Globally Correlated Nominal Fluctuations

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Abstract

Cyclical fluctuations in nominal variables—aggregate price levels and nominal interest rates—are documented to be substantially more synchronized across countries than cyclical fluctuations in real output. A transparent mechanism that can account for this striking feature of the nominal environment is highlighted. It is based on (small) cross-country spillovers of shocks and an interaction between Taylor rules and no-arbitrage conditions. The mechanism is quantitatively important for a wide range of plausible parameterizations and is found to be robust to modifications of the economic environment that help account for other important features of domestic and international aggregate fluctuations.

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1 Introduction

A growing empirical literature shows that national inflation rates contain a large ‘global’ component (Ciccarelli and Mojon, 2010; Mumtaz and Surico, forthcoming; Neely and Rapach, 2008). Put simply, in the last half a century or so movements in inflation rates have been highly synchronized across countries. A large literature has also established existence of the ‘world business cycle’—recurrent fluctuations in real economic activity that tend to coincide across countries—and characterized its salient features.\(^1\) Recently, a few studies have investigated how the two sides of international comovements, nominal and real, are empirically related to each other. Mumtaz, Simonelli, and Surico (2011) use a dynamic factor model to jointly identify a world cycle in both output growth and inflation. They find, among other things, that the ‘world’ factor generally contributes more to domestic inflation than output growth and that its importance in domestic inflation has grown over time. Wang and Wen (2007) document that cross-country correlations of inflation rates are much higher than cross-country correlations of cyclical movements in output. Both studies thus suggest that cross-country movements in the nominal environment are more synchronized than cross-country movements in real activity.

A part of the reason for the strong cross-country comovement of inflation uncovered by the literature is the similarity in inflation trends across industrialized economies, which occurred perhaps due to similar low-frequency changes in monetary policy: low inflation in the 1960s, high in the 1970s, gradually declining in the 1980s, and low again since the mid-1990s. Surprisingly though, as documented in the first part of this paper, fluctuations in the nominal environment are substantially more synchronized across countries than fluctuations in real activity even when we focus only at medium-term business-cycle frequencies; that is, after removing low-frequency movements from the data. To the extent that domestic monetary policy is more likely to be able to control the domestic nominal environment than the real economy, it is not at all obvious why this should be the case.\(^2\) Accounting for this feature of the data within a theoretical framework therefore seems important for our understanding of how domestic nominal variables are determined in an international environment—an issue that has recently concerned policy makers.\(^3\) In the second part of the paper we highlight a quantitatively important mechanism that can account for this empirical regularity, yet appears consistent with a number of other important aspects of domestic and international aggregate fluctuations, both nominal and

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\(^2\)In addition, as discussed below, quantitative-theoretical models had a difficulty generating high cross-country comovements of nominal variables.

\(^3\)See, for instance, Bean (2006), Bernanke (2007), Besley (2008), Mishkin (2007), and Sentance (2008).
real. The mechanism is based on cross-country spillovers of shocks and an interaction between Taylor-type rules and domestic no-arbitrage conditions.

We focus on two key nominal variables, the aggregate price level and the short-term nominal interest rate—the monetary policy instrument. Our empirical observations are based on a sample of the largest industrial economies.\(^4\) Using business cycle components—medium-term fluctuations in the data with periodicity of approximately 8 to 32 quarters—of aggregate price levels, short-term nominal interest rates, and real GDP obtained with a band-pass filter, we find that the fluctuations in the three variables are similar in terms of their volatility and persistence, but markedly different in terms of their cross-country comovements.\(^5\) Specifically, for the period 1960.Q1–2006.Q4 the average bilateral correlation of price levels is 0.52, that of short-term nominal interest rates 0.57, while that of real GDP is only 0.25. Moreover, the bilateral correlations of the two nominal variables vary substantially less across country pairs than those of real GDP. This empirical regularity is broadly robust to the inclusion of other economies as the required data become available, the exclusion of the Bretton Woods period of fixed exchange rates, the exclusion of commodity prices from price indexes, and to splitting the sample into two subsamples in 1984, the year generally associated with the start of the so-called “Great Moderation”—a period of relatively low output volatility and a greater focus by central banks on nominal stability. Furthermore, the findings are statistically significant.

A large literature argues that monetary policy of major central banks is reasonably well approximated by the so-called ‘Taylor rule’—a parsimonious feedback rule whereby the central bank sets the short-term nominal interest rate in response to movements in domestic output and changes in the domestic price level.\(^6\) The high cross-country correlations of short-term nominal interest rates can thus potentially be accounted for by the high cross-country correlations of prices. But in equilibrium, prices and nominal interest rates are jointly determined. How, then, do responses of national central banks to domestic economic conditions lead to substantially stronger cross-country comovements of the two nominal variables than of output?

\(^4\)Namely, Australia, Canada, Germany, Japan, the United Kingdom, and the United States for the period 1960.Q1–2006.Q4. In addition, from 1970.Q1 our sample includes also Austria and France.

\(^5\)Focusing on the price level allows us to remove low-frequency movements from the data with only one transformation. In contrast, filtering inflation leads to a double transformation of the price level, as inflation is already a transformation—first difference—of the data. In addition, it allows us to treat the price level in the same way as output, which in modern business cycle theory is usually filtered with either the Hodrick-Prescott or a band-pass filter to arrive at business-cycle frequencies.

The mechanism presented in the second part of the paper follows in equilibrium mainly from two, empirically plausible, assumptions: (i) Taylor rules provide a reasonable description of monetary policy in major industrialized economies and (ii) there are positive spillovers of shocks across countries. No-arbitrage conditions between returns on domestic nominal bonds and real capital (its marginal product) then provide a link between expected future states of the economy, monetary policy, and current prices and interest rates.

We start by examining the mechanism within a fairly parsimonious baseline model—a two-country economy where in each country a representative individual maximizes welfare subject to stylized representations of the national income and product account identities, as in Backus, Kehoe, and Kydland (1994) and Heathcote and Perri (2002). This is a useful benchmark for at least three reasons: (i) a large class of dynamic general equilibrium models in international macroeconomics are various extensions of this model; (ii) the mechanism in this model is quantitatively important; and (iii) the model allows a transparent description and understanding of the mechanism. The only shocks in this model are technology (i.e., total factor productivity, TFP) shocks. A number of studies document positive spillovers, although of different strength, of such shocks across countries (e.g., Backus et al., 1992; Heathcote and Perri, 2002; Rabanal, Rubio-Ramirez, and Tuesta, forthcoming). In addition, Ahmed, Ickes, Wang, and Yoo (1993) and Crucini, Kose, and Otrok (2011) provide empirical support that TFP shocks play a dominant role in driving the world business cycle.

We find that the mechanism is quantitatively important for even some of the smallest estimates of spillovers of such shocks found in the literature. In addition, the mechanism turns out to be quantitatively important for a broad range of plausible parameter values of the Taylor rule, shopping time technology, import share of GDP, and the elasticity of substitution between domestic and foreign goods in final expenditures. It is also robust to alternative assumptions about the structure of international asset markets. This is because domestic no-arbitrage conditions for real and nominal assets, rather than cross-country no-arbitrage conditions, are of first-order importance for the workings of the

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7Two studies have previously investigated the implications of two-country business cycle models for cross-country correlations of nominal variables. Kollmann (2001) constructs a model with sticky prices and wages, and technology and money supply shocks with cross-country spillovers. The model does not generate high cross-country correlations for both prices and interest rates. A baseline version with only technology shocks and no nominal rigidities produces very low cross-country correlations for both nominal variables. Wang and Wen (2007) demonstrate that neither a prototypical sticky-price model nor a sticky-information model (both set off by money supply shocks) can generate at the same time high cross-country correlations of inflation and realistic dynamics of inflation in relation to domestic output. Both papers model monetary policy as an exogenous stochastic process for money growth, rather than as a Taylor rule.
mechanism.

In a recursive competitive equilibrium the absence of arbitrage between returns on a country’s real capital and nominal bonds, together with a Taylor rule, implies that the country’s current price level and the nominal interest rate depend on the country’s expected output and real returns to capital (i.e., its marginal product) in all future periods. Intuitively, agents anticipate future responses of the central bank to the state of the economy and the current interest rate and the price level reflect these expectations. Due to positive spillovers of technology shocks across countries, a persistent domestic technology shock affects not only current and future productivity in the domestic economy, but also future productivity in the foreign economy—over time productivity in the foreign economy is expected to catch up with productivity in the domestic economy. Thus, although current output (determined in equilibrium in large part by the current level of technology) in the two economies may be different, future output and marginal product of capital are expected to converge to similar paths, leading to similar responses of current prices and nominal interest rates. Interestingly, even though prices are fully flexible and forward looking, in equilibrium the price level turns out to be relatively smooth and its changes, that is inflation, are persistent.

As is well known, the baseline model, like many other models, falls short of accounting for a number of features of international and domestic aggregate fluctuations. Two features that are of particular relevance, given our focus on nominal fluctuations, are: (i) the high volatility of nominal exchange rates in combination with smooth price levels (the so-called Mussa, 1986, puzzle) and (ii) the (strikingly similar across countries) lead-lag pattern of correlations of the domestic price level and nominal interest rate with domestic output, which we refer to as the ‘domestic nominal business cycle’. Recent studies (Canzoneri, Cumby, and Diba, 2007; Atkeson and Kehoe, 2008; Alvarez, Atkeson, and Kehoe, 2009; Backus, Gavazzoni, Telmer, and Zin, 2010) argue that exchange rate and interest rate fluctuations over the business cycle are structurally related to cyclical distortions in standard asset-market Euler equations that occur due to various ‘frictions’, such as limited participation, time-varying risk aversion, or time-varying uncertainty.

We are agnostic about the exact form of such distortions and capture their effects by introducing ‘wedges’, in the form of distortionary taxes, into Euler equations in the baseline model. We then choose their joint stochastic processes with technology shocks to match the observed exchange rate dynamics and the domestic nominal business cycle. Such extensions, while making the model consistent with the two aforementioned facts, do not overturn the basic implications of the mechanism for the cross-country correlations of
the two nominal variables relative to that of output. We also show how other distortions in the equilibrium conditions of the baseline model affect the mechanism, thus outlining how the mechanism may work in various specific extensions with detailed frictions that other researchers may consider.

The outline of the rest of the paper is as follows. Section 2 documents the empirical regularities, Section 3 introduces the baseline model, Section 4 describes its calibration, Section 5 presents quantitative findings, Section 6 explains the mechanism, Section 7 conducts sensitivity analysis, Section 8 studies the robustness of the mechanism to the aforementioned extensions, and Section 9 concludes.

## 2 Properties of nominal business cycles

Our empirical analysis is mainly based on quarterly data for real GDP, price levels measured by the consumer price index, and short-term nominal interest rates, usually yields on 3-month government bills, which we take as a proxy for interest rates controlled by monetary policy. The data are for Australia, Canada, Germany, Japan, the United Kingdom, and the United States, for the period 1960.Q1-2006.Q4. In addition, we include Austria and France from 1970.Q1.\(^8\) All statistics discussed in this section are for business-cycle components of the three variables obtained with the Christiano and Fitzgerald (2003) band-pass filter. Before applying the filter, the series for real GDP and price levels were transformed by taking natural logarithms. Their fluctuations are thus measured as percentage deviations from ‘trend’.

### 2.1 International nominal business cycles

We calculate the bilateral cross-country correlations for the two nominal variables (i.e., the correlations of a country’s variable with the same variable of each of the other countries) and compare them with those for real GDP. Figure 1 provides a graphical representation of the main finding. It plots the bilateral correlations of the price levels and the nominal interest rates against the bilateral correlations of output for the six-country sample. As we can see, all but one point lie above the 45-degree line, meaning that for almost all country

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\(^8\)For other developed economies, the required data are jointly available only from either late 1970s or early 1980s. However, we prefer to trade off the number of countries for series that include both the relatively stable 1960s, as well as the volatile 1970s. Furthermore, most of the economies for which the data are available from either late 1970s or early 1980s are European economies that participated in the EMS. Including those countries into our sample would therefore make the sample biased towards economies that operated under a fixed exchange rate for most of our sample period (see the discussion below on how this may potentially affect our empirical findings).
pairs, the bilateral correlations of the two nominal variables are higher than those of real GDP. In addition, the correlations for the two nominal variables are also less dispersed.

The individual correlations are reported in Table 1, for the six-country sample going back to 1960.Q1, and in Table 2 for the eight-country sample, which goes back to 1970.Q1. In the six-country sample, for all 15 pairs the bilateral correlation of nominal interest rates is higher than that of output, and in all but one case the bilateral correlation of prices is also higher. The mean (in the cross-section) bilateral correlations of the nominal interest rates and the price levels are 0.57 and 0.52, respectively—about twice the mean bilateral correlation of real GDP, which is 0.27. In addition, the coefficients of variation (i.e., the standard deviations divided by the corresponding mean) of the bilateral correlations of the nominal interest rates and the price levels are 0.22 and 0.28, respectively, while that of the bilateral correlations of real GDP is 0.89. This clearly indicates that, as mentioned above, the bilateral correlations of the two nominal variables are substantially less dispersed in the cross-section than those of real GDP.

For each country pair Table 1 also reports (in parentheses) the 5th percentiles for \( \phi_{ij}^R = \text{corr}(R_i, R_j) - \text{corr}(GDP_i, GDP_j) \) and \( \phi_{ij}^p = \text{corr}(p_i, p_j) - \text{corr}(GDP_i, GDP_j) \), respectively. The percentiles are obtained by bootstrapping from the sample (see Hardle, Horowitz, and Kreiss, 2001) and provide a test of statistical significance that the observed cross-country correlations of the two nominal variables are higher than those of real GDP. A value of the 5th percentile greater than zero indicates that with 95% probability the ‘true’ bilateral correlation of the nominal interest rates (or prices) for a given country pair is greater than that of output. The percentiles are also computed for the mean values of the bilateral correlations in the cross-section. We see that the bilateral correlations of the nominal interest rates are significantly higher than those of output in 11 cases out of 15 and the correlations of prices are higher in 10 cases. In addition, the mean bilateral correlations for both nominal variables are significantly higher than that for output.

These findings broadly hold also in the eight-country sample (2). Here in 19 cases out of 28 the bilateral correlations of the nominal interest rates are higher than those of output (15 significantly) and in 22 cases the bilateral correlations of prices are higher (15 significantly). The mean bilateral correlations of the nominal interest rates and the price levels are both 0.59, while that of real GDP is only 0.43 and these differences are statistically significant. Finally, the coefficients of variation are around 0.2 for the two nominal variables, and slightly above 0.5 for real GDP.

Even though the two nominal variables differ markedly from output in terms of their cross-country comovements, they are comparably volatile and persistent. For example, the mean standard deviation of output in the sample of the six countries is 1.39, while
the mean standard deviation of the price level is 1.28 and that of the nominal interest rate is 1.31\textsuperscript{9}; and the mean first-order autocorrelation coefficient of output is 0.92, while that of the price level is 0.94 and that of the nominal interest rate is 0.91.

2.1.1 Robustness checks

For a part of our sample period—the Bretton Woods years—national monetary policies were constrained by governments’ obligations to maintain fixed exchange rates with the dollar. It is well known that under fixed exchange rates the domestic economy is not insulated from nominal shocks originating abroad.\textsuperscript{10} In order to check that the high cross-country correlations of prices and nominal interest rates are not driven by the Bretton Woods agreement, we report in Tables 1 and 2 also the mean bilateral correlations and coefficients of variation for the period 1974.Q1-2006.Q4, which excludes the Bretton Woods years. The summary statistics are, however, little changed.

Global commodity price shocks may be another source of the strong cross-country comovements of the price indexes (and thus, through a Taylor-type rule, of the nominal interest rates). We therefore computed the cross-country correlations of CPI stripped off energy and food prices for those countries for which such data series are long enough. These are Austria, Canada, France, Germany, Japan, and the United States. The data, which are available from 1970.Q1, come from Mumtaz and Surico (forthcoming).\textsuperscript{11} We found that for 10 out of the 15 country pairs the correlations of prices are still higher than the correlations of real GDP. The mean bilateral correlation of the price levels is 0.6, while that of output is 0.5, and the difference is statistically significant. This finding is in line with the results of Mumtaz and Surico (forthcoming) who report that, except for the 1970s, there is little empirical relationship between oil and other commodity prices on one hand and changes in headline aggregate price indexes on the other.

After the ‘Great Inflation’ of the 1970s, most central banks in industrialized economies adopted a much tougher stance on inflation. In order to check if this policy change affects the cross-country correlations, we split the sample into two subsamples in 1984, the year broadly associated with the start of the so-called ‘Great Moderation’—a period of relatively low output variability and substantially more stable nominal environment. The summary statistics for the cross-section are contained in Tables 1 and 2. We see that although the mean cross-country correlations of all three variables declined after 1984,

\textsuperscript{9}The standard deviations of the nominal interest rates are for fluctuations measured in percentage points.

\textsuperscript{10}Some researchers (e.g., Eichengreen, 1996), however, argue that during the Bretton Woods period central banks were able to retain a significant degree of monetary autonomy by imposing various capital controls, and thus were able to control the domestic nominal environment.

\textsuperscript{11}We thank Paolo Surico for providing us with the data.
those of the two nominal variables remained substantially (and statistically significantly) higher than that of output. For example, in the eight-country sample, the post-1984 mean bilateral correlation of the nominal interest rates is 0.46, that of the price levels is 0.45, while that of real GDP is only 0.19.

2.2 Domestic nominal business cycles

Kydland and Prescott (1990) have pointed out that a key characteristic of the nominal side of the U.S. business cycle is a countercyclical behavior of prices—i.e., the aggregate price level is negatively correlated with output over the business cycle. Backus and Kehoe (1992) and Mumtaz et al. (2011) extend this finding to a number of developed economies. The left-hand side panel of Figure 2 confirms this. The figure plots the correlation of a country’s price level in period \( t+j \) with its output in period \( t \), for \( j \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} \). We see that for all economies in our sample, the contemporaneous correlation (i.e., that for \( j = 0 \)) is negative. Another striking feature of the data is the systematic phase shift of the price level—in all countries the price level is more negatively correlated with future output than with current output.\(^{12}\)

In the right-hand side panel of Figure 2 we extend this analysis to the nominal interest rate. As in the case of the price level, the dynamics of the nominal interest rate exhibit a clear phase shift—the nominal interest rate is strongly negatively correlated with future output and positively correlated with past output. Although this dynamics of the nominal interest rate is well known for the United States (e.g., King and Watson, 1996), as in the case of the price level, it is striking that it extends to other developed economies. As a robustness check on the mechanism investigated, we therefore explore if the mechanism can be consistent with both, the high cross-country correlations of prices and nominal interest rates, as well as with the lead-lag patterns in Figure 2.

3 Baseline model

The baseline model economy consists of two countries, each populated by a representative individual who maximizes welfare subject to stylized representations of the national income and product account identities. In each country, there are two nominal assets: money and one-period bonds. Monetary policy is conducted according to a Taylor-type rule. The real side of the economy is founded on Backus et al. (1994) and Heathcote and Perri (2002).

\(^{12}\) A related finding has been documented for inflation rates by Fuhrer and Moore (1995) and Galí and Gertler (1999) for the United States and by Wang and Wen (2007) for other countries.
Before proceeding, we set our notation and terminology. A world economy consists of two countries, denoted 1 and 2, which are populated by equal measures of identical, infinitely lived consumers. Producers in each country use country-specific capital and labor to produce a single good, which we refer to as a ‘local’ good. The good produced in country 1 is labeled by $a$, while that produced in country 2 is labeled by $b$. These are the only goods traded in the world economy. Within each country, goods $a$ and $b$ are combined to form a good that can be used for local consumption and investment, and which we refer to as an ‘expenditure’ good.

### 3.1 Preferences and technology

Preferences of the representative consumer in country $i$ are characterized by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U (c_{it}, 1 - n_{it} - s_{it}) ,$$

where $U (c, 1 - n - s) = [c^\mu (1 - n - s)^{1-\mu}]^{1-\gamma} / (1 - \gamma)$, with $0 < \mu < 1$ and $\gamma \geq 0$, and where $c_{it}$ is consumption, $n_{it}$ is time spent working, and $s_{it}$ is time spent in transaction-related activities. This ‘shopping time’ is given by the following parametric representation

$$s_{it} = \kappa_1 \left( \frac{p_{it} c_{it}}{m_{it}} \right)^{\kappa_2} ,$$

where $\kappa_1 > 0$, $\kappa_2 \geq 1$, $p_{it}$ is the domestic price level (i.e., the price of country $i$’s expenditure good in terms of country $i$’s money), and $m_{it}$ is domestic nominal money balances.

Consumers supply labor and capital to domestically located, perfectly competitive producers, who have access to an aggregate Cobb-Douglas production function $z_{it} H (k_{it}, n_{it}) = z_{it} k_{it}^\alpha n_{it}^{1-\alpha} = y_{it}$. Here, $z_{it}$ is a country-specific technology level, $k_{it}$ is capital, $y_{it}$ is output of the local good (either $a$ or $b$), and $0 < \alpha < 1$ is the capital share in production. Technologies in the two countries follow a joint first-order autoregressive process

$$\lambda_{t+1} = A_0 + A \lambda_t + \varepsilon_{t+1} , \quad \varepsilon_{t+1} \sim N (0, \Sigma) ,$$

where $\lambda_t = [\ln z_{1t}, \ln z_{2t}]'$. Market clearing for goods $a$ and $b$ requires

$$a_{1t} + a_{2t} = y_{1t} \quad \text{and} \quad b_{1t} + b_{2t} = y_{2t} ,$$

where $a_{1t}$ is the amount of good $a$ used by country 1, while $a_{2t}$ is the amount used by country 2. Similarly, $b_{1t}$ is the amount of good $b$ used by country 1, while $b_{2t}$ is the amount
used by country 2.

Consumption and investment are composites of foreign and domestic goods

\[ c_{1t} + x_{1t} = G(a_{1t}, b_{1t}) \quad \text{and} \quad c_{2t} + x_{2t} = G(b_{2t}, a_{2t}), \]  

(5)

where \( x_{it} \) is investment, and \( G(a, b) = (\omega_1 a^{-\rho} + \omega_2 b^{-\rho})^{-(1/\rho)} \), with \( 0 < \omega_1 < 1, \omega_2 = 1-\omega_1, \) and \( \rho \geq -1 \). Here, \( \omega_1 \) determines the extent to which there is a home bias in domestic expenditures and \( \rho \) controls the elasticity of substitution between domestic and foreign goods. Investment contributes to capital accumulation according to the law of motion

\[ k_{i,t+1} = (1-\delta) k_{it} + x_{it}, \]  

(6)

where \( 0 < \delta < 1 \) is a depreciation rate.

The prices of goods \( a \) and \( b \) in terms of the expenditure good of country 1 are determined competitively, and therefore given by the marginal products of these two goods

\[ q_{1t}^a = \frac{\partial G(a_{1t}, b_{1t})}{\partial a_{1t}} \quad \text{and} \quad q_{1t}^b = \frac{\partial G(a_{1t}, b_{1t})}{\partial b_{1t}}. \]  

(7)

Similarly, the prices of the two goods in terms of country 2’s expenditure good are given by

\[ q_{2t}^a = \frac{\partial G(b_{2t}, a_{2t})}{\partial a_{2t}} \quad \text{and} \quad q_{2t}^b = \frac{\partial G(b_{2t}, a_{2t})}{\partial b_{2t}}. \]  

(8)

Using these prices, we can measure output of the two countries in terms of their respective expenditure goods as \( q_{1t}^a y_{1t} = q_{1t}^a z_{1t} H(k_{1t}, n_{1t}) \) and \( q_{2t}^b y_{2t} = q_{2t}^b z_{2t} H(k_{2t}, n_{2t}) = q_{2t}^b y_{2t} \). This is the definition of real GDP employed by Heathcote and Perri (2002) which we adopt. We thus use the following notation \( GDP_{1t} \equiv q_{1t}^a y_{1t} \) and \( GDP_{2t} \equiv q_{2t}^b y_{2t} \). Total expenditures in each country are related to GDP as

\[ c_{1t} + x_{1t} + (q_{1t}^a a_{2t} - q_{1t}^b b_{1t}) = GDP_{1t} \quad \text{and} \quad c_{2t} + x_{2t} + (q_{2t}^b b_{1t} - q_{2t}^a a_{2t}) = GDP_{2t}, \]  

(9)

where the expressions in the parentheses are net exports, denoted by \( nx_{1t} \) and \( nx_{2t} \), respectively. These equalities follow from combining the resource constraints (5) with the goods-market-clearing conditions (4), and from using the constant-returns-to-scale property of the \( G(.,.) \) functions, together with the pricing functions (7) and (8). Each resulting equality is then pre-multiplied by the price of the local goods to obtain equations (9).
3.2 Monetary policy

A central bank in each country controls the nominal rate of return \( R_{it} \) on a one-period domestically traded bond, which pays one unit of country \( i \)'s money in all states of the world in period \( t+1 \). The central bank sets the rate of return according to a feedback rule

\[
R_{it} = (1 - \phi) \left[ R + \nu_y (\ln GDP_{it} - \ln GDP) + \nu_\pi (\pi_{it} - \pi) \right] + \phi R_{i,t-1},
\]

(10)

where \( \pi_{it} \equiv \ln p_{it} - \ln p_{i,t-1} \) is the inflation rate, and a variable’s symbol without a time subscript represents the variable’s steady-state value. In line with the literature we allow for the possibility that the central bank ‘smooths’ the nominal interest rate by putting a weight \( 0 < \phi < 1 \) on the past interest rate. The central bank then elastically supplies, through lump-sum transfers \( v_{it} \) to consumers, whatever amount of nominal money balances the consumers demand. Demand for money is implicitly defined by a first-order condition for money

\[
u_{nt} s_{nt} = (u_{ct} + u_{nt} s_{ct}) [1 + 1/(1 + R_t)],
\]

(11)

which follows from the consumer’s problem described in the definition of the equilibrium below. Here, \( s_{nt} = s(.)' (p_t/m_t)^2 c_t \), and \( u_{nt} \), for example, is the derivative of the utility function with respect to \( n_t \) (for simplicity of notation the country subscripts have been dropped). The nominal money stock thus evolves as

\[
m_{it} = m_{i,t-1} + v_{it}.
\]

(12)

We do not justify this monetary policy rule in terms of its welfare implications. We simply take it as the most parsimonious, empirically plausible, approximation of monetary policy in industrialized economies, as suggested by the literature, and study its implications for the cross-country behavior of nominal variables.

3.3 Consumer’s budget constraint

Consumers hold money in order to economize on shopping time. In addition, they accumulate capital, a one-period internationally traded bond \( f_{it} \), which pays one unit of good \( a \) in all states of the world in period \( t + 1 \), and the domestically traded bond, which we denote by \( d_{it} \).\(^{13}\) Measured in terms of the domestic expenditure good, the consumer’s

\(^{13}\)As in Heathcote and Perri (2002), the denomination of the internationally traded bond has only second-order effects on equilibrium, which are not captured by our computational method. The denomination of the bond thus does not affect the computed equilibrium allocations and prices. We could also extend the model by allowing consumers in country \( i \) to hold the nominal bond of country \( j \), but this would only clutter the model without affecting the computed equilibrium. The existence of such a bond
budget constraint is

\[
\frac{q^{a}_{it}f_{it}}{1 + r^{f}_{it}} + \frac{d_{it}}{p_{it}} + \frac{m_{it}}{p_{it}} + c_{it} + x_{it} = q^{c}_{it}(r^{k}_{it}k_{it} + w_{it}n_{it}) + q^{a}_{it}f_{i,t-1} + \frac{d_{i,t-1}}{p_{it}} + \frac{m_{i,t-1}}{p_{it}} + v_{it},
\]

(13)

where \( r^{f}_{it} \) is the real rate of return (in terms of good \( a \)) on the internationally traded bond, \( r^{k}_{it} \) is the rate of return on domestic capital, equal in equilibrium to the marginal product of capital \( z_{it}H_{k}(k_{it}, n_{it}) \), \( w_{it} \) is the wage rate, and \( q^{c}_{it} \) is equal to \( q^{a}_{1t} \) in the case of country 1, and to \( q^{b}_{2t} \) in the case of country 2.

### 3.4 Terms of trade and exchange rates

Following Backus et al. (1994), terms of trade \( e \) are defined as the price of good \( b \) in terms of good \( a \)

\[
e_{t} \equiv q^{b}_{1t}/q^{a}_{1t} = q^{b}_{2t}/q^{a}_{2t},
\]

(14)

where the equality holds in equilibrium. And following Heathcote and Perri (2002), the real exchange rate is defined as the price of the expenditure good of country 2 relative to the price of the expenditure good of country 1, i.e. \((1/q^{a}_{1t})/(1/q^{a}_{2t}) = q^{b}_{2t}/q^{a}_{1t}\), which, by applying relationship (14), is equal to \( q^{b}_{2t}/q^{b}_{1t} \). An increase in this ratio represents an appreciation of the real exchange rate from country 1’s perspective as less of this country’s expenditure good (relative to the amount of country 2’s expenditure good) is needed to purchase one unit of good \( a \) or \( b \). The nominal exchange rate is consequently defined as

\[
n_{e} = (q^{a}_{2t}/q^{a}_{1t})(p_{2t}/p_{1t}).
\]

(15)

### 3.5 Recursive competitive equilibrium

In each country, the consumer chooses state-contingent plans for \( c_{it}, x_{it}, k_{i,t+1}, m_{it}, d_{it}, f_{it}, n_{it}, \) and \( s_{it} \) in order to maximize (1) subject to (2), (6), and (13), taking all prices as given. In all states of the world, the prices of capital and labor services, and of the two local goods \( a \) and \( b \), are given by their respective marginal products. In period \( t \) the state of the world economy is defined by the vector of technology levels \( \lambda \), a vector of domestic endogenous state variables \( \Upsilon_{i} = (p_{i,t-1}, R_{i,t-1}, k_{it}, \vartheta_{i,t-1}, f_{i,t-1}) \), and a vector of foreign state variables \( \Upsilon_{j} = (p_{j,t-1}, R_{j,t-1}, k_{jt}, \vartheta_{j,t-1}, f_{j,t-1}) \), where \( \vartheta_{i,t-1} = d_{i,t-1} + m_{i,t-1} \), and similarly for \( \vartheta_{j,t-1} \).

would imply a no-arbitrage condition involving the nominal exchange rate, but this no-arbitrage condition (in its first-order approximation) holds even without such a complication. Further, as consumers of country \( i \) do not ‘shop’ in country \( j \), they would never want to hold money of country \( j \). We can thus also abstract from cross-border money holdings.
The equilibrium of the world economy is then characterized by a set of pricing functions for each country \( \{ r_k^i(\lambda, \Upsilon_i, \Upsilon_j), \, w_i(\lambda, \Upsilon_i, \Upsilon_j), \, q_a^i(\lambda, \Upsilon_i, \Upsilon_j), \, q_b^i(\lambda, \Upsilon_i, \Upsilon_j), \, p_i(\lambda, \Upsilon_i, \Upsilon_j), \, R_i(\lambda, \Upsilon_i, \Upsilon_j) \} \), a set of aggregate decision rules for each country \( \{ n_i(\lambda, \Upsilon_i, \Upsilon_j), \, k_i(\lambda, \Upsilon_i, \Upsilon_j), \, m_i(\lambda, \Upsilon_i, \Upsilon_j), \, d_i(\lambda, \Upsilon_i, \Upsilon_j), \, f_i(\lambda, \Upsilon_i, \Upsilon_j) \} \), and a pricing function for the rate of return on the internationally traded bond \( r_f^i(\lambda, \Upsilon_i, \Upsilon_j) \), such that the allocations and prices generated by these functions satisfy the consumer’s optimization problem, the resource constraints (5), the goods-market-clearing conditions (4), a market-clearing condition for domestically traded bonds \( d_{it} = 0 \), a market-clearing condition for the internationally traded bond \( f_1 + f_2 = 0 \), and the monetary policy rule (10).  

4 Calibration

Table 3 summarizes the parameter values for our benchmark experiment. Results of a thorough sensitivity analysis are reported in Section 7. As preferences and technology are the same as in Backus et al. (1994), the parameters of utility and production functions, and of the stochastic process for technology shocks, are either the same as in their paper, or are calibrated to the same targets. We therefore do not discuss them here and refer the reader to their paper for details.

The parameters of the shopping time function (2) are chosen so that the money demand function in the model has the same interest rate elasticity and implies the same average velocity of money as its empirical counterpart estimated for the United States. The money demand function in the model is given implicitly by the consumer’s first-order condition for money holdings, which in steady state has the form

\[
\kappa_1 \kappa_2 \left( \frac{p c}{m} \right)^{\kappa_2} \frac{p}{m} = \frac{1}{w} \left( \frac{R}{1 + R} \right).
\]

Setting the curvature parameter \( \kappa_2 \) equal to 1, the money demand function becomes

\[
\frac{m}{p} = \left[ \kappa_1 c w \left( 1 + \frac{1}{R} \right) \right]^{0.5},
\]

which has interest elasticity equal to \(-0.5\), in line with a number of empirical studies (see Lucas, 2000). We set the level parameter \( \kappa_1 \) equal to 0.0054, which implies annual velocity

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14 We compute log-linear approximations to the equilibrium decision rules and pricing functions in the neighborhood of the model’s non-stochastic steady-state, using the method described by Hansen and Prescott (1995). Before computing the equilibrium, all nominal variables are transformed so that they are stationary. Following Heathcote and Perri (2002) we also impose a tiny quadratic cost of adjusting holdings of the internationally traded bond in the consumer’s optimization problem in order to ensure stationarity of international bond holdings.
of money equal to 6.1—the average U.S. annual velocity of M1 in the period 1959-2006.

The estimates of the parameters of the monetary policy rule (10) vary greatly in the literature, depending on the countries considered, periods covered, and the exact specification of the rule. For our benchmark experiment we set the weight on inflation $\nu_\pi$ equal to 1.5 and the weight on output $\nu_y$ equal to 0.125—the values used by Taylor (1993). In addition, we set the steady-state inflation rate $\pi$ equal to 0.0091—the average quarterly inflation rate in the United States between 1959 and 2006—and the smoothing coefficient $\phi$ equal to 0.75, which is within the range of estimates obtained in the literature (e.g., Clarida et al., 2000; Sack and Wieland, 2000).

5 Quantitative findings

This section reports quantitative findings for the benchmark calibration. We organize it in two parts: cross-country correlations and correlations of domestic variables with domestic output. Although the results for real variables are well known, we report them for completeness. As we will see later, certain features of the dynamics of the real economy have implications for the dynamics of the nominal variables.

5.1 International business cycle

Table 4 reports the cross-country correlations of the price levels, the nominal interest rates, and output for the model and the data (the cross-sectional averages). As in the case of the data, the artificial series generated by the model are filtered with the Christiano and Fitzgerald (2003) filter. The statistics for the model are averages for 100 runs of the model.

We see that the baseline model generates the main feature of the international nominal business cycle: The cross-country correlations of the price levels and the nominal interest rates are substantially higher than that of output. In addition, in line with the data, the cross-country correlations of the price levels and the nominal interest rates are similar. Furthermore, the model’s quantitative predictions are reasonably close to the data as well. In particular, in the model the cross-country correlations of the price levels and the nominal interest rates are 0.69 and 0.68, respectively, while the cross-country correlation of real GDP is only 0.23. In the data the mean values of these correlations are, respectively, 0.52, 0.57, and 0.27 for the six country sample going back to 1960.Q1, and 0.59, 0.59, 0.59.

Taylor uses the weight on output equal to 0.5. This value is scaled down by four in our calibration in order to make it consistent with the nominal interest rate and inflation in our model, which are measured at quarterly rates, rather than annual rates as in his paper.
and 0.43 for the eight-country sample starting in 1970.Q1. It is important to stress that this result was not generated by deliberate choice of parameter values. Recall that the parameterization of the real side of the economy is based on Backus et al. (1994), while the parameterization of the nominal side is based on empirical properties of money demand functions and ‘standard’ values of the Taylor rule.

### 5.2 Domestic business cycle

Table 5 reports the usual, in the literature, statistics for the model’s domestic business cycle and compares them with those of the U.S. economy, which we take as representative (Backus and Kehoe, 1992; Backus et al., 1994; Zimmermann, 1997). In particular, we report the standard deviations of key domestic variables, relative to that of real GDP, and their correlations with real GDP at various leads and lags. We also report the J-curve—a dynamic relationship between net exports and the terms of trade.

We see that the model accounts for about 80 percent of GDP fluctuations and, in line with the data, produces consumption about half as volatile as GDP, investment about three times as volatile as GDP, and net exports about 25 percent as volatile as GDP. Hours, however, are somewhat less volatile in the model than in the data. In addition, in line with the data, consumption, investment, and hours are procyclical, while net exports are countercyclical. Furthermore, the model generates a J-curve—net exports are negatively correlated with future terms of trade and positively correlated with past terms of trade.

As for the two nominal variables, the model correctly generates a countercyclical price level and produces standard deviations of the price level and the nominal interest rate, relative to that of GDP, similar to those for the U.S. economy. However, the model fails to produce the empirical lead-lag pattern of the price level and the comovement between output and the nominal interest rate—in the model price level lags output negatively and the nominal interest rate is negatively correlated with output contemporaneously without any phase shift. This failure is not surprising—these are well known anomalies, at least for the United States, and we would therefore not expect the baseline model to account for them. However, as a robustness check, in Subsection 7.3, we make the model consistent with the observed dynamics of the two nominal variables in relation with domestic output and ask if the model can still produce higher cross-country correlations of the two nominal variables than that of output.

Table 5 also reports the cyclical behavior of the nominal exchange rate. This is for completeness as the exchange rate and the two nominal interest rates are structurally related in the model (more on this below). It is clear that the exchange rate is substantially
less volatile in the model than in the data and that its lead-lag relationship with real GDP is opposite to that in the data.\textsuperscript{16} Again, capturing the observed exchange rate dynamics within theoretical models is a well-known challenge. As a robustness check, in Subsection 7.4 we repeat our experiments while aligning the model more closely with the observed exchange rate behavior.

6 The mechanism

We can gain understanding of our main result by plotting the responses of the model’s variables to a 1% positive technology shock. These responses are contained in Figure 3. As the focus is on nominal variables, we describe the responses of the real variables only briefly and refer the reader to Backus et al. (1994) and Heathcote and Perri (2002). In this section it is useful to abstract from the effects of nominal variables on the real economy, which in our model occur (through the first-order condition for money) due to an inflation tax. This tax affects the real money balances held by the consumer and thus the allocation of time between transactions, work, and leisure. These effects are small for the benchmark calibration and taking them into account would unnecessarily complicate the description of the mechanism without changing the main insight.

6.1 Real variables

Because the shocks in the two countries are correlated, a 1% increase in technology in country 1 leads, on impact, to an increase in technology in country 2 by 0.258\%, where 0.258 is the correlation coefficient of the $\varepsilon$’s. More importantly, due to spillovers, technology in country 2 gradually catches up with technology in country 1. As a result of a higher current and expected future technology level, consumption in both countries increases, but it increases by less in country 2 than in country 1. There are two reasons for this. First, the net present value of country 2’s future income is smaller than that of country 1. This is because technology in country 2 does not reach the level of technology in country 1 for a while. Second, there is intertemporal trade between the two countries: in order to take advantage of higher total factor productivity, country 1 increases investment by borrowing from country 2. Country 2 is thus giving up some of its current consumption in return for higher future consumption. This intertemporal trade is reflected in the decline of net exports of country 1, and the increase in the real return on the internationally traded bond. Because of the initially higher technology level in country 1, GDP is initially

\textsuperscript{16}As the ratio of the price levels in the data is relatively little volatile, the dynamics of real and nominal exchange rates in the data are similar.
higher in country 1 than in country 2 (also labor is higher due to intertemporal substitution). However, as technology in country 2 catches up with technology in country 1, GDP in country 2 catches up with GDP in country 1. As a result of initially higher output in country 1, the price of good \( a \) falls, reflecting its abundance in the world market relative to good \( b \). The terms of trade of country 1 therefore worsen, following the technology shock.

6.2 Nominal variables

The dynamics of the price level and the nominal interest rate are determined by the interaction between Taylor rules and no-arbitrage conditions.

6.2.1 Domestic and international no-arbitrage relations

The Euler equations for the accumulation of capital, and domestic and foreign bonds in country \( i \) are, respectively,

\[
E_t \left[ Q_{it} \left( 1 + r_{k,t+1}^k - \delta \right) \right] = 1,
\]

\[
E_t \left[ Q_{it} \left( \frac{1}{1 + \pi_{i,t+1}} \right) \left( 1 + R_{it} \right) \right] = 1,
\]

\[
E_t \left[ Q_{it} \left( \frac{a_{i,t}^a}{a_{i,t}^{a+1}} \right) \left( 1 + r_f^i \right) \right] = 1,
\]

where \( Q_{it} \equiv \beta (U_{c,t+1} - U_{lt,s,t+1})/(U_{ct} - U_{lt,t+1}s_{ct}) \) is country \( i \)'s stochastic discount factor. It is convenient to log-linearize these conditions around the model’s non-stochastic steady state (as preferences are standard time-additive expected utility CRRA preferences, little is lost by log-linearization)

\[
E_t \hat{Q}_{it} + E_t \hat{r}_{k,t+1}^k = 0, \tag{16}
\]

\[
\hat{R}_{it} + E_t \hat{Q}_{it} - E_t \hat{\pi}_{i,t+1} = 0, \tag{17}
\]

\[
\hat{r}_{f,t}^i + E_t \hat{Q}_{it} + E_t \hat{a}_{i,t+1}^a - \hat{q}_{it} = 0, \tag{18}
\]

where \( \hat{r}_{k,t+1}^k \equiv (r_{k,t+1}^k - r^k)/(1 + r^k - \delta), \hat{R}_{it} \equiv (R_{it} - R)/(1 + R), \hat{\pi}_{it} \equiv (\pi_{it} - \pi)/(1 + \pi), \hat{r}_{f,t}^i \equiv (r_{f,t} - r^f)/(1 + r^f) \) are percentage deviations of the gross rates from steady state, and \( \hat{Q}_{it} \equiv \log Q_{it} - \log Q_{i} \) is the percentage deviation of the stochastic discount factor. Combining equations (16) and (17), and (16) and (18), then gives, respectively, a no-arbitrage condition for domestic real and nominal assets, and for real domestic and
international assets

\[ E_t \hat{r}_{i,t+1} = \hat{R}_{it} - E_t \hat{\pi}_{i,t+1}, \]  
(19)

\[ E_t \hat{r}^k_{i,t+1} = \hat{r}^f_t + E_t \hat{q}_a_{i,t+1} - \hat{q}_a^t, \]  
(20)

In addition, combining equation (20) for country 1 with that for country 2 gives a relationship between the return to capital in the two countries

\[ E_t \hat{r}^k_{1,t+1} + E_t (\hat{q}_2_{t+1}^a - \hat{q}_1_{t+1}^a) - (\hat{q}_2^a - \hat{q}_1^a) = E_t \hat{r}^k_{2,t+1}, \]

where \( E_t (\hat{q}_2_{t+1}^a - \hat{q}_1_{t+1}^a) - (\hat{q}_2^a - \hat{q}_1^a) \) is the expected change in the real exchange rate. Notice, that due to the expected changes in the real exchange rate, the expected real returns on capital in the two countries do not need to be equalized. In addition, using a log-linearized version of equation (15) together with equation (19), we can re-write equation (21) in its nominal form as

\[ \hat{R}_{it} + E_t \hat{n} \hat{r}_{t+1} - \hat{n} \hat{r}_t = \hat{R}_{2t}. \]

This is, of course, the standard ‘uncovered interest rate parity’ condition.

6.2.2 Equilibrium prices and nominal interest rates

Abstracting from the small inflation tax effects, we can think of the real variables as being determined independently of the nominal variables. Thus, given the equilibrium real quantities and prices, whose dynamics were described above, the equilibrium nominal interest rate and the price level in country \( i \) are determined by the no-arbitrage condition for real and nominal assets (19) and the (log-linearized) Taylor rule

\[ \hat{R}_{it} = \tilde{\nu}_\pi \hat{\pi}_{it} + \tilde{\nu}_y \hat{Y}_{it}, \]  
(23)

where \( \tilde{\nu}_\pi \equiv \nu_\pi (1 + \pi)/(1 + R), \tilde{\nu}_y \equiv \nu_y/(1 + R), \) and \( \hat{Y}_{it} \equiv \log GDP_{it} - \log GDP, \) and where for expositional reasons we set \( \phi \) equal to zero (the argument, however, holds even for non-zero \( \phi \)). Combining the equilibrium conditions (19) and (23) yields a first-order difference equation in inflation

\[ E_t \hat{r}^k_{i,t+1} + E_t \hat{\pi}_{i,t+1} = \tilde{\nu}_\pi \hat{\pi}_{it} + \tilde{\nu}_y \hat{Y}_{it}. \]

(24)
For $\nu_\pi > 1$, equation (24) can be solved by forward substitution, excluding unstable equilibria that lead to either hyperinflations or hyperdeflations. For a given $p_{t-1}$, this gives $p_t$ as a difference between the expected discounted sum of future real returns to capital and the expected discounted sum of current and future output

$$\hat{p}_{it} = \hat{p}_{i,t-1} + E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j \hat{r}_{i,t+j}^k \right] - \tilde{\nu}_y E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j \hat{Y}_{i,t+j-1} \right], \quad (25)$$

where the ‘discount factor’ is the inverse of the weight on inflation in the Taylor rule. The current price level thus reflects the expected future paths of output and the real return to capital.

Intuitively, period-$t$ price level has to be consistent, according to the Taylor rule, with period-$t$ output and the nominal interest rate, which (through the no-arbitrage condition (19)) has to be consistent with the expected real return to capital and the price level in period $t + 1$. Period-$(t + 1)$ price level in turn has to be consistent with period-$(t + 1)$ output and the nominal interest rate, which has to be consistent with the expected real return to capital and the price level in period $t + 2$, and so on. As a result of this recursion, period-$t$ price level reflects all future states of the real economy.

How is the equilibrium price level supported? As the monetary authority supplies money elastically through lump-sum transfers, which in our setting are equivalent to open market operations (see Cooley and Hansen, 1995), the price level is supported by nominal money balances required by consumers to achieve real money balances dictated by the first-order condition for money (11).

Substituting the price level from equation (25) into the Taylor rule (23) then gives a similar expression for the nominal interest rate

$$\hat{R}_{it} = E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\nu_\pi} \right)^{j-1} \hat{r}_{i,t+j}^k \right] - \tilde{\nu}_y E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j \hat{Y}_{i,t+j} \right]. \quad (26)$$

Notice the slight differences in the indexes, compared with equation (25).

### 6.2.3 ‘Stickiness’ of prices and the role of international asset markets

Notice from equation (25) that by increasing the weight on inflation in the Taylor rule, we can make the price level arbitrarily smooth—i.e., reduce its responsiveness to shocks. The price level can thus appear fairly ‘sticky’ despite prices being fully flexible and forward-

\[^{17}\text{Hyperinflations in the model are costly because they make agents spend an increasingly larger amount of time in transaction-related activities, while hyperdeflations are costly because they lead to depletion of capital.}\]
looking. Notice also that we have used only the domestic no-arbitrage condition to arrive at equations (25) and (26). The structure of international asset markets (i.e., the degree of their completeness), which affects the form of the cross-country no-arbitrage conditions, affects domestic prices and the nominal interest rate only to the extent to which it affects the dynamics of $y_{it}$ and $r^k_{it}$.

6.2.4 Responses of prices and nominal interest rates to technology shocks

As follows from equation (21), due to real exchange rate movements, cross-country borrowing and lending does not necessarily equate the returns to capital in the two economies. This is indeed the case in our benchmark experiment, as we see in Figure 3: The return to capital in country 1 increases on impact, while the return to capital in country 2 increases only gradually as technology in country 2 catches up with technology in country 1. The expected discounted sums of the rates of return in the two countries nevertheless increase on impact, as in both countries the return to capital is expected to stay above its steady-state level for much of the duration of the technology shock. A similar argument also applies to the expected discounted sums of output. Thus, although output differs across the two countries between the impact period and the time when country 2 catches up with country 1, the discounted sums increase on impact in both countries. Because the price level and the nominal interest rate depend on the difference between the expected discounted sums of returns to capital and GDP, the sign of their responses depends on the relative weight on GDP in the Taylor rule. It turns out that, for our benchmark experiment, the weight on GDP is sufficiently large, leading to a fall in price levels and nominal interest rates in the two countries following the technology shock in country 1. Notice that the changes in the price levels are gradual but sustained, implying a highly persistent inflation rate—the first, second, and third-order autocorrelation coefficients of the inflation rate, in the simulation of the model, are 0.87, 0.65, and 0.35, respectively.

The movements in the nominal interest rates have implications for exchange rate dynamics through the uncovered interest rate parity (22). As the nominal interest rate of country 1 is below that of country 2 in the plots in Figure 3, the nominal exchange rate is increasing (i.e., appreciating from the perspective of country 1).

7 Sensitivity analysis

In order to check robustness of the quantitative finding, we conduct a thorough sensitivity analysis. Due to space constraints, we report results only for those experiments that lead to noticeable changes in the main result. These are for the parameters of the monetary

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policy rule \((\nu_\pi, \nu_y, \phi)\) and the degree of spillovers of technology shocks \((A_{12})\). Varying the elasticity of substitution between home and foreign goods \((\sigma)\), the steady-state import share of GDP \((b_1/y_1)\), and the shopping-time parameters \((\kappa_1, \kappa_2)\) has only a limited effect. In addition, as should be expected from the discussion in the previous section, changes in the structure of international asset markets (in particular, we imposed financial autarky—i.e., no international borrowing or lending) has only a limited effect.

### 7.1 Parameters of the Taylor rule

Figure 4 plots the cross-country correlations for output and the two nominal variables for alternative values of \(\nu_y\), which we vary between -0.05 and 0.25—a range that covers most of the estimates found in the literature.\(^{18}\) We see that except for a small interval between 0.025 and 0.06, the cross-country correlations of the price level and the nominal interest rate are higher than that of real GDP.

In the top panels of Figure 5 we plot the international correlations for alternative weights on inflation. We plot these correlations for two alternative weights on output: our benchmark weight of 0.125, and a zero weight. In empirical Taylor rules, \(\nu_\pi\) is usually in the range from 0.8 to 2.5. In the model, however, when \(\nu_\pi\) is too close to one, the equilibrium becomes indeterminate. This is a common feature of this class of models. We therefore restrict \(\nu_\pi\) to be in the interval from 1.05 to 2.5. As can be seen, except for the case of a zero weight on output, together with the weight on inflation being close to our lower bound, the cross-country correlations of the two nominal variables are higher than that of output. The large increase in the cross-country correlation of output in the right-hand side panel is due to substantial inflation tax effects that kick in when the weight on output is relatively large and the weight on inflation is relatively small.

Some specifications of Taylor rules include a smoothing coefficient while others do not. When the coefficient is included its estimates are usually in the range between 0.5 and 0.9 (see Woodford, 2003, chapter 1). The mid-panels of Figure 5 therefore report how the cross-country correlations change when \(\phi\) is varied between 0 and 0.99. We see that the main result is robust to such variation.

### 7.2 Spillovers

The estimates of the spillover term in the transition matrix \(A\) vary in the literature. Backus et al. (1992) estimate this term to be 0.088, our benchmark value, while Heathcote and Perri

\(^{18}\)These values are the values reported in the literature, divided by four in order to make them consistent with the inflation and interest rates in the model, which are expressed at a quarterly rate.
(2002) obtain an estimate around 0.025. Baxter and Crucini (1995) find little evidence for non-zero spillovers. We therefore vary $A_{12}$ between 0 and 0.1. In all these experiments the diagonal elements of $A$ are adjusted so that the highest eigenvalue is the same as in the benchmark experiment, thus keeping persistence of the shocks constant. We see in the bottom panels of Figure 5 that except for the case of no spillovers, nominal variables are correlated more strongly across countries than output. Crucially, the gap between the cross-country correlations of the two nominal variables and that of GDP opens up rapidly as we move away from the case of no spillovers. The quantitative implications of the mechanism thus do not rely on unrealistically large spillovers. For example, even for a modest degree of spillovers, such as that found by Heathcote and Perri (2002), the model generates a gap between the cross-country correlations of the two nominal variables and that of real GDP close to that observed in the data (for the benchmark value of 0.088 the gap is too wide). For example, in the case of prices, the gap is about 0.35, when $\nu_y = 0$, and about 0.2, when $\nu_y = 0.125$. However, as should be clear from the description of the mechanism, the spillover term must be non-zero for the main result to hold at least qualitatively.

8 Domestic nominal business cycle and exchange rate dynamics

This section derives expressions for equilibrium prices and nominal interest rates that hold in the presence of various distortions, and carries out quantitative experiments for two types of distortions that help align the model with the observed properties of domestic nominal business cycles and exchange rate dynamics.

8.1 The mechanism in the presence of distortions

For researchers who would like to imbed the mechanism highlighted above into more complex environments, it may be helpful to outline how various distortions may affect the workings of the mechanism. We do so by drawing on the insight from business cycle accounting (Chari, Kehoe, and McGrattan, 2007) that the distortionary effects of various frictions can be summarized by appropriately constructed distortionary taxes. In an extension of this approach to monetary models, Sustek (forthcoming) shows that equilibrium inflation dynamics (when monetary policy is conducted according to a Taylor rule)
can be characterized by a generalized version of the log-linear equation (24). Specifically,
\[- \chi_k \hat{\tau}^k_{it} + E_t \hat{m}_k p_{i,t+1} + \chi_b \hat{\tau}^b_{it} + E_t \hat{\pi}_{i,t+1} = \hat{\nu}_\pi \hat{\pi}_{it} + \hat{\nu}_y \hat{Y}_{it}, \]  
(27)
where $\hat{m}_k p_{i,t+1}$ is the marginal product of capital, $\hat{\tau}^k_{it}$ is a tax on capital, which creates a wedge between the market return to capital and its marginal product, $\hat{\tau}^b_{it}$ is a tax on nominal bonds, which creates a wedge in the Euler equation for domestic bonds (17), and $\chi_b > 0$ and $\chi_k > 0$ are constants. Here, the taxes on capital and bonds are the only ‘wedges’ that show up in the equilibrium conditions that determine the price level and the nominal interest rate. Wedges in other equilibrium conditions have only indirect effects on these two variables by affecting either output (in the case when the weight on output in the Taylor rule is nonzero) or the marginal product of capital.

Sustek (forthcoming) demonstrates that the tax on capital captures, for example, distortionary effects of nominal price rigidities, while the tax on bonds captures distortionary effects of asset market frictions, such as limited participation. The tax on bonds can also reflect time-varying volatility or time-varying risk aversion. The price level in the presence of these distortions is given by
\[\hat{p}_{it} = \hat{p}_{i,t-1} + E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\hat{\nu}_\pi} \right)^j \hat{m}_k p_{i,t+j} \right] - \hat{\nu}_y E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\hat{\nu}_\pi} \right)^j \hat{Y}_{i,t+j-1} \right] \]
\[- \chi_k E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\hat{\nu}_\pi} \right)^j \hat{\tau}^k_{i,t+j-1} \right] + \chi_b E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1}{\hat{\nu}_\pi} \right)^j \hat{\tau}^b_{i,t+j-1} \right].\]
A similar expression can be also derived for the nominal interest rate. The degree of cross-country comovements of prices and nominal interest rates then depends on how strongly the four discounted sums co-move across countries. For example, if a given shock is expected to be propagated in the two countries in a similar way (meaning its distortions lead to similar movements in one or both wedges) the nominal variables in the two countries will move in the same direction.

8.2 Domestic nominal business cycle

This subsection investigates the quantitative strength of the mechanism once a tax on domestic bonds is introduced into the model. Recall that the baseline model does not generate the observed lead-lag pattern neither for the nominal interest rate nor for the price level. As noted in the Introduction, recent research argues that the observed dynamics of interest rates are structurally related to variations of a wedge in the standard
Euler equation for bonds. Although there have been a few attempts to provide a theory of such a relationship, a well established model is not yet available. We therefore remain agnostic about the sources of the variation and proceed by introducing a tax on adjusting domestic bonds $\tau_{it}^b$ into the budget constraint in each country. We then choose its stochastic process so as to replicate the lead-lag pattern of the nominal interest rate for the United States, which we take as representative of the patterns in Figure 2 and ask if the model generates both the observed lead-lag pattern of the price level and higher cross-country correlations of the two nominal variables than that of output.

The budget constraint of the representative consumer in country $i$ is now

$$\frac{q_{it}^a f_{it}}{1 + r_t^f} + (1 + \tau_{it}^b) \left[ \frac{d_{it}}{p_{it} (1 + R_{it})} - \frac{d_{i,t-1}}{p_{it}} \right] + \frac{m_{it}}{p_{it}} + c_{it} + x_{it} = \frac{q_{it}^c (r_{it}^k k_{it} + w_{it} n_{it})}{p_{it}} + q_{it}^a f_{i,t-1} + \frac{m_{i,t-1}}{p_{it}} + \frac{v_{it}}{p_{it}} + T_{it},$$

where $T_{it}$ is the proceeds from taxing the accumulation of domestic bonds, which are rebated back to the consumer in a lump sum way. The Euler equation for domestic bonds becomes

$$E_t \left[ Q_{it} \left( \frac{1 + \tau_{i,t+1}^b}{1 + \tau_{it}^b} \right) \left( \frac{1}{1 + \pi_{i,t+1}} \right) (1 + R_{it}) \right] = 1.$$ 

We postulate a joint stochastic process for the tax and technology shocks

$$\begin{bmatrix} \ln z_{1,t+1} \\ \tau_{1,t+1}^b \\ \ln z_{2,t+1} \\ \tau_{2,t+1}^b \end{bmatrix} = \Lambda_0 + \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{13} & \Lambda_{14} & \Lambda_{11} & \Lambda_{12} \\ \Lambda_{23} & \Lambda_{24} & \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \ln z_{1t} \\ \tau_{1t}^b \\ \ln z_{2t} \\ \tau_{2t}^b \end{bmatrix} + \varepsilon_{t+1}. \quad (28)$$

Notice that we impose symmetry across the two countries in the stochastic process, set the steady-state value of $\tau_{it}^b$ equal to zero, and let $\varepsilon_{t+1} \sim N(0, \Omega)$, where the elements of $\Omega$ related to the innovations in technology are the same as those in $\Sigma$, the covariance matrix in the stochastic process (3); those related to the innovations in the tax set equal to zero.

This stochastic process has eight parameters that need to be calibrated: $\Lambda_{11}, \Lambda_{12}, \Lambda_{13}, \Lambda_{14}, \Lambda_{21}, \Lambda_{22}, \Lambda_{23},$ and $\Lambda_{24}$. We choose their values by minimizing the distance between eight moments in the data and the same moments in the model: $\text{corr}(R_{1t}, R_{1,t-1}),$ the persistence of the nominal interest rate; $\text{corr}(R_{1,t-3}, GDP_{1t}),$ $\text{corr}(R_{1,t-1}, GDP_{1t}),$ $\text{corr}(R_{1,t+1}, GDP_{1t}),$ $\text{corr}(R_{1,t+3}, GDP_{1t}),$ every other cross-correlation coefficient in the row for the nominal interest rate in Table 5, panel B; and $\text{corr}(\ln z_{1t}, \ln z_{1,t-1}),$ $\text{corr}(\ln z_{1t}, \ln z_{2,t-1}),$ and $\text{corr}(\ln z_{1t}, \ln z_{2,t-3}),$ in order to ensure that technology shocks in the extended model
have approximately the same persistence and spillovers as in the baseline model—i.e., estimating the stochastic process for technology shocks (3) on time series for $\ln z_{1t}$ and $\ln z_{2t}$ generated by the stochastic process (28) yields approximately the same autocorrelations and spillovers. The resulting values of the eight parameters are contained in panel A of Table 6. All other parameters are as in Table 3.

Although we do not take a firm stand on the interpretation of $\tau^b$, it is interesting that the calibration implies a positive, and relatively large, $\Lambda_{21}$. This means that after a positive technology shock the tax on domestic bonds increases, making the bond relatively less attractive. This is consistent with interpreting the tax as capturing counter-cyclical risk premia—following a positive technology shock (a boom period) a short-term government bond becomes relatively less attractive—flight out of quality.

Panels B and C of Table 6 report the results. Recall that in the baseline model, the price level lags output negatively, while in the data it leads negatively. The extended model, in contrast, generates the correct phase shift of the price level while still producing a negative contemporaneous correlation between the price level and output. In addition, it still produces higher cross-country correlations of the two nominal variables than that of output. It is also important to realize that because $\tau^b$ affects only the two nominal variables (it shows up only in the Euler equation for bonds), the desirable business cycle properties of real variables in the baseline economy are preserved in the extended economy.

### 8.3 Exchange rate dynamics

In a similar way to that in the previous subsection we also investigate the robustness of the mechanism to aligning the model with the observed exchange rate dynamics. In this case the tax is imposed on the accumulation of the real international bond $f_{it}$. As such it distorts the Euler equation (18), which now becomes

$$E_t \left[ Q_{it} \left( \frac{1 + \tau^f_{i,t+1}}{1 + \tau^f_{it}} \right) q^a_{t+1}(1 + r^f_t) \right] = 1.$$  

As a result, the log-linear uncovered interest rate parity condition (22) also changes to

$$\hat{R}_{it} + (\Delta \tau^f_{1,t+1} - \Delta \tau^f_{2,t+1}) + (E_t \hat{\mu} \hat{e}_{t+1} - \hat{\mu} \hat{e}_t) = \hat{R}_{2t},$$

where $\Delta \tau^f_{i,t+1} = E_t \hat{\tau}_{i,t+1} - \hat{\tau}_{i,t}$, and the ‘hat’ denotes a percentage deviation from steady-state. The interpretation of the tax is as in the case of the tax on domestic bonds, except that it now applies to the international asset market. It is important to note that as the tax does not show up in the Euler equation for domestic bonds, it affects the the price level and the nominal interest rate only indirectly by affecting output and the real return to capital (by affecting the real
exchange rate and thus international trade and borrowing and lending).

As in the previous case we postulate a VAR(1) process for technology and the tax in the two countries. The Λ’s are again chosen by minimizing a distance between moments in the data and in the model: \( \text{corr}(\ln z_{1t}, \ln z_{1,t-1}), \text{corr}(\ln z_{1t}, \ln z_{2,t-1}), \text{corr}(\ln z_{1t}, \ln z_{2,t-3}) \), for the same reasons as before; \( \text{corr}(\ln \text{ner}_{1t}, \ln \text{ner}_{1,t-1}) \), nominal exchange rate persistence; \( \text{corr}(\text{ner}_{1,t-3}, \text{GDP}_{1t}), \text{corr}(\text{ner}_{1,t-1}, \text{GDP}_{1t}), \text{corr}(\text{ner}_{1,t+1}, \text{GDP}_{1t}), \text{corr}(\text{ner}_{1,t+3}, \text{GDP}_{1t}) \); the lead-lag pattern of the nominal exchange rate in relation to output; and \( \text{std}(\text{ner}_t)/\text{std}(	ext{GDP}_t) \), its volatility relative to that of real GDP. We also allow \( \varepsilon^2_t \) to be non-zero and include it among the parameters of the stochastic process to be calibrated—this way we have as many parameters as moments. All other parameters are as in Table 3.

We consider two versions of this extension: without and with capital adjustment costs, \( \varphi (k_{t+1} - k_t)^2 \), which reduce the volatility of investment and net exports in response to volatility of the real exchange rate.\(^{19}\) In the case with capital adjustment costs, we also include \( \text{std}(x_t)/\text{std}(\text{GDP}_t) \) among the moments and include \( \varphi \) among the parameters chosen to match the moments. In light of our discussion of the mechanism with distortions, it is worth pointing out that the capital adjustment cost works like a tax on capital—it creates a wedge in the Euler equation for capital (Chari et al., 2007).

The parameters of the VAR process, and of the capital adjustment cost, are reported in Table 7. The table also contains the results for the cross-country correlations, as well as for the dynamics of the exchange rate, investment, and net exports in relation to domestic GDP. We see that in both cases the cross-country correlations of the two nominal variables are higher than the cross-country correlation of GDP, while the model accounts for about 61% of nominal exchange rate volatility (for a given volatility of real GDP). This is about 2.5 times more than in the baseline model.

Finally, it is important to point out that, for the reasons discussed in Section 6, price levels are relatively smooth even when the baseline model is extended as we have done here. The mechanism is thus consistent with a key feature of international relative price movements pointed out by Mussa (1986): the ratio of price levels is smooth whereas the nominal exchange rate is volatile, implying (as can be seen from equation (15)) a volatile real exchange rate. In the case without capital adjustment costs, the (absolute, that is not relative to GDP) standard deviation of the nominal exchange rate is 2.00, while that of the ratio of the price levels is only 0.15. In the case with capital adjustment costs, the standard deviation of the nominal exchange rate is 2.17, while that of the ratio of the

\(^{19}\)There is a trade-off between achieving realistic volatility of the exchange rate on one hand and of investment and net exports on the other. Matching the volatility of the exchange rate exactly produces almost five times as volatile investment and net exports as in the data. It also prevents the model from generating realistic lead-lag relationship between the exchange rate and real GDP.
price levels is 1.08.

9 Concluding remarks

This study makes the empirical contribution of documenting that, at business cycle frequencies, fluctuations in aggregate price levels and nominal interest rates are substantially more synchronized across countries than are fluctuations in output. This is an intriguing empirical regularity both from a theoretical point of view, as well as from the perspective of the policy debate about how the domestic nominal environment is determined in a globalized world.

We then ask what is the key mechanism that brings about this striking empirical regularity. To this end, we employ an international business cycle model that includes nominal assets and, in each country, a monetary authority who follows a rule with considerable empirical support. For a benchmark calibration, the cross-country correlation of output is slightly lower than that in the data, while the cross-country correlations of prices and nominal interest rates are slightly higher. Even though the answer to our question very much involves dynamics, it is quite transparent within our abstraction. Due to spillovers over time of shocks across countries, expected future responses of national central banks to fluctuations in domestic output and inflation generate movements in current prices and interest rates that strongly co-move across countries even when output does not. International nominal business cycles are thus highly synchronized even when national monetary policies focus squarely on domestic output and inflation. A key element of our finding is that even a modest degree of spillovers, in the range of the smaller estimates found in the literature, is sufficient to generate correlations such as those in the data.

Having refrained from making our model more detailed than necessary for our question while maintaining transparency of our answer, naturally there are some deviations in the data relative to the model. The most notable ones, in our context, are in the forms of lead-lag properties of prices and nominal interest rates as well as exchange-rate volatility exceeding that implied by the model. A question is, then, can these deviations be reduced significantly without invalidating our findings? To provide at least a preliminary answer to that question, we borrow insights from the recent business cycle accounting literature. This literature attributes business cycle fluctuations to a small number of wedges. For our purpose, the most relevant ones are time-varying wedges in Euler equations for financial assets. Choosing their stochastic processes appropriately makes the model also consistent with the key features of domestic nominal business cycles and with exchange rate dynamics while maintaining our key relative-correlations finding. It is well known that more than
one ‘deep’ model feature may map into the same wedge. We relate these wedges to recent literature on interest rate and exchange rate dynamics and interpret them as capturing various distortions in asset markets or time-varying risk premia. A promising avenue for future research is to focus on exploring which mechanisms can distort the Euler equations over the business cycle in a similar way as the wedges in our model. More generally, model features from which we have abstracted may imply mechanisms that interact to strengthen or weaken that emphasized in this paper.
References


Appendix: Data sources

For all countries, data on real GDP and the price level (consumer price index) come from the International Financial Statistics (IFS) database. For Germany, the consumer price index for the period 1960.Q1-1991.Q4 is for West Germany only. Wherever possible, the nominal interest rate is the yield on a 3-month government bond. For Austria we use the yield on a 1-year government bond, and for France and Japan we use a money market rate. The interest rate data for Japan, the United Kingdom, and the United States come from the IFS database; for Australia, Canada, and Germany from the Global Financial Data database; for Austria from Datastream; and for France from the IFS database for the period 1970.Q1-1999.Q1, and from Datastream for the period 1999.Q2-2006.Q4.
Figure 1: Cross-country comovement of nominal variables vs cross-country comovement of real GDP—the six-country sample, 1960.Q1-2006.Q4.
Table 1: Cross-country correlations, 1960.Q1-2006.Q4

(a) Real GDP

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Excluding Bretton Woods period
Mean = 0.25  CV = 1.21

(b) Nominal interest rates

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Excluding Bretton Woods period
Mean = 0.57  CV = 0.26

(c) Price levels

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Mean = 0.52  CV = 0.28
Excluding Bretton Woods period
Mean = 0.50  CV = 0.29

Note: The numbers in parentheses are the 5th percentiles for $\text{corr}(R_i, R_j) - \text{corr}(GDP_i, GDP_j)$ obtained by bootstrapping.

Note: The numbers in parentheses are the 5th percentiles for $\text{corr}(p_i, p_j) - \text{corr}(GDP_i, GDP_j)$ obtained by bootstrapping.
Table 2: Cross-country correlations, 1970.Q1-2006.Q4

(a) Real GDP

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mean = 0.43  CV = 0.54

Excluding Bretton Woods period
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post-1984
mean = 0.19  CV = 2.07

(b) Nominal interest rates

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Excluding Bretton Woods period
mean = 0.55  (0.13)  CV = 0.25

post-1984
mean = 0.46  (0.17)  CV = 0.48

Note: The numbers in parentheses are the 5th percentiles for \( \text{corr}(R_{i}, R_{j}) - \text{corr}(\text{GDP}_{i}, \text{GDP}_{j}) \) obtained by bootstrapping.

(c) Price levels

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mean = 0.59  (0.08)  CV = 0.23

Excluding Bretton Woods period
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post-1984
mean = 0.45  (0.16)  CV = 0.56

Note: The numbers in parentheses are the 5th percentiles for \( \text{corr}(p_{i}, p_{j}) - \text{corr}(\text{GDP}_{i}, \text{GDP}_{j}) \) obtained by bootstrapping.
Figure 2: Correlations of nominal variables in period $t + j$ with real GDP in period $t$. 
Table 3: Baseline calibration

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<tr>
<td>$\kappa_2$</td>
<td>1.0</td>
<td>Curvature parameter</td>
</tr>
<tr>
<td><strong>Monetary policy rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.0091</td>
<td>Steady-state inflation rate</td>
</tr>
<tr>
<td>$\nu_y$</td>
<td>0.125</td>
<td>Weight on GDP</td>
</tr>
<tr>
<td>$\nu_\pi$</td>
<td>1.5</td>
<td>Weight on inflation</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.75</td>
<td>Smoothing coefficient</td>
</tr>
<tr>
<td><strong>Process for technology shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_0 =$</td>
<td>$\begin{bmatrix} 0.00072 &amp; 0.00072 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$A =$</td>
<td>$\begin{bmatrix} 0.906 &amp; 0.088 \ 0.088 &amp; 0.906 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\text{Var } \varepsilon_1 = \text{Var } \varepsilon_2 =$</td>
<td>0.00852^2</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(\varepsilon_1, \varepsilon_2) =$</td>
<td>0.258</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: International business cycle$^a$

<table>
<thead>
<tr>
<th></th>
<th>Correlation $(p_1, p_2)$</th>
<th>Correlation $(R_1, R_2)$</th>
<th>Correlation $(GDP_1, GDP_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model economy</td>
<td>0.69</td>
<td>0.68</td>
<td>0.23</td>
</tr>
<tr>
<td>Six-country sample, 1960.Q1-2006.Q4</td>
<td>0.52</td>
<td>0.57</td>
<td>0.27</td>
</tr>
<tr>
<td>Eight-country sample, 1970.Q1-2006.Q4</td>
<td>0.59</td>
<td>0.59</td>
<td>0.43</td>
</tr>
</tbody>
</table>

$^a$ The entries for the model are averages for 100 runs of the length of 188 periods each. As in the case of the data, the series for output and prices in the model are in logs and all series are filtered with the Christiano-Fitzgerald (2003) band-pass filter.
### Table 5: Domestic business cycle

#### A. Model economy

<table>
<thead>
<tr>
<th>$v_{t+j}$</th>
<th>Rel. Correlations of GDP in period $t$ with variable $v$ in period $t+j$:</th>
<th>st.dev.</th>
<th>$j = -4$</th>
<th>$j = -3$</th>
<th>$j = -2$</th>
<th>$j = -1$</th>
<th>$j = 0$</th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
<th>$j = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP $(qy)$</td>
<td>1.21 0.02 0.29 0.62 0.89 1.00 0.89 0.62 0.29 0.02</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Consumption $(c)$</td>
<td>0.53 -0.06 0.20 0.51 0.80 0.95 0.90 0.69 0.42 0.17</td>
<td>0.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment $(x)$</td>
<td>3.35 0.04 0.29 0.60 0.85 0.93 0.79 0.63 0.52 0.18</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Hours $(n)$</td>
<td>0.43 0.08 0.34 0.66 0.90 0.97 0.82 0.52 0.17 -0.10</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net exports $(nx)$</td>
<td>0.25 0.05 -0.13 -0.35 -0.54 -0.61 -0.53 -0.33 -0.11 0.06</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price level $(p)$</td>
<td>1.00 0.47 0.34 0.12 -0.13 -0.37 -0.52 -0.57 -0.54 -0.46</td>
<td>0.46</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Nominal interest rate $(R)$</td>
<td>0.67 0.02 -0.24 -0.55 -0.82 -0.95 -0.88 -0.66 -0.37 -0.12</td>
<td>0.12</td>
<td></td>
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</tr>
<tr>
<td>Nom. exchange rate $(ner)$</td>
<td>0.75 -0.42 -0.35 -0.21 -0.03 0.16 0.29 0.34 0.33 0.28</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**J-curve**

<table>
<thead>
<tr>
<th>$J$-curve</th>
<th>Correlations of terms of trade $(e_t)$ with net exports $(nx_{t+j})$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$j = -4$</td>
</tr>
<tr>
<td></td>
<td>-0.50</td>
</tr>
</tbody>
</table>

#### B. U.S. economy

<table>
<thead>
<tr>
<th>$v_{t+j}$</th>
<th>Rel. Correlations of GDP in period $t$ with variable $v$ in period $t+j$:</th>
<th>st.dev.</th>
<th>$j = -4$</th>
<th>$j = -3$</th>
<th>$j = -2$</th>
<th>$j = -1$</th>
<th>$j = 0$</th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
<th>$j = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.48 0.22 0.50 0.75 0.93 1.00 0.93 0.75 0.50 0.22</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.52 0.15 0.37 0.58 0.73 0.80 0.78 0.68 0.53 0.34</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>2.99 0.35 0.58 0.78 0.91 0.94 0.86 0.68 0.43 0.17</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hours</td>
<td>0.90 0.07 0.19 0.46 0.71 0.88 0.95 0.91 0.76 0.55</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net exports</td>
<td>0.25 -0.48 -0.52 -0.56 -0.58 -0.58 -0.52 -0.39 -0.21 -0.01</td>
<td>0.01</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Price level</td>
<td>0.82 -0.70 -0.77 -0.78 -0.73 -0.60 -0.43 -0.23 -0.03 0.16</td>
<td>0.16</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.73 -0.66 -0.47 -0.22 0.06 0.31 0.49 0.58 0.59 0.53</td>
<td>0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nom. exchange rate</td>
<td>3.09 0.23 0.21 0.21 0.20 0.19 0.17 0.14 0.08 0.01</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

**J-curve**

<table>
<thead>
<tr>
<th>$J$-curve</th>
<th>Correlations of terms of trade $(e_t)$ with net exports $(nx_{t+j})$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$j = -4$</td>
</tr>
<tr>
<td></td>
<td>-0.09</td>
</tr>
</tbody>
</table>

---

*The entries are averages for 100 runs of the length of 188 periods each. Except for net exports and the nominal interest rate, all artificial series are in logs; the nominal interest rate is expressed at annual rates. Before computing the statistics, the artificial series were filtered with the Christiano-Fitzgerald (2003) band-pass filter.

*Standard deviations are measured relative to that of GDP; the standard deviation of GDP is in absolute terms.

*Except for net exports and the nominal interest rate, all data series are in logs; net exports are measured as a fraction of trend GDP and the nominal interest rate is expressed at annual rates. All statistics are based on series filtered with the Christiano-Fitzgerald (2003) band-pass filter. Consumption is measured as the sum of nondurables, services, and government expenditures; investment as the sum of fixed private investment and consumer durables; hours as total hours in nonagricultural establishments; and terms of trade as the ratio of import and export price deflators.
Figure 3: Responses to a 1% technology shock in country 1 for the baseline calibration; rates of return are measured as percentage point deviations from steady state at annual rates; all other variables as percentage deviations.
Figure 4: Sensitivity analysis: varying the weight on GDP in the Taylor rule.
Weight on inflation in the Taylor rule

\[ \nu_y = 0 \quad \quad \quad \nu_y = 0.125 \]

Interest rate smoothing

\[ \nu_y = 0 \quad \quad \quad \nu_y = 0.125 \]

Spillovers

\[ \nu_y = 0 \quad \quad \quad \nu_y = 0.125 \]

Figure 5: Sensitivity analysis
Table 6: Extension with a time-varying wedge in the Euler equation for domestic bonds

A. Parameters of the transition matrix of the VAR(1) process$^a$

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda_{11}$</th>
<th>$\Lambda_{12}$</th>
<th>$\Lambda_{13}$</th>
<th>$\Lambda_{14}$</th>
<th>$\Lambda_{21}$</th>
<th>$\Lambda_{22}$</th>
<th>$\Lambda_{23}$</th>
<th>$\Lambda_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.075</td>
<td>0.642</td>
<td>0.18</td>
<td>-0.44</td>
<td>0.808</td>
<td>-0.112</td>
<td>0.999</td>
<td>0.496</td>
</tr>
</tbody>
</table>

B. Domestic nominal business cycle

<table>
<thead>
<tr>
<th>$v_{t+j}$ std$^b$</th>
<th>Correlations of GDP in period $t$ with variable $v$ in period $t + j$:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$j = -4$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$p$</td>
<td>0.50</td>
<td>0.06</td>
</tr>
<tr>
<td>$R$</td>
<td>0.85</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

C. Cross-country correlations

<table>
<thead>
<tr>
<th>($p_1,p_2$)</th>
<th>($R_1,R_2$)</th>
<th>($GDP_1,GDP_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>0.91</td>
<td>0.41</td>
</tr>
</tbody>
</table>

$^a$The parameters are chosen by minimizing the distance between data and model moments. The moments include: $\text{corr}(R_{1t},R_{1,t-1}), \text{corr}(R_{1,t-3},GDP_{1t}), \text{corr}(R_{1,t-1},GDP_{1t}),\text{corr}(R_{1,t+1},GDP_{1t}), \text{corr}(R_{1,t+3},GDP_{1t}), \text{corr}(\ln z_{1t},z_{1,t-1}),\text{corr}(\ln z_{1t},z_{2,t-1}), \text{and corr}(\ln z_{1t}, z_{2,t-3})$.

$^b$ Standard deviations are divided by that of $GDP_{1t}$. 

Table 7: Extension with a time-varying wedge in the Euler equation for foreign bonds

A. New parameters\textsuperscript{a}

\begin{table}[h]
\centering
\begin{tabular}{cccccccccc}
\hline
& $\Lambda_{11}$ & $\Lambda_{12}$ & $\Lambda_{13}$ & $\Lambda_{14}$ & $\Lambda_{21}$ & $\Lambda_{22}$ & $\Lambda_{23}$ & $\Lambda_{24}$ & $\text{Var } \varepsilon_t$ & $\varphi$ \\
\hline
$\varphi = 0$ & 0.707 & 0.007 & 0.297 & -0.005 & -2.352 & -0.003 & -0.004 & -0.001 & 4.8e\text{-4} & 0 \\
$\varphi > 0$ & 0.742 & 0.007 & 0.263 & -0.004 & -3.258 & -0.003 & -0.004 & -0.001 & 7.29e\text{-4} & 0.23 \\
\hline
\end{tabular}
\end{table}

B. Cyclical behavior of the nominal exch. rate, investment, and net exports

\begin{table}[h]
\centering
\begin{tabular}{lcccccccc}
\hline
$\varphi$ & $v_{t+j}$ & Correlations of \textit{GDP} in period $t$ with variable $v$ in period $t+j$: & std\textsuperscript{b} & $j = -4$ & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & \textit{ner} & 1.86 & -0.14 & 0.04 & 0.25 & 0.37 & 0.30 & 0.07 & -0.17 & -0.29 & -0.24 \\
 & \textit{x} & 5.78 & -0.17 & 0.06 & 0.40 & 0.65 & 0.67 & 0.42 & 0.05 & -0.23 & -0.32 \\
 & \textit{nx} & 1.03 & 0.15 & -0.04 & -0.27 & -0.41 & -0.35 & -0.11 & 0.16 & 0.32 & 0.29 \\
0.23 & \textit{ner} & 1.89 & -0.28 & -0.09 & 0.22 & 0.46 & 0.49 & 0.28 & -0.03 & -0.27 & -0.32 \\
 & \textit{x} & 4.93 & -0.19 & 0.07 & 0.44 & 0.73 & 0.77 & 0.52 & 0.11 & -0.23 & -0.36 \\
 & \textit{nx} & 0.88 & 0.18 & -0.03 & -0.32 & -0.51 & -0.48 & -0.21 & 0.13 & 0.37 & 0.39 \\
\hline
\end{tabular}
\end{table}

C. Cross-country correlations

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
\textit{(p}_1, \textit{p}_2) & \textit{(R}_1, \textit{R}_2) & \textit{(GDP}_1, \textit{GDP}_2) \\
\hline
$\varphi = 0$ & 0.94 & 0.92 & 0.45 \\
$\varphi = 0.23$ & 0.65 & 0.30 & 0.19 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{a}The parameters are chosen by minimizing the distance between data and model moments. The moments include: \text{corr}(\textit{ner}_1, \textit{ner}_{1,t-1}), \text{corr}(\textit{ner}_{1,t-3}, \textit{GDP}_{1t}), \text{corr}(\textit{ner}_{1,t-1}, \textit{GDP}_{1t}), \text{corr}(\textit{ner}_{1,t+1}, \textit{GDP}_{1t}), \text{corr}(\textit{ner}_{1,t+3}, \textit{GDP}_{1t}), \text{corr}(\textit{ln } z_{1t}, z_{1,t-1}), \text{corr}(\textit{ln } z_{1t}, z_{2,t-1}), \text{corr}(\textit{ln } z_{1t}, z_{2,t-3}), \text{and } \text{std}(\textit{ner}_{1t})/\text{std}(\textit{GDP}_{1t})$, and in the case of $\varphi > 0$ also \text{std}(\textit{x}_{1t})/\text{std}(\textit{GDP}_{1t})$.

\textsuperscript{b}Standard deviations are measured relative to that of $\textit{GDP}_{1t}$.