z^\ell)^{-\theta} \right\}$ is the joint density function over the vector $z$. Given the properties of the Fréchet distribution, the price of the final good $\ell$ at time $t$ is
\[
P^\ell_t = \Gamma \left[ \sum_{\ell'=1}^N \left[ \frac{x_{\ell,t}^{'\ell'}}{A^{\ell'}} \right]^{-\theta} \right]^{-1/\theta}
\] (22)
where $\Gamma$ is a constant given by the value of a Gamma function evaluated at $1 + (1 - \eta/\theta)$ and we assume that $1 + \theta > \eta$. The share of total expenditure in market $\ell$ on goods from $\ell'$ is given by
\[
\pi_{t}^{\ell,\ell'} = \frac{\left[ x_{t}^{\ell'} / A^{\ell'} \right]^{-\theta}}{\sum_{\ell''=1}^N \left[ x_{t}^{'\ell''} / A^{\ell''} \right]^{-\theta}}.
\] (23)

Note that this equilibrium condition is more general than the one in the main text. In this setup, $\pi$ reflects the international, interregional, and intersectoral expenditure on intermediate goods, whereas in the main text $\pi$ represents only the international and interregional expenditure shares, and the intersectoral expenditure share is modeled using a constant input-output share.

**Market Clearing**

Let $X^\ell_t$ denote the total expenditure on final goods in $\ell$. Then, the goods market clearing condition is given by
\[
X^\ell_t = (1 - \gamma) \sum_{\ell'=1}^N \pi_{t}^{\ell,\ell'} X^{\ell'}_t + w^\ell_t L^\ell_t.
\] (24)
Labor market clearing in $\ell$ is
\[ L^\ell_t = \frac{\gamma}{w^\ell_t} \sum_{e=1}^{N} \pi^{e,\ell}_{t} X^{e}_{t}, \] (25)
and we further assume balanced trade.

It is easy to verify that all the equilibrium conditions of the static temporary equilibrium of the one-sector model map to Alvarez and Lucas (2007), and therefore their theorems apply.
APPENDIX 3: PROOFS

**Proposition 1** Consider the temporary equilibrium at state $L_t$, $w(L_t, \Theta)$, and a temporary equilibrium at state $L_{t+1}$, $w(L_{t+1}, \Theta)$. Denote the change in the temporary equilibrium from one state $L_t$ to $L_{t+1}$ by $\dot{w}(L_{t+1}, \Theta)$ where $\dot{L}_{t+1} = L_{t+1} / L_t$. Given the allocation at state $L_t$, $\Omega(L_t, \Theta)$, the solution to the change in the temporary equilibrium from state $\dot{L}_{t+1}$ does not require information on the level of $\Theta$.

**Proof:** Suppose the economy is at a temporary equilibrium at $L_t$ and given $\Theta$. Consider a change in $L_t$ to $L_{t+1}$ and denote by $\dot{L}_{t+1} = L_{t+1} / L_t$. Expressing the equilibrium conditions that define a temporary equilibrium under $L_t$ and under $L_{t+1}$ in relative change, equations (5) to (10), results in the following set of equilibrium conditions:

$$x_{t+1}^{nj} = \left[ (\bar{w}_{t+1}^{nj})(1-\xi^n) (\bar{r}_{t+1}^{nj}) \xi^n \right]^{n_{nj}} \prod_{k=1}^{J} \left[ \bar{p}_{t+1}^{nk} \right]^{n_{nj,kn}}, \quad (26)$$

$$\bar{p}_{t+1}^{nj} = \left[ \sum_{i=1}^{N} \pi_t^{n_{ij}} \left( \bar{x}_{t+1}^{ij} \right)^{-\theta^j} \right]^{-1/\theta^j}, \quad (27)$$

$$x_{t+1}^{nj} = \sum_{k=1}^{J} \gamma_{nk,tn} \sum_{i=1}^{N} \pi_t^{n_{ik}} x_t^{ik} + \sum_{k=1}^{J} \bar{w}_{t+1}^{nk} \bar{L}_{t+1} \bar{w}_{t}^{nk} L_t^{nk} + \bar{\lambda}_{t+1}, \quad (28)$$

where $\chi_{t+1} = \sum_{i=1}^{N} \sum_{k=1}^{J} \gamma_{nk,tn} \sum_{i=1}^{N} \pi_t^{n_{ik}} x_t^{ik} + \sum_{k=1}^{J} \bar{w}_{t+1}^{nk} \bar{L}_{t+1} \bar{w}_{t}^{nk} L_t^{nk} + \bar{\lambda}_{t+1}$. By inspecting equations (26) to (29), we can see that with information on the temporary equilibrium under $L_t$ and the allocation $\Omega(L_t, \Theta)$ —that is, $\pi_t^{n_{ij}}, w_{t}^{nk} L_t^{nk}$, and $r_{t}^{nk} H^{nk}$— we can solve for $\bar{w}_{t+1}^{nj}$, and $\bar{p}_{t+1}^{nj}$, given $\dot{L}_{t+1}$, without knowing the level of $\Theta$. Finally, if we denote the change in the temporary equilibrium from one state $L_t$ to $L_{t+1}$ by $\dot{w}(\dot{L}_{t+1}, \Theta)$, then the solution to the change in the temporary equilibrium is given by $\dot{w}(\dot{L}_{t+1}, \Theta)$.

**Proposition 2** Let $Y_{t+1}^{ik} = \exp \left( V_{t+1}^{ik} - V_t^{ik} \right)^{1/\nu}$. Conditional on an initial allocation of the economy, $(L_0, \mu_{-1}, \Omega(L_0, \Theta))$, the solution to the sequential equilibrium in relative time differences does not require information on the level of the fundamentals, $\Theta$, and solves the following system of equations:

$$\dot{h}_{t+1}^{nj,ik} = \frac{h_t^{nj,ik} \left[ Y_{t+2}^{ik} \right]^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_t^{nj,mh} \left[ Y_{t+2}^{mh} \right]^{\beta}}, \quad (30)$$
\[ Y_{t+1}^{nj} = \left[ \omega^{nj}(\hat{L}_{t+1}) \right]^{1/\nu} \sum_{i=1}^{N} \sum_{j=0}^{J} \mu_{t}^{nj,ik} Y_{t+2}^{ik}, \]  

(31)

\[ L_{t+1}^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{ik,nj} L_{t}^{ik}, \]  

(32)

for all \( j, n, i \) and \( k \) at each \( t \), where \( \{ \omega^{nj}(\hat{L}_{t+1}) \}_{n=1}^{N} \) \( \{ j, j = 1 \} \) is the solution to the temporary equilibrium given \( \hat{L}_{t+1} \).

**Proof:** Consider the fraction of workers who reallocate from market \( n, j \) to \( i, k \), at \( t + 1 \); that is, equilibrium condition (3) at \( t + 1 \) :

\[ \mu_{t+1}^{nj,ik} = \frac{\exp \left( \beta V_{t+2}^{ik} - \tau^{nj,ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+2}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}. \]

Taking the relative time differences of this equation, we get

\[ \frac{\mu_{t+1}^{nj,ik}}{\mu_{t}^{nj,ik}} = \frac{\exp \left( \beta V_{t+2}^{ik} - \tau^{nj,ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+2}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}, \]

Using the fact that mobility costs do not change over time, this expression can be expressed as

\[ \frac{\mu_{t+1}^{nj,ik}}{\mu_{t}^{nj,ik}} = \frac{\exp \left( \beta V_{t+2}^{ik} - \beta V_{t+1}^{ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+2}^{mh} - \beta V_{t+1}^{mh} \right)^{1/\nu}}, \]

which is equivalent to

\[ \frac{\mu_{t+1}^{nj,ik}}{\mu_{t}^{nj,ik}} = \frac{\exp \left( V_{t+2}^{ik} - V_{t+1}^{ik} \right)^{3/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{nj,mh} \exp \left( V_{t+2}^{mh} - V_{t+1}^{mh} \right)^{3/\nu}}. \]

Using the definition of \( Y_{t+1}^{ik} \) we get

\[ \mu_{t+1}^{nj,ik} = \frac{\mu_{t}^{nj,ik} \left[ Y_{t+2}^{ik} \right]^{3/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{nj,mh} \left[ Y_{t+2}^{mh} \right]^{3/\nu}}, \]

which is equilibrium condition (11) in the main text. Now take the equilibrium condition (2) in time differences at region \( n \) and sector \( j \):

\[ V_{t+1}^{nj} - V_{t}^{nj} = U(C_{t+1}^{nj}) - U(C_{t}^{nj}) + \nu \log \left[ \frac{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+2}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{t+1}^{mh} - \tau^{nj,mh} \right)^{1/\nu}} \right]. \]
Multiplying and dividing each term in the numerator by \( \exp\left(\beta V_{t+1}^{m} - \tau^{n,j,m}h\right)^{1/\nu} \) and using (3) and the fact that mobility costs do not change over time, we obtain

\[
V_{t+1}^{n} - V_{t}^{n} = U(C_{t+1}^{n}) - U(C_{t}^{n}) + \nu \log \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t+1}^{n,j,mh} \exp \left(\beta V_{t+2}^{m} - \beta V_{t+1}^{m}\right)^{1/\nu} \right].
\]

Taking exponential from both sides, using the definition of \( Y_{t+1}^{i,k} \) and Assumption 1, we obtain

\[
Y_{t+1}^{n} = \left[ \hat{\omega}_{n}^{n,j}(\hat{L}_{t+1}) \right]^{1/\nu} \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n,j,ik} \left[ Y_{t+2}^{i,k} \right]^{\beta}, \tag{34}
\]

where \( \hat{\omega}_{n}^{n,j}(\hat{L}_{t+1}) = \hat{w}_{i+1}^{n,j}(\hat{L}_{t+1})/\hat{P}_{t+1}^{n,j}(\hat{L}_{t+1}) \) solves the temporary equilibrium. Note that by Proposition 1, the sequence of temporary equilibria given \( \hat{\Theta} \) does not depend on the level of \( \Theta \). The equilibrium conditions (33) and (34) do not depend on the level of \( \Theta \) either. Therefore, given a sequence \( \hat{\Theta} = \{\hat{\Theta}_{t}\}_{t=1}^{\infty} \), the solution to the change in the sequential equilibrium of the model given \( \hat{\Theta} \) does not require \( \Theta \).

We now present more details on how to solve for a counterfactual sequential equilibrium without information on the level of fundamentals \( \Theta \), stated in Proposition 3, as well as the proof of this Proposition.

**Proposition 3** Consider a sequence of changes in fundamentals, \( \hat{\Theta} = \{\hat{\Theta}_{t}\}_{t=1}^{\infty} \). Conditional on the initial allocation of the economy, \( (L_{0}, \mu_{-1}, \Omega(L_{0}, \Theta)) \), and the baseline sequential competitive equilibrium in time differences at \( t = 0 \), \( (Y_{1}, \mu_{0}) \), the solution to the counterfactual sequential equilibrium in relative time differences given an unanticipated \( \hat{\Theta} \) does not require information on the level of fundamentals \( \Theta \).

**Proof:** The proof of the proposition is similar to that in Proposition 2. In the following Lemma, we introduce the system of dynamic equations that must be solved in order to compute for the counterfactual sequential equilibrium in relative time differences. As in Proposition 2, the expressions derived in the Lemma do not require the level of \( \Theta \), only its changes.

**Lemma 1:** Denote by \( Y_{t+1}^{i,k}(\hat{\Theta}_{t}) \equiv \exp \left( V_{t+1}^{i,k}(\hat{\Theta}_{t}) - V_{t}^{i,k}(\hat{\Theta}_{t}) \right)^{1/\nu}, \forall t > 1 \), and by \( Y_{1}^{i,k}(\hat{\Theta}_{t}) \equiv \exp \left( V_{1}^{i,k}(\hat{\Theta}_{t}) - V_{0}^{i,k} \right)^{1/\nu} \); solving for counterfactuals requires solving the following system of equations:

For period \( t = 1 \):

\[
\mu_{1}^{n,j,ik}(\hat{\Theta}) = \frac{\partial_{0}^{n,j,ik}(\hat{\Theta}) \left( Y_{2}^{i,k}(\hat{\Theta}) \right)^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \partial_{0}^{n,j,mh}(\hat{\Theta}) \left( Y_{2}^{m,h}(\hat{\Theta}) \right)^{\beta}} \tag{35}
\]
\[ Y_{1}^{nj}(\hat{\Theta}) = \left( \frac{\hat{w}_{1}^{nj}(\hat{\Theta})}{\hat{P}_{1}^{n}(\hat{\Theta})} \right)^{1/\nu} \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \hat{\phi}_{0}^{nj,ik}(\hat{\Theta}) \left( Y_{2}^{nj}(\hat{\Theta}) \right)^{\beta} \right] \]

\[ L_{1}^{nj}(\hat{\Theta}) = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{ik,nj} L_{0}^{ik}, \]

For period \( t \geq 2 \):

\[ \mu_{t}^{nj,ik}(\hat{\Theta}) = \frac{\mu_{t-1}^{nj,ik}(\hat{\Theta}) \left( Y_{t+1}^{ik}(\hat{\Theta}) \right)^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{nj,mh}(\hat{\Theta}) \left( Y_{t+1}^{mh}(\hat{\Theta}) \right)^{\beta}} \]

\[ Y_{t}^{nj}(\hat{\Theta}) = \left( \frac{\hat{w}_{t}^{nj}(\hat{\Theta})}{\hat{P}_{t}^{n}(\hat{\Theta})} \right)^{1/\nu} \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{nj,ik}(\hat{\Theta}) \left( Y_{t+1}^{nj}(\hat{\Theta}) \right)^{\beta} \right] \]

\[ L_{t+1}^{nj}(\hat{\Theta}) = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{ik,nj}(\hat{\Theta}) L_{t}^{ik}(\hat{\Theta}), \]

for all \( j, n, i \) and \( k \) at each \( t \), where \( \{ \hat{\omega}^{nj}(\hat{L}_{t+1}) \}_{n=1,j=1}^{N,J} \) is the solution to the temporary equilibrium given \( \hat{L}_{t+1} \), and

\[ \hat{\phi}_{0}^{nj,ik}(\hat{\Theta}) = \mu_{0}^{nj,ik} \left( \frac{Y_{1}^{ik}(\hat{\Theta})}{Y_{1}^{nj}(\hat{\Theta})} \right)^{\beta}. \]

**Proof:** Take the lifetime utility at period \( t = 0 \) for the economy with no shock,

\[ V_{0}^{nj} = \log(w_{0}^{nj}/P_{0}^{n}) + \nu \log \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{1}^{mh} - \tau_{nj,mh} \right)^{1/\nu} \right], \]

add and subtract \( \beta V_{1}^{mh}(\hat{\Theta}) \), to obtain

\[ V_{0}^{nj} = \log(w_{0}^{nj}/P_{0}^{n}) + \nu \log \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( V_{1}^{mh} - V_{1}^{mh}(\hat{\Theta}) \right)^{\beta/\nu} \exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{nj,mh} \right)^{1/\nu} \right], \]

define

\[ \phi_{1}^{mh} \equiv \exp \left( V_{1}^{mh} - V_{1}^{mh}(\hat{\Theta}) \right)^{\beta/\nu}; \]

then

\[ V_{0}^{nj} = \log(w_{0}^{nj}/P_{0}^{n}) + \nu \log \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{mh} \exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{nj,mh} \right)^{1/\nu} \right]. \]

Take the lifetime utility at period \( t = 1 \) in the counterfactual economy,

\[ V_{1}^{mh}(\hat{\Theta}) = \log(w_{1}^{nj}(\hat{\Theta})/P_{1}^{n}(\hat{\Theta})) + \nu \log \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{2}^{mh}(\hat{\Theta}) - \tau_{nj,mh} \right)^{1/\nu} \right], \]
and take the difference between $V_{n}^{rj}(\hat{\Theta})$ and $V_{0}^{n}$, to get

$$V_{1}^{n,j}(\hat{\Theta}) - V_{0}^{n,j} = \log \left( \frac{w_{1}^{n,j}(\hat{\Theta}) / P_{1}^{n}(\hat{\Theta})}{w_{0}^{n,j} / P_{0}^{n}(\hat{\Theta})} \right) + \nu \log \left[ \frac{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{2}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right)}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{mn} \exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right)} \right].$$

(46)

Note that we can re-write $\mu_{0}^{n,j,ik}$ as

$$\mu_{0}^{n,j,ik} = \frac{\exp \left( \beta V_{1}^{ik} - \tau_{n,j,ik} \right) \nu}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right) \phi_{1}^{ik}}$$

$$= \frac{\exp \left( \beta V_{1}^{ik}(\hat{\Theta}) - \tau_{n,j,ik} \right) \nu}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{mh} \exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right) \nu}.$$

(47)

Then, operating over equation (46), we have

$$V_{1}^{n,j}(\hat{\Theta}) - V_{0}^{n,j} = \log \left( \frac{w_{1}(\hat{\Theta})^{n,j} / P_{1}^{n}(\hat{\Theta})}{w_{0}^{n,j} / P_{0}^{n}(\hat{\Theta})} \right) + \nu \log \left[ \frac{\sum_{m=1}^{N} \sum_{h=0}^{J} \nu \exp \left( V_{2}^{mh}(\hat{\Theta}) - V_{1}^{ik}(\hat{\Theta}) \right)}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{mh} \exp \left( V_{2}^{mh}(\hat{\Theta}) - V_{1}^{ik}(\hat{\Theta}) \right)} \right].$$

Finally, replacing $\vartheta_{0}^{n,j,ik} = \frac{\mu_{0}^{n,j,ik}}{\phi_{1}^{ik}} = \mu_{0}^{n,j,ik} \left( \frac{Y_{1}^{ik}(\hat{\Theta})}{Y_{2}^{ik}(\hat{\Theta})} \right)^{\beta}$, noting that allocations at period $t = 0$, in particular real wages, are the same across equilibria, and exponentiating and rearranging, we have equation (36).

Now take

$$\mu_{1}^{n,j,ik}(\hat{\Theta}) = \frac{\exp \left( \beta V_{2}^{ik}(\hat{\Theta}) - \tau_{n,j,ik} \right) \nu}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{2}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right) \nu}$$

and divide by $\mu_{0}^{n,j,ik}$ and using (48),

$$\frac{\mu_{1}^{n,j,ik}(\hat{\Theta})}{\mu_{0}^{n,j,ik}} = \frac{\exp \left( \beta V_{2}^{ik}(\hat{\Theta}) - \tau_{n,j,ik} \right) \nu}{\exp \left( \beta V_{1}^{ik}(\hat{\Theta}) - \tau_{n,j,ik} \right) \phi_{1}^{ik}}$$

$$= \frac{\exp \left( \beta V_{2}^{mk}(\hat{\Theta}) - \tau_{n,j,mh} \right) \nu}{\exp \left( \beta V_{1}^{mk}(\hat{\Theta}) - \tau_{n,j,mh} \right) \phi_{1}^{mh}} \frac{\exp \left( \beta V_{2}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right) \nu}{\exp \left( \beta V_{1}^{mh}(\hat{\Theta}) - \tau_{n,j,mh} \right) \phi_{1}^{mh}},$$

then

$$\mu_{1}^{n,j,ik}(\hat{\Theta}) = \frac{\mu_{0}^{n,j,ik}}{\phi_{1}^{ik}} \exp \left( \beta V_{2}^{ik}(\hat{\Theta}) - V_{1}^{ik}(\hat{\Theta}) \right) \frac{\beta}{\nu}$$

and replacing $\vartheta_{0}^{n,j,ik} = \frac{\mu_{0}^{n,j,ik}}{\phi_{1}^{ik}}$ and using the definition of $Y_{2}^{ik}(\hat{\Theta}_{t})$, we get equation (35).

The proof of the other expressions is similar to the ones in the proof of Proposition 2 and thus we omit them here.
APPENDIX 4: EQUILIBRIUM CONDITIONS IN RELATIVE CHANGES

In this appendix, we present the set of equilibrium conditions of our model in relative time differences. All the equations were introduced earlier, but to ease the exposition and facilitate the understanding of our model, we present them again here. Define the operator $^\gamma$ over a variable $y_{t+1}$ as $^\gamma y_{t+1} = y_{t+1}^{1-\xi^n}$ (1). The equilibrium conditions of our model in relative time differences are

cost of the input bundle ($NJ$ equations):

$$^\gamma x_{nj}^{t+1} = (\hat{w}_{nj}^{t+1})^{(1-\xi^n)} (\hat{r}_{t+1})^\gamma^n J_k=1 \hat{p}_{nk}^{t+1} \gamma_{nj,nk}^{t+1} ,$$

(49)

price index ($NJ$ equations):

$$\hat{P}_{nj}^{t+1} = \left[ \sum_{i=1}^N \pi_{nj,i}^{t+1} \left( ^\gamma x_{ji}^{t+1} \right)^{\theta_i^t} \left( \hat{A}_{ji}^{t+1} \right)^{1/\theta_i^t} \right]^{-1/\theta_i^t} ,$$

(50)

trade shares ($N^2J$ equations):

$$\pi_{nj,ij}^{t+1} = \pi_{nj,ij}^t \left( \frac{^\gamma x_{ji}^{t+1} \hat{r}_{t+1}^{t+1}}{\hat{P}_{nj}^{t+1}} \right)^{-\theta_i^t} \left( \hat{A}_{ji}^{t+1} \right)^{\theta_i^t} ,$$

(51)

market clearing in final goods ($NJ$ equations):

$$X_{nj}^{t+1} = \sum_{k=1}^J \gamma_{nk,nj}^{t+1} \sum_{i=1}^N \pi_{nj,ik}^{t+1} X_{ik}^{t+1} + \alpha^t \left[ \sum_{k=1}^J u_{nj}^{nk} T_{nk}^{t+1} u_{nk}^{nk} L_{nk}^{t+1} + \nu^t X_{lt+1} \right] ,$$

(52)

labor mobility shares ($N^2(J+1)^2$ equations):

$$\mu_{nj,ik}^{t+1} = \frac{\mu_{nj}^{t+1} \left[ Y_{nj}^{t+2} \right]^{\beta^t} \sum_{m=1}^N \sum_{h=0}^N \mu_{nj}^{t, mh} \left[ Y_{mh}^{t+2} \right]^{\beta^t}}{\sum_{m=1}^N \sum_{h=0}^N \mu_{nj}^{t, mh} \left[ Y_{mh}^{t+2} \right]^{\beta^t}} ,$$

(53)

value function changes ($N(J+1)$ equations)$^{45}$:

$$Y_{nj}^{t+1} = \sum_{i=1}^N \sum_{k=0}^J \left[ \hat{w}_{nj}^{t+1} / \hat{P}_{nj}^{t+1} \right]^{1/x_t^t} \mu_{nj,ik}^{t+1} \left[ Y_{ik}^{t+2} \right]^{\beta^t} ,$$

(54)

labor reallocation dynamics ($N(J+1)$ equations):

$$L_{nj}^{t+1} = \sum_{i=1}^N \sum_{k=0}^J \mu_{nj,ik}^{t+1} L_{ik}^{t+1} .$$

(55)

$^{45}$Note that, with an abuse of notation, for $j = 0$, $\left( \hat{w}_{nj}^{t+1} / \hat{P}_{nj}^{t+1} \right) = 1$. 47
Finally, labor market clearing has to hold in equilibrium ($NJ$ equations):

$$w_{nj}^{t+1} L_{nj}^{t+1} = \gamma_{nj}^n (1 - \xi^n) \sum_{i=1}^{N} \pi_{nj}^{ij, nj} X_{nj+1}^{ij},$$

(56)

where $\chi_{t+1} = \sum_{i=1}^{N} \sum_{k=1}^{J} \hat{r}_{ik}^{t+1} H^{ik}$. We also have that in equilibrium $\hat{w}_{nj}^{t+1} = \hat{w}_n^{t+1} = \hat{r}_{nj}^{t+1} = \hat{r}_n^{t+1}$ for all $n$ such that $\pi_{nj,nk} = 0$. This condition means that if there is free mobility across sectors in a given region $n$, factor prices will equalize across sectors in that region. In the context of our model, this happens in all countries outside the United States.

We take as given $L_{nj}^{0}$, $w_0^{nj} L_0^{nj}$, $\mu_{-1}^{nj, ik}$, $\pi_0^{nj, ij}$, $r_0 H_0^{nj}$, $\chi_0$, for all $n, j, i, k$. We need to find $\hat{w}_{nj}^{t+1}$ ($((NJ) \times t)$, where the term in parentheses denotes the number of elements, $\hat{r}_{nj}^{t+1}$ ($((NJ) \times t)$, $\hat{P}_{t+1}^{nj}$ ($((NJ) \times t)$, $\pi_{t+1}^{nj, ij}$ ($((N^2 J) \times t)$, $X_{t+1}^{nj}$ ($((NJ) \times t)$, $\mu_{t+1}^{nj, ik}$ ($((N^2 (J+1)^2) \times t)$, $Y_{t+1}^{nj}$ ($((N(J+1)) \times t)$), and $L_{t+1}^{nj}$ ($((N(J+1)) \times t)$ for all $t > 0$. 

48
APPENDIX 5 : SOLUTION ALGORITHM

Part I: Solving for the sequential competitive equilibrium

The strategy to solve the model under the assumption that there are no policy changes in the economy from period 0 onward and given an initial value for the migration shares ($\mu_{-1}$) and an initial distribution of labor ($L_0$), is as follows$^{46}$:

1. Initiate the algorithm at $t = 0$ with a guess for the path of $\left\{Y_{t+1}^{n_j(0)}\right\}_{t=0}^T$, where the superscript (0) indicates it is a guess. The path should converge to $Y_{T+1}^{n_j(0)} = 1$ for a sufficiently large $T$.

   Take as given the set of initial conditions $L_0^{n_j}$, $\pi_0^{n_i,n_j}$, $w_0^{n_j}L_0^{n_j}$, $r_0^{n_j}H_0^{n_j}$.

2. For all $t \geq 0$, use $\left\{Y_{t+1}^{n_j(0)}\right\}_{t=0}^T$ and $\mu_{-1}^{n_j,i,k}$ to solve for the path of $\left\{\mu_t^{n_j,i,k}\right\}_{t=0}^T$ using equation (53).

3. Use the path for $\left\{\mu_t^{n_j,i,k}\right\}_{t=0}^T$ and $L_0^{n_j}$ to get the path for $\left\{L_{t+1}^{n_j}\right\}_{t=0}^T$ using equation (55).

4. Solving for the temporary equilibrium:
   (a) For each $t \geq 0$, given $\hat{L}_{t+1}^{n_j}$, guess a value for $\hat{w}_{t+1}^{n_j}$.
   (b) Obtain $\hat{x}_{t+1}^{n_j}$, $\hat{P}_{t+1}^{n_j}$, and $\pi_{t+1}^{n_j,i,j}$ using equations (49), (50), and (51).$^{47}$
   (c) Use $\pi_{t+1}^{n_j,i,j}$, $\hat{w}_{t+1}^{n_j}$, and $\hat{L}_{t+1}^{n_j}$ to get $X_{t+1}^{n_j}$ using equation (52).
   (d) Check if the labor market is in equilibrium in equation (56), and if not, go back to step (a) and adjust the initial guess for $\hat{w}_{t+1}^{n_j}$ until labor markets clear.
   (e) Repeat steps (a) through (d) for each period $t$ and obtain paths for $\left\{\hat{w}_{t+1}^{n_j}, \hat{P}_{t+1}^{n_j}\right\}_{t=0}^T$.

5. For each $t$, use $\mu_t^{n_j,i,k}$, $\hat{w}_{t+1}^{n_j}$, $\hat{P}_{t+1}^{n_j}$, and $Y_{t+2}^{n_j(0)}$ to solve backwards for $Y_{t+1}^{n_j(1)}$ using equation (54). This delivers a new path for $\left\{Y_{t+1}^{n_j(1)}\right\}_{t=0}^T$, where the superscript 1 indicates an updated value for $Y$.

6. Take the path for $\left\{Y_{t+1}^{n_j(1)}\right\}_{t=0}^T$ as the new set of initial conditions.

7. Check if $\left\{Y_{t+1}^{n_j(1)}\right\}_{t=0}^T \approx \left\{Y_{t+1}^{n_j(0)}\right\}_{t=0}^T$. If not, go back to step 1 and update the initial guess.

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$^{46}$If the economy is initially in a stationary equilibrium, then this step can be skipped.

$^{47}$Notice that $\hat{w}_{t}^{n_j} = \hat{w}_{t}^{n_i} = \hat{r}_{t}^{n_j} = \hat{r}_{t}^{n_i}$ for all $n$ such that $\tau^{n_j,n,k} = 0$, and $\hat{r}_{t}^{n_j} = \hat{w}_{t}^{n_i}\hat{L}_{t}^{n_j}$ or all $n$ such that $\tau^{n_j,n,k} \neq 0$. 

49
Part II: Counterfactuals

Take an exogenous change in policy $\hat{\Theta} = \Theta'/\Theta$. We assume that the change in policy occurs at time $t \geq 1$, but no change occurs at $t = 0$. The new policy path becomes known at time $t = 1$, but it was unanticipated beforehand. Define $y(\hat{\Theta})$ as the variable $y_t$ under the policy change $\hat{\Theta}$.

Given the timing assumption for the shocks, $L^j_0(\hat{\Theta}) = L^j_0$ since workers reallocate at the end of the period. Therefore, $w^j_0(\hat{\Theta}) = w^j_0$, $P^j_0(\hat{\Theta}) = P^j_0$, and $\pi^{n,j,ij}_0(\hat{\Theta}) = \pi^{n,j,ij}_0$. To solve for the counterfactual equilibrium, do the following:

1. Initiate the algorithm at $t = 0$ with a guess for the path of $Y^{n,j}_{t+1}(\hat{\Theta})$, where the superscript $(0)$ indicates it is a guess. Note that the sequence for $Y^{n,j}_{t+1}(\hat{\Theta})$ is given by

\[ Y^{n,j}_{nj}(\hat{\Theta}) = \exp(V^{n,j}_{nj}(\hat{\Theta}) - V^{n,j}_{nj}(0)) = \frac{\exp(V^{n,j}_{nj}(\hat{\Theta}) - V^{n,j}_{nj}(0))}{\exp(V^{n,j}_{nj}(0))}. \]

The path should converge to $Y^{n,j}_{T+1}(0)(\hat{\Theta}) = 1$ for a sufficiently large $T$. Take as given the set of initial conditions $L^j_0$, $\mu^{-1}_{-1}$, $\pi^{n,j,ij}_0$, $w^j_0 I^j_0$, $r^j_0 H^j_0$ and the solution to the sequential competitive equilibrium with no shocks computed previously.

2. For all $t \geq 0$, use $\left\{Y^{n,j}_{t+1}(\hat{\Theta})\right\}^T_{t=0}$ and $\mu^{-1}_{-1}$ to solve for the path of $\left\{\mu^{n,j,ik}_{t}(\hat{\Theta})\right\}^T_{t=0}$ using equations:

For period $t = 1$:

\[ \mu^{n,j,ik}_{1}(\hat{\Theta}) = \frac{\phi^{n,j,ik}_0 \left(Y^{n,j}_{2}(\hat{\Theta})\right)^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi^{n,j,mh}_0 \left(Y^{n,j}_{mh}(\hat{\Theta})\right)^{\beta}}, \]

where

\[ \phi^{n,j,ik}_0 = \mu^{-1}_{-1} \left(Y^{n,j}_{1}(\hat{\Theta}) \frac{Y^{n,j}_{1}(\hat{\Theta})}{\mu^{n,j,ik}_{1}(\hat{\Theta})}\right)^{\beta}. \]

Note that in the denominator we use the value of $Y^{n,j}_{1}$ from the economy with no policy change computed previously.

For period $t \geq 2$:

\[ \mu^{n,j,ik}_{t}(\hat{\Theta}) = \frac{\mu^{n,j,ik}_{t-1}(\hat{\Theta}) \left(Y^{n,j,ik}_{t+1}(\hat{\Theta})\right)^{\beta}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu^{n,j,mh}_{t-1}(\hat{\Theta}) \left(Y^{n,j,ik}_{t+1}(\hat{\Theta})\right)^{\beta}}. \]

3. Use the path for $\left\{\mu^{n,j,ik}_{t}(\hat{\Theta})\right\}^T_{t=0}$ and $L^j_0(\hat{\Theta})$ to get the path for $\left\{L^j_0(\hat{\Theta})\right\}^T_{t=0}$ using the
4. Solving for the temporary equilibrium

(a) For each \( t \geq 0 \), given \( \hat{L}^n_{t+1}(\hat{\Theta}) \), guess a value for \( \hat{w}^n_{t+1}(\hat{\Theta}) \).

(b) Obtain \( \hat{\pi}^{n,j}_{t+1}(\hat{\Theta}), \hat{w}^n_{t+1}(\hat{\Theta}), \) and \( \hat{I}^n_{t+1}(\hat{\Theta}) \) using equations (49), (50), and (51).

(c) Use \( \pi^{n,j}_{t+1}(\hat{\Theta}), \hat{w}^n_{t+1}(\hat{\Theta}), \) and \( \hat{I}^n_{t+1}(\hat{\Theta}) \) to get \( X^n_{t+1}(\hat{\Theta}) \) using equation (52).

(d) Check if the labor market is in equilibrium in equation (56), and if not go back to step (a) and adjust the initial guess for \( \hat{w}^n_{t+1}(\hat{\Theta}) \) until labor markets clear.

(e) Repeat steps (a) though (d) for each period \( t \) and obtain paths for \( \{\hat{w}^n_{t+1}(\hat{\Theta}), \hat{I}^n_{t+1}(\hat{\Theta})\}^T_{t=0} \).

5. For each \( t \), use \( \mu_t^{n,j,ik}(\hat{\Theta}), \hat{w}^n_{t}(\hat{\Theta}), \hat{P}^n_{t}(\hat{\Theta}), \) and \( Y^{n,j(0)}_{t+2}(\hat{\Theta}) \) to solve for backwards \( Y^{n,j(1)}_{t+1}(\hat{\Theta}) \) using equations:

For periods \( t \) where \( t \geq 2 \)

\[
Y^{n,j(1)}_{t}(\hat{\Theta}) = \left(\frac{\hat{w}^n_{t}(\hat{\Theta})}{\hat{P}^n_{t}(\hat{\Theta})}\right)^{1/\nu} \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \mu^{n,j,ik}_{t-1}(\hat{\Theta}) \left(Y^{n,j(0)}_{t+1}(\hat{\Theta})\right)^{\beta} \right].
\]

For period 1:

\[
Y^{n,j(1)}_{1}(\hat{\Theta}) = \left(\frac{\hat{w}^n_{1}(\hat{\Theta})}{\hat{P}^n_{1}(\hat{\Theta})}\right)^{1/\nu} \left[ \sum_{m=1}^{N} \sum_{h=0}^{J} \mu^{n,j,ik}_{0}(\hat{\Theta}) \left(Y^{n,j(0)}_{2}(\hat{\Theta})\right)^{\beta} \right].
\]

This delivers a new path for \( \{Y^{n,j(1)}_{t+1}(\hat{\Theta})\} \), where the superscript 1 indicates an updated value for \( Y \).

6. Take the path for \( \{Y^{n,j(1)}_{t+1}(\hat{\Theta})\} \) as the new set of initial conditions.

7. Check if \( \{Y^{n,j(1)}_{t+1}(\hat{\Theta})\} \simeq \{Y^{n,j(0)}_{t+1}(\hat{\Theta})\} \). If not, go back to step 1 and update the initial guess.

\[48\text{Note that } L^n_t(\hat{\Theta}) = L^n_{t+1}(\hat{\Theta}) \text{ given our timing assumptions.}\]
APPENDIX 6: DATA AND ESTIMATION

List of sectors and countries We calibrate the model to the 50 U.S. states, 37 other countries including a constructed rest of the world, and a total of 22 sectors classified according to the North American Industry Classification System (NAICS) for the year 2000. The list includes 12 manufacturing sectors, 8 service sectors, wholesale and retail trade, and the construction sector. Our selection of the number of sectors and countries was guided by the maximum level of disaggregation at which we were able to collect the production and trade data needed to compute our model. The 12 manufacturing sectors are Food, Beverage, and Tobacco Products (NAICS 311–312); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (NAICS 313–316); Wood Products, Paper, Printing, and Related Support Activities (NAICS 321–323); Petroleum and Coal Products (NAICS 324); Chemical (NAICS 325); Plastics and Rubber Products (NAICS 326); Nonmetallic Mineral Products (NAICS 327); Primary Metal and Fabricated Metal Products (NAICS 331–332); Machinery (NAICS 333); Computer and Electronic Products, and Electrical Equipment and Appliance (NAICS 334–335); Transportation Equipment (NAICS 336); Furniture and Related Products, and Miscellaneous Manufacturing (NAICS 337–339). The 8 service sectors are Transport Services (NAICS 481-488); Information Services (NAICS 511–518); Finance and Insurance (NAICS 521–525); Real Estate (NAICS 531-533); Education (NAICS 61); Health Care (NAICS 621–624); Accommodation and Food Services (NAICS 721–722); Other Services (NAICS 493, 541, 55, 561, 562, 711–713, 811–814). We also include the Wholesale and Retail Trade sector (NAICS 42-45), and the Construction sector, as mentioned earlier.

The countries in addition to the United States are Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, Slovak Republic, Slovenia, South Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.

International trade, production, and input shares across countries International trade flows across sectors and the 38 countries including the United States for the year 2000, $X^i_{n;i}$, where $n, i$ are the 38 countries in our sample, are obtained from the World Input-Output Database (WIOD). The WIOD provides world input-output tables from 1995 onward. National input-output tables of 40 major countries in the world and a constructed rest of the world are linked through international trade statistics for 35 sectors. For three countries in the database, Luxembourg,
Malta, and Latvia, value added and/or gross output data were missing for some sectors; thus, we decided to aggregate these three countries with the constructed rest of the world, which gives us the 38 countries (37 countries and the United States) we used in the paper. From the world input-output table, we know total purchases made by a given country from any other country, including domestic sales, which gives us the bilateral trade flows.\footnote{In a few cases (12 of 30,118 observations), the bilateral trade flows have small negative values due to negative change in inventories. Most of these observations involve bilateral trade flows between the constructed rest of the world and some other countries, and in two cases, bilateral trade flows of Indonesia. We input zero trade flows when we observe these small negative bilateral trade flows that in any way represent a negligible portion of total trade.}

We construct the share of value added in gross output $\gamma_{nj}$, and the material input shares $\gamma_{nj,nk}$ across countries and sectors using data on value added, gross output data, and intermediate consumption from the WIOD.

The sectors, indexed by $ci$ for sector $i$ in the WIOD database, were mapped into our 22 sectors as follows: Food Products, Beverage, and Tobacco Products (c3); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4–c5); Wood Products, Paper, Printing, and Related Support Activities (c6–c7); Petroleum and Coal Products (c8); Chemical (c9); Plastics and Rubber Products (c10); Nonmetallic Mineral Products (c11); Primary Metal and Fabricated Metal Products (c12); Machinery (c13); Computer and Electronic Products, and Electrical Equipment and Appliance (c14); Transportation Equipment (c15); Furniture and Related Products, and Miscellaneous Manufacturing (c16); Construction (c18); Wholesale and Retail Trade (c19–c21); Transport Services (c23–c26); Information Services (c27); Finance and Insurance (c28); Real Estate (c29–c30); Education (c32); Health Care (c33); Accommodation and Food Services (c22); Other Services (c34).

Regional trade, production data, and input shares

Interregional Trade Flows The sectoral bilateral trade flows across the 50 U.S. states, $X_{n_i}^{0,j,i}$ for all $n, i = U.S. states$, were constructed by combining information from the WIOD database and the 2002 Commodity Flow Survey (CFS). From the WIOD database we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. We use information from the CFS for the year 2002, which is the closest available year to 2000, to compute the bilateral expenditure shares across U.S. states, as well as the share of each state in sectoral total expenditure. The CFS survey for the year 2002 tracks pairwise trade flows across all 50 U.S. states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). These commodities were mapped into our 22 NAICS sectors by using the CFS tables for the year 2007, which present such...
mapping. The 2007 CFS includes data tables that cross-tabulate establishments by their assigned NAICS code against commodities (SCTG) shipped by establishments within each of the NAICS codes. These tables allow for mapping of NAICS to SCTG and vice versa. Having constructed the bilateral trade flows for the NAICS sectors, we first compute how much of the total U.S. domestic sales in each sector is spent by each state. To do so, we multiply the total U.S. domestic sales in each sector by the expenditure share of each state in each sector. Then we compute how much of this sectoral expenditure by each state is spent on goods from each of the 50 U.S. states. We do so by applying the bilateral trade shares computed with the 2002 CFS to the regional total spending in each sector. The final product is a bilateral trade flows matrix for the 50 U.S. states across sectors, where the bilateral trade shares across U.S. states are the same as those in the 2002 CFS, and the total U.S. domestic sales match those from the WIOD for the year 2000.

Regional production data and input shares

We compute the share of value added in gross output \( \gamma^{n,i} \), and the material input shares \( \gamma^{n,i,m,nk} \) for all \( n, i = \text{U.S. states} \), for each state and sector in the United States for the year 2000, using data on value added, gross output, and intermediate consumption. We obtain data on sectoral and regional value added from the Bureau of Economic Analysis (BEA). Value added for each of the 50 U.S. states and 22 sectors is obtained from the Bureau of Economic Analysis (BEA) by subtracting taxes and subsidies from GDP data. Gross outputs for the U.S. states in the 12 manufacturing sectors are computed from our constructed bilateral trade flows matrix as the sum of domestic sales and total exports.\(^{50}\) With the value-added data and gross output data for all U.S. states and sectors, we compute the share of value added in gross output \( \gamma^{n,i} \). For the eight service sectors, the wholesale and retail trade sector, and the construction sector, we have only the aggregate U.S. gross output computed from the WIOD database; thus, we assume that the share of value added in gross output is constant across states and equal to the national share of value added in gross output; that is, \( \gamma^{n,i} = \gamma^{US,j} \) for each nonmanufacturing sector \( j \), and \( n = \text{U.S. states} \).

While material input shares are available by sector at the country level, they are not disaggregated by state in the WIOD database. We assume therefore that the share of materials in total intermediate consumption varies across sectors but not across regions. Note, however, that the material-input shares in gross output are still sector and region specific as the share of total material expenditure in gross output varies by sector and region.\(^{50}\)

\(^{50}\) In a few cases (34 observations), gross output was determined to be a bit smaller than value added (probably due to some small discrepancies between trade and production data—for instance, a few missing trade shipments in the CFS database); in these cases we constrain value added to be equal to gross output.
Trade between U.S. states and the rest of the world. The bilateral trade flows between each U.S. state and the rest of the countries in our sample were computed as follows. In our paper, local labor markets have different exposure to international trade shocks because there is substantial geographic variation in industry specialization. Labor markets that are more important in the production in a given industry should react more to international trade shocks in that industry. Therefore, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, following ADH, we assume that the share of each state in total U.S. trade with any country in the world in each sector is determined by the regional share of total employment in that industry. The employment shares used to compute the bilateral trade shares between the U.S. states and the rest of the countries are constructed using employment data across sectors and states from the BEA.\textsuperscript{51} Using this procedure, we obtain $X_{nj;ij}^0$ for all $n = \text{U.S. states}$, $i \neq \text{U.S. states}$, and $n \neq \text{U.S. states}$, $i = \text{U.S. states}$.

Bilateral trade shares Having obtained the bilateral trade flows $X_{nj;ij}^0$ for all $n, i$, we construct the bilateral trade shares $\pi_{n;j;ij}^0$ as $\pi_{n;j;ij}^0 = X_{nj;ij}^0 / \sum_{m=1}^{N} X_{nj;mj}^0$.

Share of final goods expenditure The share of income spent on goods from different sectors is calculated as follows,

$$\alpha^j = \frac{\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma_{nk,nj} \sum_{j=1}^{N} \pi_{ik,nk} X_{ik}^{nk} \sum_{n=1}^{N} w_{nk} L_{nk} + \sum_{n=1}^{N} \nu_{n} \chi}{\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma_{nk,nj} \sum_{j=1}^{N} \pi_{ik,nk} X_{ik}^{nk}}$$

where $\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma_{nk,nj} \sum_{j=1}^{N} \pi_{ik,nk} X_{ik}^{nk}$ denotes total spending in intermediate goods across all countries and regions, and $\sum_{n=1}^{N} \sum_{k=1}^{J} w_{nk} L_{nk} + \sum_{n=1}^{N} \nu_{n} \chi$ is the total world income.

Share of labor compensation in value added Disaggregated data on labor compensation are generally very incomplete. Therefore, we compute the share of labor compensation in value added, $1 - \xi^i$, at the national level and assume that it is constant across sectors. For the United States, data on labor compensation and value added for each state for the year 2000 are obtained from the BEA. For the rest of the countries, data are obtained from the OECD input-output table for 2000 or the closest year. For India, Cyprus, and the constructed rest of the world, labor compensation data were not available. In these cases, we input the median share across all countries from the other 34 countries that are part of the rest of the world.

\textsuperscript{51}In 22 cases, data are missing, and in these cases we search for employment data in the closest available year. Still, in three cases (Alaska in the plastics and rubber industry, and North Dakota and Vermont in the petroleum and coal industry, we could not find employment data) thus, we input zero employment. The 19 cases in which we find employment data in years different from 2000 represent in total less than 0.01% of U.S. employment in 2000.
The initial labor mobility matrix and the initial distribution of labor To determine the initial distribution of workers in the year 2000 by U.S. states and sectors (and unemployment), we use the 5% Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. As we mentioned before, information on industry is classified according to the NAICS, which we aggregate to our 22 sectors and unemployment. We restrict the sample to people between 25 and 65 years of age who are either unemployed or employed in one of the sectors included in the analysis. Our sample contains over 5 million observations.

We combine information from the PUMS of the American Community Survey (ACS) and the Current Population Survey (CPS) to construct the initial matrix of quarterly mobility across our states and sectors ($\mu_{-1}$). Our goal is to construct a transition matrix describing how individuals move between state-sector pairs from one quarter to the next (from $t$ to $t+1$). The ACS has partial information on this; in particular, the ACS asks people about their current state and industry (or unemployment) and the state in which they lived during the previous year. We use the year 2001 since this is the first year for which data on interstate mobility at a yearly frequency are available. After selecting the sample as we did before in terms of age range and the industries in our analysis, we have almost 450 thousand observations. We find that around 2% of the U.S. population moves across states in a year in this time period. Unfortunately, the ACS does not have information on workers’ past employment status or the industry in which people worked during the previous period, so we resort to other data for this information.

We use the PUMS from the monthly CPS to obtain information on past industry of employment (or unemployment) at the quarterly frequency. The main advantages of the CPS are that it is the source of official labor market statistics and has a relatively large sample size at a monthly frequency. In the CPS, individuals living at the same address can be followed month to month for a small number of periods. We match individuals surveyed three months apart and compute their employment or unemployment status and work industry, accounting for any change between interviews as a transition. The main limitation with the CPS is that individuals who move to

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52. While unemployment in the Census is defined similarly to the Current Population Survey (CPS), design and methodological differences in the Census tend to overestimate the number of unemployed workers relative to the CPS.

53. The ACS interviews provide a representative sample of the U.S. population for every year since 2000. For the year 2001, the sample consists of 0.5% of the U.S. population. The survey is mandatory and is a complement to the decennial Census.

54. The 2000 Census asked people about the state in which they lived 5 years before but not the previous year; thus, we do not use the Census data despite the much larger sample.

55. In particular, the CPS collects information on all individuals at the same address for four consecutive months, stops for eight months, and then surveys them again for another four months.

56. We observe individuals three months apart using, on the one hand, their first and fourth interviews, and on the other, their fifth and eighth interviews.
a different residence, which of course includes interstate moves, cannot be matched. Our 3-month
match rate is close to 90%. As the monthly CPS does not have information on interstate moves,
we use this information to compute the industry and unemployment transitions within each state—
that is, a set of 50 transition matrices, each with $23 \times 23$. After restricting the sample as discussed
earlier, in any given month we have around 12,000 observations for the entire United States. To
more precisely estimate the transitions, we use all months from October 1998 to September 2001,
leading to a sample of over 400,000 matched records. Since the CPS uses the Standard Industry
Classification, we translate this into NAICS, using the crosswalk in Table 3.

Table 2 summarizes the information used to construct a quarterly transition matrix across state,
industry, and unemployment. The letter $x$ in the table denotes information available in the matched
CPS, and the letter $y$ denotes information available in the ACS. The information missing from the
above discussion is the past industry history of interstate movers. To have a full transition matrix,
we assume that workers who move across states and are in the second period in state $i$ and sector
$j$ have a past industry history similar to workers who did not switch states and are in the second
period in state $i$ and sector $j$.\footnote{Mechanically, we distribute the interstate movers according to the intersectoral mobility matrix for the state in which they currently live.}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{State A} & \textbf{State B} \\
\hline
\textbf{Ind 1} & \textbf{x} & \textbf{x} & \textbf{x} & \textbf{x} \\
\textbf{Ind 2} & \textbf{x} & \textbf{x} & \textbf{x} & \textbf{x} \\
\textbf{Ind J} & \textbf{x} & \textbf{x} & \textbf{x} & \textbf{x} \\
\hline
\textbf{Total} & \textbf{y} & \textbf{y} & \textbf{y} & \textbf{y} \\
\hline
\end{tabular}
\caption{Information Available on ACS and CPS}
\end{table}

As mentioned earlier, information on interstate mobility in the ACS is for moves over the year.
To calculate quarterly mobility we assume that interstate moves are evenly distributed over the year
and we rule out more than one interstate move per year. In this case, our adjustment consists of
keeping only one-fourth of these interstate moves and imputing three-fourths as non-moves. After
this correction, we impute the past industry history for people with interstate moves from state $i$ to

\footnote{Mortality, residence change, and nonresponse rates are the main drivers of the 10% mismatch rate.}
state $n$ and industry $j$ according to the intrastate sectoral transition matrix for state $n$ conditional on industry $j$.

Our computed value for the initial labor transition matrix is consistent with aggregate magnitudes of interstate and industry mobility for the yearly frequency estimated in Molloy et al. (2011) and Kamborouf and Manovskii (2008). We obtain a mobility transition matrix with over 1.3 million elements.\textsuperscript{59}

**Predicting import changes from China** To identify the China shock, we use the international trade data from ADH.\textsuperscript{60} Specifically, we use data measuring the value of trade between several countries from 1991-2007. ADH retrieve these data from the UN Comrade Database and concord them from six-digit Harmonized System (HS) product codes to a 1987 Standard Industrial Classification (SIC) manufacturing industry code scheme.\textsuperscript{61} Their scheme is essentially the same as the SIC 1987 classification scheme, except for a few four-digit industries that did not map directly from the HS-codes. These industries are aggregated into other four-digit industry codes so that each of the ADH’s resulting 397 industries maps directly from a HS trade code.\textsuperscript{62} Once the data are in this SIC 1987 structure, the authors deflate the import values into real 2007 US dollars using the personal consumption expenditure deflator and aggregate the country-level data into importing and exporting regions. The final data are reported over two importing regions (the United States and an aggregate of eight other developed countries —namely, Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland— and four exporting regions (China and other low-income countries). For our purposes, we use the two data series that measure imports from China by the United States, and imports from China by the other advanced economies.

To make these data comparable with the rest of our analysis, we developed a crosswalk to map the data from ADH’s SIC coding into our NAICS sectors. Because their SIC codes include only manufacturing industries, they only intersect with 13 of our 22 NAICS sectors —our 12 manufacturing sectors and also the information and communications sector.\textsuperscript{63} Table 3 shows the exact mapping between the two industry schemes. The SIC 1987 codes are a hierarchical system,

\textsuperscript{59}With the exception of one element, all zero transitions occur out of the diagonal. In fact, the diagonal of the matrix typically accumulates the largest probability transition values, which just reflects the fact that staying in your current labor market is a high probability event. However, we do find that one of the estimated transition probabilities in the diagonal is zero. Only in this case we replace this value with the minimum value of the other elements in the diagonal and re-normalize such that the conditional transition probabilities add to one.

\textsuperscript{60}The data for their analysis is publicly available on David Dorn’s website http://www.ddorn.net/data.htm.

\textsuperscript{61}For more details about this crosswalk, see ADH’s Online Data and Theory Appendix.

\textsuperscript{62}Details about the industry coding scheme (referred to as sic87dd by the authors) can be found on Dorn’s website.

\textsuperscript{63}Because of the different definitions between SIC and NAICS, some industries classified as manufacturing in SIC are now part of the information and communications sector in NAICS. The value of imports for these industries is very small and we drop them from our calculations.
where the first two numbers represent the broader groups, and as extra digits are added the industry becomes more narrowly defined. Many of the SIC codes matched our sectors on the two-digit level, in other words, the broad groups were the same.

After this redefinition of sectors, we compute the changes in the level of imports from China between 2000 and 2007 by the United States and the other advanced economies. The change in U.S. imports from China during this period can, in part, be the result of domestic U.S. shocks, but we are looking for a measure of changes in imports that are mostly the result of shocks that originate in China. Inspired by ADH’s instrumental variable strategy, we run the following regression

\[ \Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j, \]

where \( j \) is one of our 12 manufacturing sectors, and \( \Delta M_{USA,j} \) and \( \Delta M_{other,j} \) are the changes in real U.S. imports from China and imports by the other advanced economies from China between 2000 and 2007.

The coefficient of the regression is estimated \( a_2 = 1.27 \) with a robust standard error of 0.011.

This regression is related to the first-stage regression in AHD’s two-stage least square estimation. Using this result we construct the changes in U.S. imports from China for each industry that are predicted by the change in imports in other advanced economies from China. We then calibrate, within our model, the changes in the 12 manufacturing TFP measures in China that match the predicted changes in U.S. imports for the 12 manufacturing sectors.

Table 3: Concordance SIC87dd - NAICS

<table>
<thead>
<tr>
<th>NAICS</th>
<th>NAICS Sector Description</th>
<th>SIC87dd Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Food, Beverage, and Tobbaco Products</td>
<td>20**, 21**</td>
</tr>
<tr>
<td>2</td>
<td>Textiles and Apparel Products</td>
<td>22**, 23**, 31**</td>
</tr>
<tr>
<td>3</td>
<td>Wood, Paper, Printing and Related Products</td>
<td>24** exc. 241*, 26**, 274*-279*</td>
</tr>
<tr>
<td>4</td>
<td>Petroleum and Coal Products</td>
<td>29**</td>
</tr>
<tr>
<td>5</td>
<td>Chemical</td>
<td>28**</td>
</tr>
<tr>
<td>6</td>
<td>Plastics and Rubber Products</td>
<td>30**</td>
</tr>
<tr>
<td>7</td>
<td>Nonmetallic Mineral Products</td>
<td>32**</td>
</tr>
<tr>
<td>8</td>
<td>Primary and Fabricated Metal Products</td>
<td>33**, 34**</td>
</tr>
<tr>
<td>9</td>
<td>Machinery</td>
<td>351*-356*, 3578-3599</td>
</tr>
<tr>
<td>10</td>
<td>Computer, Electrical, and Appliance</td>
<td>3571-3577, 365*-366*, 3812-3826, 3829, 386*-387*, 361*-364*, 367*-369*</td>
</tr>
<tr>
<td>11</td>
<td>Transportation Equipment</td>
<td>37**</td>
</tr>
<tr>
<td>12</td>
<td>Furniture and Miscellaneous Products</td>
<td>25**, 3827, 384*-385*, 39**</td>
</tr>
<tr>
<td>16</td>
<td>Information and Communication</td>
<td>271*-273*</td>
</tr>
</tbody>
</table>

Note: an entire broad group was mapped into the NAICS sector by substituting the last one or two digits with an asterisk. All intervals listed in the table are inclusive.
Reduced-form analysis  In the previous paragraphs we described how we followed ADH to compute the change in U.S. imports from China. We now take one step forward and reproduce some of the results in ADH but under our definition of labor market and under our sample selection criteria.\footnote{That is, we use U.S. states instead of commuting zones, and we use 12 manufacturing sectors classified by NAICS instead of their 397 SIC manufacturing industries that they use. Moreover, we restrict the sample to people within ages 25 to 65 that are in the labor force, while ADH use people 16 to 64 that worked the previous year.}

We follow the same methodology as ADH to impute the U.S. total imports to state-industry units, except where ADH used commuting zones and SIC codes we use states and our 12 manufacturing sectors. Total U.S. manufacturing imports are allocated to states by weighting total imports according to the number of employees in a certain local industry relative to the total national employment. Following the example of ADH, we use County Business Patterns (CBP) data for the year 2000 from the census bureau to measure local industry employment. The CBP is a county-level, annual data set that provides details on local firm-level employment by industry. The data is compiled from the Census Bureau’s Business Register, and includes almost all employment at known companies.

To avoid giving away identifiable information about specific firms, the census bureau will sometimes report county-industry level data in an interval instead of one point. ADH establish a methodology of imputing employment within these intervals, which we follow to get the most accurate estimate of local industry employment. ADH start by using the employment distribution of known firms within a particular size interval and the aggregated employment in a firm’s industry to narrow the employment interval. Once the possibility of values is narrowed, they set employment to the midpoint of the bracket and run a regression using a sample of similar firms. Finally, they add up and proportionally adjust the imputed numbers based on the aggregate employment in that industry.\footnote{For more details on the imputation process, see the ADH online data dictionary.}

To actually perform the imputations we use ADH’s publicly available code, and only adapt a few lines at the end that aggregate employment to state-sector levels instead of commuting zone-industry levels.

Once we have the 12-sector state-level industry employment data, we allocate the national import data to the worker level using the following formula proposed by ADH (see their equation 3):

$$\Delta IPW_{uit} = \sum_j \frac{L_{ijt}}{L_{ujt}} \frac{\Delta M_{ucjt}}{L_{it}}.$$  

The expression above states that the change in U.S. imports per worker from china is defined based on each state’s industry employment structure in the starting year. Following ADH’s no-
tation, \( L_{it} \) is the total employment at state \( i \) at time \( t \), \( j \) represents one of our 12 manufacturing sectors, and the \( u \) stands for a U.S. related variable (as opposed to a variable constructed using other countries imports, for which they use an \( o \)). For example, \( \Delta M_{ucjt} \) means the change in U.S. imports from China for industry \( j \) time \( t \).\(^{66}\)

We also followed ADH in constructing our dependent variable: the change in local manufacturing employment as a share of the working age population. Data for local manufacturing employment comes from the 2000 census 5% PUMS and from the 2006, 2007, and 2008 ACS 1% PUMS. To make the data samples more comparable, we followed ADH in pooling 2006-2008 ACS samples together and treating them all as 2007. Both the census and ACS data come from the Minnesota IPUMS service. Industry data from these sources is originally coded according to census industry codes under a NAICS classification that we aggregate to our 22 NAICS sectors. As in our study, we restrict the sample to those individuals between ages 25 to 65 that are either employed or unemployed.\(^ {67}\)

As a last step, we augment the microdata weights by multiplying the PUMS sampling weights with the ADH labor weight (see data ADH Data appendix for details). We finish by collapsing the data to the state-level and taking the difference in the share of manufacturing labor as a percent of the labor force (ages 25 to 65) between 2000 and 2007. We use the constructed variables to run a regression relating the change in local manufacturing employment from 2000 to 2007 (\( \Delta L^m_{it} \)) to the change in imports per worker (\( \Delta IPW_{uit} \)):

\[
\Delta L^m_{it} = b_1 + b_2 \Delta IPW_{uit} + e_{it}
\]

In this regression the unit of observation is a U.S. state. We include D.C. as a state but exclude Hawaii and Alaska since they are not part of ADH analysis. As in ADH, we perform a Two Stage Least Squares regression instrumenting \( \Delta IPW_{uit} \) with \( \Delta IPW_{oit} \), which is other advanced economies’ change in imports from China per worker.\(^ {68}\)

In addition, we also run the following regression,

\[
\Delta \bar{u}_{it} = c_1 + c_2 \Delta IPW_{uit} + e_{it}
\]

where \( \bar{u}_{it} \) is the change in the unemployment rate of state \( i \) for the age groups in our sample. ADH

\(^{66}\)In ADH’s this equation varied over commuting zones (\( i \)) and SIC industries (\( j \)).

\(^{67}\)ADH restrict the sample to those individuals ages 16 to 64 who had worked in the past year and were not institutionalized.

\(^{68}\)Note that, as in ADH, the formula for \( \Delta IPW_{oit} \) contains the imports from other advanced economies, but the employment of the different U.S. states and sectors. We calibrated our model with data on other countries from the WIOD. Unfortunately, the WIOD does not contain data from New Zealand and Switzerland. Therefore, our definition of other advanced economies uses data from Australia, Denmark, Finland, Germany, Japan, and Spain. Thus, we only use these 6 countries instead of the 8 used by ADH.
perform a similar regression in their Table 5. Once again, we perform the same type of regression but using our definitions and time period and do not have additional controls in the regression.

Table 4: Reduced-form regression results

<table>
<thead>
<tr>
<th></th>
<th>(\Delta L_{it}^m)</th>
<th>(\Delta u_{it})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>data</td>
<td>model</td>
</tr>
<tr>
<td>(\Delta IPW_{uit})</td>
<td>-1.718</td>
<td>-1.124</td>
</tr>
<tr>
<td></td>
<td>(0.194)</td>
<td>(0.368)</td>
</tr>
<tr>
<td>Obs</td>
<td>49</td>
<td>50</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.51</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Results from Two Stage Least Squares using \(IPW_{oit}\) (imports of other advanced economies per worker) as instrument. Regressions in columns 1 and 2 have the change in the share of manufacturing employment as the dependent variable and regressions in columns 3 and 4 have the change in the share of the population unemployed as the dependent variable. Data stands for the regression using observed data and model stands for the same regression using model generated data given our counterfactual experiment. Changes are between 2000 to 2007. Estimated standard error in parentheses. Model includes the 50 U.S. states, where D.C. has been merged to Virgina. Data include the 48 U.S. continental states and D.C. as a separate state. All regressions include a constant but no other controls. Results slightly differ from ADH due to different time periods, the use of additional controls in the regression, the definition of geographic area and industries used, and sample selection criteria.

Table 4 presents the results. As in ADH, we find that the change in \(IPW_{uit}\), negatively affects the share of employment in manufactures and positively affects unemployment. Our estimates of \(b_2\) are \(-1.71\) with a robust standard error of 0.19.\(^{69}\) The regression results in columns (1) and (3) are somewhat different from those reported by ADH. Our reduced-form results using our data are largely aligned with theirs, both in terms of the sign and significance. The differences stem from the different time periods we use (we use only changes between 2000 to 2007 while in several of ADH’s specifications they use 1990 to 2007), the use of additional controls in the regressions, the definition of geographic areas and industries (we use U.S. states and NAICS sectors), and sample selection criteria (population ages and labor force).

In columns (2) and (4), we run the same type of regressions but with model generated data. The

\(^{69}\) Using ADH’s codes and data we are able to replicate their results exactly. We are particularly interested in their estimates of column 2 of their Table 2, which under their definitions of commuting zones and SIC industries delivers \(b_2 = -0.72\) with their codes and data. Unfortunately, we cannot directly use their data to aggregate to our definitions of sectors and U.S. states. We obtained the data from the original sources and followed ADH’s steps. With this data and under their definitions of commuting zones, SIC industries and sample selection, we estimate \(b_2 = -0.8\) and significant. Keeping their definitions of SIC industries and sample selection but using U.S. states instead of commuting zones, we estimate \(b_2 = -0.97\) and significant. On the other hand, keeping their commuting zones and sample selection but aggregating industries to our 12 NAICS sectors we estimate \(b_2 = -1.07\) and significant. Finally, changing both the geographic and industry definitions to ours, but keeping their sample selection criteria we find \(b_2 = -1.51\) and significant. Thus, the differences in the definitions that we use tend to amplify the estimated coefficient relative to theirs.
coefficients we estimate with the model generated data are close to those estimated with actual
data, displaying the same sign and significance. Our estimate of the effects of Chinese import
penetration on unemployment is positive, as in ADH. However, this is a relative effect. States with
a relatively higher import penetration will tend to have a relatively higher unemployment rate.
However, we know from our model that unemployment tends to fall on average on almost all states.
APPENDIX 7: WELFARE

1. The Option Value and Welfare Equations

In this appendix, we derive equation (14). The lifetime utility of being in a particular market is given by

$$v_{nj}^t = \log C_{nj}^t + \max_{\{ik\}_{i=1,k=0}^{N,J}} \left\{ \beta E \left[ v_{ik}^{t+1} \right] - \tau_{nj,ik} + \nu e_i^t \right\}. \quad (57)$$

The first term is the period utility in market $nj$ at time $t$, and the second term captures the value of staying in that labor market, and the third term is the option value. As we showed in Appendix 1, taking the expected value of this equation, we can write the expected lifetime utility of being at market $nj$ at time $t$ as

$$V_{nj}^t = \log C_{nj}^t + \beta V_{nj}^{t+1} + \nu \log \left[ \sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left( \beta \left( V_{ik}^{t+1} - V_{nj}^{t+1} \right) - \tau_{nj,ik} \right)^{1/\nu} \right], \quad (58)$$

where the second term on the right hand side of equation (58) is the option value.

From equation (3) we know that

$$\mu_{t,nj} = \frac{\exp \left( \beta V_{nj}^{t+1} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta V_{mh}^{t+1} - \tau_{nj,mh} \right)^{1/\nu}},$$

and therefore the option value is given by

$$\nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left( \beta \left( V_{mh}^{t+1} - V_{nj}^{t+1} \right) - \tau_{nj,mh} \right)^{1/\nu} = -\nu \log \mu_{t,nj}. \quad (59)$$

Plugging this equation into the value function, we get

$$V_{nj}^t = \log C_{nj}^t + \beta V_{nj}^{t+1} - \nu \log \mu_{t,nj}.$$  

Finally, iterating this equation forward we obtain

$$V_{nj}^t = \sum_{s=t}^{\infty} \beta^{s-t} \log C_{nj}^s - \nu \sum_{s=t}^{\infty} \beta^{s-t} \log \mu_{s,nj}. \quad (60)$$

The expected lifetime utilities with and without change in fundamentals are given by,

$$V_{nj}^t(\Theta) = \sum_{s=t}^{\infty} \beta^{s-t} \log \left( \frac{C_{nj}^s(\Theta)}{(\mu_{s,nj}(\Theta))^{\nu}} \right) \quad (61)$$

$$V_{nj}^t = \sum_{s=t}^{\infty} \beta^{s-t} \log \left( \frac{C_{nj}^s}{(\mu_{s,nj})^{\nu}} \right) \quad (62)$$

64
We define the compensating variation in consumption for market \( nj \) at time \( t \) to be the scalar \( \delta_{nj}^{nt} \) such that

\[
V_{nt}^{nj}(\hat{\Theta}) = \sum_{s=t}^{\infty} \beta^{s-t} \log \left( \frac{C_{s}^{nj}}{(\mu_{s}^{nj,nj})^{\nu}} \delta_{nj}^{nt} \right) \tag{63}
\]

Re-arranging we have,

\[
\log \delta_{nj}^{nt} = (1 - \beta) \sum_{s=t}^{\infty} \beta^{s-1} \log \left( \frac{C_{s}^{nj}(\hat{\Theta})/C_{s}^{nj}}{(\mu_{s}^{nj,nj}(\hat{\Theta})/\mu_{s}^{nj,nj})^{\nu}} \right) \tag{64}
\]

which is our measured of consumption equivalent change in welfare in equation (14).

2. Welfare Effects of Changes in Fundamentals

In this section, we discuss the welfare effects from changes in fundamentals in our economy. To fix ideas, let \( V_{nt}^{nj}(\hat{\Theta}) \) be the present discounted value of utility at time \( t \) in market \( nj \) under the change in fundamentals \( \hat{\Theta} \), and denote by \( V_{nt}^{nj} \) to the same object without changes in fundamentals. The change in present discounted value from a change in fundamentals \( V_{nt}^{nj}(\hat{\Theta}) - V_{nt}^{nj} \) is given by

\[
V_{nt}^{nj}(\hat{\Theta}) - V_{nt}^{nj} = \sum_{s=t}^{\infty} \beta^{s-1} \log \left( \frac{C_{s}^{nj}(\hat{\Theta})/C_{s}^{nj}}{(\mu_{s}^{nj,nj}(\hat{\Theta})/\mu_{s}^{nj,nj})^{\nu}} \right) \tag{65}
\]

The change in welfare in market \( nj \) from a change in fundamentals is given by the present discounted value of the expected change in real consumption, and the change in the option value. Equation (65) shows that the change in the option value is summarized by the change in the fraction of workers that do not reallocate, \( \hat{\mu}_{nt}^{nj,nj} \), and the variance of the taste shocks \( \nu \). The intuition is that higher \( \hat{\mu}_{nt}^{nj,nj} \) means that fewer workers in market \( nj \) move to a market with higher expected value. Notice that if the cost of moving to a different labor market is infinite, then \( \hat{\mu}_{nt}^{nj,nj} = 1 \), and the option value is zero.

In our model, the change in real consumption in market \( nj \), \( C_{nt}^{nj}(\hat{\Theta})/C_{nt}^{nj} \) is given by the change in the real wage earned in that market, \( w_{nt}^{nj}/P_{nt} \), and can be expressed as\(^{70} \)

\[
C_{nt}^{nj}(\hat{\Theta})/C_{nt}^{nj} = \frac{w_{nt}^{nj}(\hat{\Theta})/w_{nt}^{nj}}{\prod_{k=1}^{J} (w_{nt}^{nk}(\hat{\Theta})/w_{nt}^{nk})^{\alpha^{k}}} \prod_{k=1}^{J} \left( \frac{w_{nt}^{nk}(\hat{\Theta})/w_{nt}^{nk}}{P_{nt}^{nk}(\hat{\Theta})/P_{nt}^{nk}} \right)^{\alpha^{k}} \tag{66}
\]

The first component denotes the unequal welfare effects for households working in different sectors within the same region \( n \); and reflects the fact that workers in sectors that pay higher wages have more purchasing power in that region. The second component is common to all households residing in region \( n \) and captures the change in the cost of living in that region. This second component

\(^{70} \hat{C}_{nt}^{n,0} = 1 \) if the household in region \( n \) at time \( t \) is unemployed.
is a measure of the change in the average real wage across labor markets in region \( n \), weighted by the importance of each sector in the consumption bundle, and it is shaped by several mechanisms in our model. Specifically,

\[
\prod_{k=1}^{J} \left( \frac{w_{t}^{nk}(\hat{\Theta})}{P_{t}^{nk}(\hat{\Theta})} \right)^{x_{nk}^{t}} = \sum_{k=1}^{J} \alpha_{k} \left( \log \left( \frac{\pi_{t}^{nk,nk}(\hat{\Theta})}{\pi_{t}^{nk,nk}} \right)^{-1/\theta} + \log \left( \frac{w_{t}^{nk}(\hat{\Theta})}{w_{t}^{nk}} \right) \right) \tag{67}
\]

The first term in equation (67) is the change in trade openness, \( \log \left( \frac{\pi_{t}^{nk,nk}(\hat{\Theta})}{\pi_{t}^{nk,nk}} \right) \), that give households in region \( n \) access to cheaper imported goods. The second term in equation (67) is the change in factor prices, \( \log \left( \frac{w_{t}^{nk}(\hat{\Theta})}{w_{t}^{nk}} \right) \) and captures the effects of migration, local factors, and intersectoral trade. To fix ideas, when we abstract from materials in the model, 

\[
\log \left( \frac{w_{t}^{nk}(\hat{\Theta})}{w_{t}^{nk}} \right) = -\xi_{n} \log \left( \frac{L_{t}^{nk}(\hat{\Theta})}{L_{t}^{nk}} \right) \left( \frac{H_{nk}(\hat{\Theta})}{H_{nk}} \right). \tag{67}
\]

Migration into region \( n \) may have a positive or negative effect on factor prices depending on how \( L_{t}^{nk} \) changes relative to the stock of structures \( H_{nk} \). In our model structures are in fixed supply, thus, migration has a negative effect on real wages because the inflow of workers strains local fixed factors and raises the relative price of structures and the cost of living in region \( n \). This is a congestion effect as in Caliendo et al. (2014).\(^{71}\) Finally, material inputs and input-output linkages impact welfare through changes in the cost of the input bundle as in Caliendo and Parro (2015).\(^{72}\)

In a one-sector model with no materials and structures, equation (67) reduces to

\[
V_{t}^{nj}(\hat{\Theta}) - V_{t}^{nj} = \sum_{s=t}^{\infty} \beta^{s} \log \left( \frac{\pi_{s}^{nn}(\hat{\Theta})}{\pi_{s}^{nn}} \right)^{-1/\theta} \left( \frac{\mu_{s}^{nn}(\hat{\Theta})}{\mu_{s}^{nn}} \right)^{\nu},
\]

which combines the welfare formulas in ACM (2010), and Arkolakis, Costinot, and Rodriguez Clare (2012).

\(^{71}\) Dix-Carneiro (2014) studies the impact of capital mobility on the reallocation of labor.

\(^{72}\) In the simpler model presented in Appendix 2, there are \( N \) labor markets indexed by \( \ell \), and households in location \( \ell \) consume local goods. In this setup, the welfare equation (65) takes the form \( W_{t}^{\ell}(\hat{\Theta}) = \sum_{s=t}^{\infty} \rho^{s} \log \left( \frac{\bar{x}_{s}^{\ell}/P_{t}^{\ell}}{\bar{x}_{s}^{\ell}} \right)^{\nu} \), and the change in real wages is given by \( \log(\bar{x}_{t}^{\ell}/\bar{x}_{t}^{\ell}) = -(1/\theta) \log(\hat{L}_{t}^{\ell}/\hat{H}_{t}^{\ell}) \).
APPENDIX 8: ADDITIONAL RESULTS

In this appendix, we present histograms showing the welfare effects and adjustment costs across labor in the manufacturing and nonmanufacturing sectors. The key findings in these figures is that welfare effects and adjustment costs are more heterogeneous in the manufacturing sectors than in the nonmanufacturing sectors.

**Fig. A8.1:** Welfare across labor markets (manufacturing)

![Histogram of welfare across labor markets (manufacturing)](image)

**Fig. A8.2:** Welfare across labor markets (nonmanufacturing sectors)

![Histogram of welfare across labor markets (nonmanufacturing sectors)](image)
Fig. A8.3: Adjustment costs (manufacturing)

Fig. A8.4: Adjustment costs (nonmanufacturing sectors)