Collateral Crises*

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Abstract

Short-term, collateralized, debt is efficient if agents are willing to lend without producing costly information about the value of the collateral. When the economy relies on this informationally-insensitive debt, information is not renewed over time. If the value of collateral is mean reverting, there is a credit boom when firms with bad collateral start borrowing as the information about their collateral depreciates.

The longer an economy remains in an information-insensitive regime, the smaller the fraction of collateral with information about their true value, and the larger the fraction of collateral that look similar. This creates fragility, since a small aggregate shock to collateral values is more likely to generate a large systemic collapse in output and consumption. Furthermore, if a crisis triggers information production, the economy takes longer to recover.

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1 Introduction

Financial intermediaries can provide efficient transactions services by designing their liabilities so that they can be used to trade without adverse selection. But, a shock can occur which causes this characteristic of information-insensitivity of bank liabilities to disappear, raising the fear of adverse selection when trading with bank liabilities; then there can be a systemic crisis or a credit crunch. In this paper we investigate the implications of this vulnerability of bank liabilities for crises and the business cycle. We show how "small" shocks can cause systemic crises at some times but not others. We also show "small" shocks can drastically affect the effectiveness of policies to solve crises and the speed of recoveries. There are business cycles in which a long credit boom is followed by a sharp systemic crisis and a slow recovery.

Financial intermediation is most importantly about the creation of short-term securities (liabilities of banks) that provide transactions services. Examples are pre-Civil War bank notes (in the U.S.), demand deposits, and, of late, sale and repurchase agreements ("repo"). These bank liabilities can be used directly for transactions or can be transformed into a fixed amount of cash easily. In order to provide transaction services, bank liabilities are designed to be information-insensitive (i.e., it is not profitable for any agent to produce information about the backing assets) so that users do not face adverse selection. To accomplish this, bank money is "backed" by collateral (designated state bonds in the case of pre-Civil War free banks, or specific bond collateral that the depositor takes possession of in repo) or by a diversified portfolio of loans (in the case of demand deposits). A crisis occurs when there is a shock causing bank liability holders to suspect that the backing collateral has deteriorated in value such that it has become information-sensitive. The bank debt holders then seek to withdraw their cash or they do not roll over the debt (e.g., in repo), causing a potential liquidation of the banking system.

While the U.S. had an extraordinary period of quiet with respect to banking crises from 1934 to 2007, banking crises are, in fact, very common and very costly. There is a connection between recessions and financial crises. Financial crises tend to occur near business cycle peaks, when economic activity is starting to weaken and banking crises are often preceded by credit booms.¹

As suggested above, our starting point is that the raison d’être of financial interme-
diation is the production of short-term bank debt for transactions purposes. Gorton 
and Pennacchi (1990) argue that intermediaries exist to create trading securities that 
are immune to adverse selection when used by agents in markets. In their setting, 
the securities used were riskless debt securities, backed by bank assets. The notion 
of “liquidity” proposed was the idea that trade was facilitated by securities that were 
immune to adverse selection. Diamond and Dybvig (1983), on the other hand, view 
“liquidity” as consumption smoothing. While both papers assumed that debt was 
optimal, Dang, Gorton, and Holmström (2011) show that debt is in fact the optimal 
trading security. The debt need not be riskless, but its defining characteristic is that it 
is “information-insensitive,” which means that it is common knowledge that it is not 
profitable to produce (costly) information about the payoff on the security. There is no 
adverse selection when trading. A shock, however, can cause previously information-
insensitive debt to become information-sensitive, possibly leading to a crisis.

In this paper we build on these micro foundations to investigate the role of such 
information-insensitive debt in a macro economy. We do not explicitly model the 
trading motive for short-term information-insensitive debt. We assume that house-
holds have a demand for such debt and, further, we assume that the short-term debt 
is issued directly by firms to households. The debt that firms issue is backed by col-
lateral. We show that, while the lack of information acquisition improves trade and 
fuels credit booms, it also causes most collateral to look similar over time, increasing 
the potential losses in case of a crisis and slowing down a potential recovery.

The amount and nature of the borrowing depends on households’ beliefs about the 
value of the collateral, and on the households’ decisions about whether to produce in-
formation about the collateral value or not. Each firm’s collateral can be either good 
or bad and its perceived quality is given by the probability that the collateral is good. 
To determine the real quality of the collateral is costly. If households have incentives 
to learn about the true quality of the collateral, firms may prefer to cut back on the 
amount borrowed to avoid costly information production, a credit constraint. How-
ever, firms whose collateral is of intermediate perceived quality may prefer to have 
information produced, and offer an interest rate that covers the cost of information 
production by households.

We focus on the dynamics of firms issuing information-insensitive debt and those is-
suing information-sensitive debt. If households know a firm has good collateral, they are willing to lend the optimal capital firms require. If they know a firm has a bad collateral, they are not willing to make any loan to the firm. We assume all collateral changes quality with certain probability, so if households do not renew the information, they decide on the loan based on the perceived quality. When the perceived quality is large enough not only firms with good collateral can borrow but also some firms with bad collateral but without information about it. In fact, consumption is highest if there is no information production, because then all firms can borrow, regardless of their true quality. In our setting opacity dominates transparency and the economy enjoys a blissful ignorance.

If there has been information-insensitive lending for a long time, there can be a significant depreciation of information in the economy and only a small fraction of collateral with known quality. In this setting we introduce aggregate shocks that may increase or decrease the average value of collateral in the economy. We show that the longer information about the collateral has not being renewed, the greater the credit boom but also the greater the fragility characterized by a larger drop in consumption when a negative aggregate shock hits. In other words, a shock of a given size can have a larger impact on consumption the longer the preceding credit boom. The reason is that a negative aggregate shock affects more collateral than the same aggregate shock when the value of collateral is known. Hence, the size of the downturn depends on how long debt has been information-insensitive in the past.

The crisis may trigger information production or not, and this will have important implications for the speed of recovery after a crisis. If the negative aggregate shock does not trigger information production (firms reduce borrowing to avoid information acquisition), a subsequent positive shock (e.g., a policy to improve collateral value) can help the economy recover fast since still most collateral looks similar and firms can borrow the optimum when the expected value of the collateral improves enough. If the negative aggregate shock does trigger information production, the economy will slowly recover since information will take time to vanish again, and for a while firms that were found to have bad collateral will not be able to borrow, until that information depreciates again.

In the next Section we present the model. In Section 3 we study debt decisions by a single firm. In Section 4 we study the aggregate and dynamic implications of infor-
mation sensitiveness. In Section 5 we show a numerical simulation to illustrate our results. In Section 6, we conclude.

2 Model

The economy is composed of a mass 1 continuum of long-lived firms. Each firm has a unit of land $X$ that is required to operate a project with a Leontief production function:

$$Y = \begin{cases} 
A \min\{K, L\} & \text{with prob. } q \\
0 & \text{with prob. } (1 - q)
\end{cases}$$

where $A > 1$. Each firm has available a fixed amount of managerial labor $L^*$ (that does not generate any disutility) and needs to borrow the capital $K$ to produce. We assume depreciation is $\delta = 1$, so firms need to borrow capital each period to produce. We also assume that the expected marginal product of capital is higher than its marginal cost $qA > 1$, hence production is efficient. Naturally, given the Leontief production function, $K^* = L^*$ is the level of capital that maximizes profits and is the amount that each firm tries to borrow.

There is also a mass 1 continuum of short-lived households, who are born with an endowment $\overline{K}$ of a numeraire good, which they can either consume or lend as capital to firms. We assume $\overline{K} > K^*$, hence households have enough endowment to finance efficient production.

Households have shares in all firms, such that firms’ profits are transferred back to households. This assumption guarantees that it is sufficient to measure welfare in the economy at a period $t$ by the expected aggregate consumption,

$$W_t = E(Y_t) - K_t + \overline{K}.$$ 

The unconstrained efficient situation $W^*$ is given by complete financing of the project to capacity. Combined with our assumption of a continuum of firms,

$$W^* = K^*(qA - 1) + \overline{K}.$$ 

Having discussed the technology, preferences and objectives that agents face, we need
to discuss the characteristics of the lending market. We assume lenders cannot observe or seize the production of the firms they finance, which means there is no lending unless there is some collateral lenders can liquidate in case of no repayment. We assume the firm can use the unit of land $X$ that is required to produce as a collateral.

The collateral can be either good (with a true value $C$) or bad (with a true value $0$), which does not affect production. In every period, with probability $\lambda$ the true quality of each unit of collateral remains unchanged and with probability $(1 - \lambda)$ there is an idiosyncratic shock such that the collateral changes quality. In this last case, the collateral becomes good with a probability $\hat{p}$. Even when the shock is observable, the realization of the new quality is not, unless a certain amount of the numeraire good $\gamma$ is used to learn about it. The idiosyncratic shock and the realization are assumed independent of the idiosyncratic production shock $q$.

Conditional on the whole history of idiosyncratic shocks and monitoring results, all agents in the economy share a belief $p$ about each collateral, which is the probability the collateral is good. Hence the value of a collateral with belief $p$ is:

$$X = \begin{cases} C & \text{with prob. } p \\ 0 & \text{with prob. } (1 - p) \end{cases}$$

In case the firm decides not to pay back its debt, the lender can seize the collateral and sell it a fair price $pC$. Recall, in the aggregate, the mass of firms is independent of the number of firms that can pay back the debt or not. This is because there is a competitive market for land, and we assume no fire sales.

We will proceed in three steps. First we analyze the borrowing decision of a single firm that has collateral which is good with probability $p$. Then, we study the aggregate output when the distribution of beliefs about the collateral quality in the economy has a constant mean $\hat{p}$ and an endogenously evolving variance. Finally, we introduce aggregate shocks that affect all collateral in the economy, in addition to the idiosyncratic shocks, changing the mean of that distribution.

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2For the moment we abstract from the possibility of asymmetric information, under which the firm may potentially know more about the collateral than the lenders.

3This is a key difference with Kiyotaki and Moore (1997). In their paper credit cycles are generated by cyclical fluctuations in the endogenous price of collateral. In this paper, they will be generated by cyclical fluctuations in the endogenous information available in the economy about the collateral.
3 A single firm

In this section we study the short-term debt contract between a single lender and a single firm and characterize the optimal debt the firm issues considering the possibility that the lender may want to produce information about the true quality of the collateral.

First we solve the optimal borrowing decision of a single firm that lives a single period, and decides whether to issue debt that triggers information production or not. Triggering information production (information-sensitive debt) is costly for the firm because it raises the cost of borrowing. However, not triggering information production (information-insensitive debt) is also costly because it may reduce the possible size of the loan. This trade-off determines the information sensitiveness of the debt and then the volume of information in the economy.

3.1 A Single Period

3.1.1 Information-Insensitive Debt

Lenders can learn the true quality of the land by paying an amount $\gamma$ of the numeraire good. When firms borrow an amount $K$ to use as capital, there are no incentives for the lender to produce information about the collateral if:

$$(1 - q)(1 - p)K \leq \gamma,$$

that is, if the expected gains of producing information (learning the collateral has no value, which occurs with a probability $(1 - p)$, saves the lender from lending $K$ and not recovering it in case the production is not successful, which happens with probability $(1 - q)$) is smaller than the cost of producing information, $\gamma$.

It is clear from the previous condition that the firm can discourage information production about collateral that has a high perceived probability of being bad (high $(1 - p)$), by reducing the size of the loan $K$. Hence, the cost for a firm to save on information production is less borrowing and lower output.

If the condition is not binding, then there are no strong incentives for lenders to produce information and $K = K^\ast$. If the condition is binding $K = \frac{\gamma}{(1 - q)(1 - p)}$. 

We assume lenders are competitive and risk neutral. Then they break even when determining the face value of the debt $R$

\[ qR + (1 - q)pC = K \quad \Rightarrow \quad R = \frac{K - (1 - q)pC}{q}. \]

Naturally, the lender has to believe that the firm prefers to repay the loan $R$ rather than not and lose the collateral. This incentive compatibility constraint is characterized by $R < pC$. If this inequality is not fulfilled, then the firm would prefer to default for sure, hand the collateral to the lender and buy it back in the market at a price $pC$. If this constraint is binding, the firm cannot obtain a loan higher than $K = pC$.

Hence, information-insensitive borrowing is characterized by the following debt size:

\[ K(p|II) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - p)}, pC \right\} \quad (1) \]

We focus on the case in which information-insensitive borrowing is characterized by three regions.\(^4\)

\[ K(p) = \begin{cases} 
K^* & \text{if } K^* \leq \frac{\gamma}{(1 - q)(1 - p)} \quad \text{(no credit constraint)} \\
\frac{\gamma}{(1 - q)(1 - p)} & \text{if } K^* > \frac{\gamma}{(1 - q)(1 - p)} \quad \text{(credit constraint)} \\
pC & \text{if } C < \frac{\gamma}{p(1 - q)(1 - p)} \quad \text{(no lending)}
\end{cases} \]

The first kink is generated by the point at which the constraint to avoid information production is binding when evaluated at the optimal loan size $K^*$, this is when financial constraints start binding more than technological constraints. The second kink is generated by the incentive compatibility constraint $R < pC$, below which the firm is only able to borrow up to the expected value of the collateral $pC$.

\(^4\)This is the more natural case when $C > K^*$ and $\gamma$ is not large (specifically $\gamma < \left( \frac{C - K^*}{C} \right)(1 - q)K^*$).

If $C > K^*$ and $\gamma > \left( \frac{C - K^*}{C} \right)(1 - q)K^*$, then there are four regions, where a second one is added such that $K = pC$. If $C < K^*$, then the first region is given by $pC$ and not $K^*$, since $pC$ is always smaller than $K^*$. In all cases the main conclusions we will derive are the same, this is why we will focus on the more natural case shown in expression 1 to discuss the model.
Expected profits under information-insensitive borrowing are:

\[ q( AK - R + pC ) + (1 - q) 0, \]

i.e., with probability \( q \) production is successful, the firm pays back the debt and keeps the collateral, which has an expected value of \( pC \); with probability \( 1 - q \) the firm does not produce and loses the collateral. Then:

\[ E(\pi|p, II) = K(p|II)(qA - 1) + pC \tag{2} \]

In the case we are focusing on, considering explicitly the kinks,

\[
E(\pi|p, II) = \begin{cases} 
K^*(qA - 1) + pC & \text{if } K^* \leq \frac{\gamma}{(1-q)(1-p)} \\
\frac{\gamma}{(1-q)(1-p)}(qA - 1) + pC & \text{if } K^* > \frac{\gamma}{(1-q)(1-p)} \\
pCqA & \text{if } C < \frac{\gamma}{p(1-q)(1-p)}
\end{cases}
\]

3.1.2 Information-Sensitive Debt

Firms with relatively low \( p \) will be borrowing constrained, since they will borrow less to restrain lenders’ incentives to produce information, and lenders charge firms for that activity. However, if the implied reduction in borrowing volume is large enough, it may be in the best interest of the firm to just give up to adverse selection and allow information production.

Still, lenders have to break even in their decision to produce information and lend. This is, lenders should charge an interest rate that makes them indifferent ex-ante between producing information or not.\(^5\)

\[ p(qR + (1 - q)C - K^*) = \gamma \quad \Rightarrow \quad R = \frac{pK^* + \gamma - (1 - q)pC}{pq}. \]

In this case, the expected borrowing of a firm with collateral that has a probability \( p \)

\(^5\)We assume this interest rate \( R \) is not renegotiable after the lender finds out the collateral is good, otherwise there would never be incentives to produce information in the first place.
of being good (net of information costs) is:

\[ K(p|IS) = \max \left\{ pK^* - \frac{\gamma}{(qA - 1)}, 0 \right\}. \] (3)

The expected profits from borrowing that triggers information production are \( p[q(AK^* - R + C) + (1 - q)0] + (1 - p)0 \), or

\[ E(\pi|p, IS) = K(p|IS)(qA - 1) + pC. \] (4)

Then,

\[ E(\pi|p, IS) = \begin{cases} pK^*(qA - 1) - \gamma + pC & \text{if } K^* + \frac{\gamma}{p} \leq C \\ pC & \text{if } K^* + \frac{\gamma}{p} > C \end{cases} \]

The kink is also determined by incentive compatibility \( R < C \). When the face value of debt is very high, the firm may decide not to repay the loan, but rather hand in the collateral and rebuy it later. In this case the lender will not obtain more than \( p(C - K^*) \). If this is less than \( \gamma \), the lender would prefer not producing information and not lending at all.

### 3.1.3 Optimal Debt

Figure 1 shows the profits under these two regimes for each possible \( p \) (after deducting the expected value of the collateral \( pC \) in all expressions). From the comparison of which one is larger we can obtain the values of \( p \) for which the firm prefers to borrow with an information-insensitive loan (II) or with an information-sensitive loan (IS).

The cutoffs highlighted in Figure 1 are determined in the following way: \( p^H \) is the \( p \) that generates the first kink of the profit function of an information-insensitive loan. This is the \( p \) below which firms have to reduce borrowing to prevent information production:

\[ p^H = 1 - \frac{\gamma}{(1 - q)K^*}. \]

The cutoff \( p^L_{II} \) is obtained from the second kink of the profit function of an information-
insensitive loan,\(^6\)

\[
p_{II}^L = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\gamma}{(1-q)(1-p)}}.
\]

Similarly, the cutoff \(p_{IS}^L\) is obtained from the kink of the profit function of an information-sensitive loan:

\[
p_{IS}^L = \frac{\gamma}{C - K^*}.
\]

Finally, the cutoffs \(p_{Ch}^C\) and \(p_{Cl}^C\) are obtained from equalizing the profit functions of information sensitive and insensitive loans and solving for the quadratic roots of the following equation:

\[
\frac{\gamma}{(1-q)(1-p)} = pK^* - \frac{\gamma}{(qA-1)}.
\]

There are only three regions of financing. Information-insensitive loans are chosen by firms with collateral with high and low values of \(p\), while information-sensitive loans

\(^6\)The positive root for the solution of \(pC = \gamma/(1-q)(1-p)\) is irrelevant since it is greater than \(p^H\), and then it is not binding given all firms with a collateral that is good with probability \(p > p^H\) can borrow the optimal level of capital \(K^*\) without triggering information production.
are chosen by firms with collateral with intermediate values of $p$.

To understand how these regions depend on the information cost $\gamma$, the four arrows in the Figure show how the different cutoffs and functions move as we reduce $\gamma$. In an extreme, when $\gamma = 0$, all collateral is information-sensitive (i.e., the IS region is $p \in [0, 1]$), which is intuitive since information is free.

Contrarily, as $\gamma$ increases, the two cutoffs $p^{Ch}$ and $p^{CI}$ get together and the IS region shrinks until it disappears (i.e., the II region is $p \in [0, 1]$) when $\gamma$ is large enough.

Having characterized how the information-sensitiveness of debt depends on collateral beliefs $p$, we can analyze expected consumption, which is our measure of welfare in the aggregate at a given period. By construction, at the cutoffs $p^{CI}$ and $p^{Ch}$, where the system changes from information-insensitive debt to information-sensitive debt, the expected consumption between the two regimes is the same. This is because, at the cutoffs

$$ W_{IS} \equiv pK^*(qA - 1) - \gamma + \bar{K} = K(qA - 1) + \bar{K} \equiv W_{II}. $$

In Figure 2 we show how financial constraints reduce aggregate consumption when the perceived quality of the collateral $p$ declines. Efficient consumption is the blue line and the financially constrained aggregate consumption is the red line.

### 3.2 More Periods

In this section we first assume two periods and then we extend the model to infinite periods. Again, we assume the idiosyncratic shock, which occurs with probability $(1 - \lambda)$, is observed but the realization is not, unless information is produced. To obtain the expected profit of the firm issuing information-sensitive debt and issuing information-insensitive debt, we proceed backwards. In period 2 we know the decisions for each possible $p$. In particular we know that for $p = 1$, which is the case of knowing the collateral is good for sure, a firm can always borrow $K^*$ without restrictions. Contrarily, for $p = 0$, which is the case of knowing the collateral is worthless for sure, we know there is no chance for the firm to borrow.

Defining $\beta$ as the discount factor, the expected discounted profits with information-
The expected discounted profits with information-sensitive debt is:

\[
E(\pi|IS) = \begin{cases} 
(1 + \beta q \lambda)pK^*(qA - 1) + pC - \gamma + \beta q(1 - \lambda)E(\pi|\hat{p}) & \text{if } K^* + \frac{\gamma}{p} \leq C \\
(1 + \beta q \lambda)pCqA + \beta q(1 - \lambda)E(\pi|\hat{p}) & \text{if } K^* + \frac{\gamma}{p} > C 
\end{cases}
\]

where \( E(\pi|\hat{p}) = K(\hat{p})(qA - 1) + \hat{p}C \) from equation (2) if \( \hat{p} > p^{Ch} \) or \( \hat{p} < p^{Cl} \), and from equation (4) if \( p^{Cl} < \hat{p} < p^{Ch} \).
In order to obtain the kinks in each case it is important that loans are one period contracts. This makes the problem a repeated game without links across periods, other than information production. Contrarily, the cutoffs that determine the switch between information-sensitive and information-insensitive debt, $p^{CI}$ and $p^{Ch}$, do depend on the probability of idiosyncratic shocks in each period.

When idiosyncratic shocks are i.i.d. over time and there is no persistence of the collateral quality (that is, $\lambda = 0$), the decision in period 1 about the debt type does not have any effect on the expected profit in period 2, not modifying the cutoffs obtained in the previous section. This extreme case makes the problem effectively a static one, independent on the number of periods.

On the other hand, when there is perfect persistence of the collateral quality (i.e., $\lambda = 1$), the decision in period 1 about the debt type is critical for the expected profit in period 2. If there is no information production, the perception about a collateral’s quality does not change in period 2. If there is information production, everybody observes the true quality and there is no need to produce information again during period 2.

More generally, when $\lambda \in (0, 1)$, the range of $p$ under which the firm issues information-sensitive debt widens if we consider two periods. Some persistence of the collateral quality decreases the cost of figuring out the true quality of the collateral, since $\gamma$ has to be faced only once to produce information relevant for the future. The effective monitoring cost becomes $\gamma / (1 + \beta q \lambda)$ when there are two periods.

The cutoffs $p^{CI}$ and $p^{Ch}$ are now determined by the following equation

$$\frac{\gamma}{(1 - q)(1 - p)} = pK^* - \frac{\gamma}{(1 + \beta q \lambda)(qA - 1)}.$$

As we increase the number of periods, the range of values $p$ that sustains information-sensitive debt widens even further. In particular, when $T = \infty$, the effective monitoring cost becomes $(1 - \beta q \lambda) \gamma$. Furthermore, as $\beta \to 1$, $q \to 1$ and $\lambda \to 1$, the effective cost approaches 0, and there is always information production. In Figure 3 we illustrate the effect of infinite periods in widening the range of $p$ that sustains information production.
3.3 Optimal Information Production Costs

The cost of information production, $\gamma$, is fixed. However, we can study the case in which a planner can choose the optimal monitoring costs $\gamma^*$ that maximizes aggregate consumption at each period $t$ given financial constraints.

**Proposition 1** Optimal monitoring costs are $\gamma^* = \infty$ if $C > K^*$, $\gamma^* = [0, \infty)$ if $C = K^*$ and $\gamma^* = 0$ if $C < K^*$.

**Proof** If $C > K^*$, as $\gamma \to \infty$, $E(\pi|p, II) = \min\left\{K^*, pC\right\} (qA - 1) + pC$, which is greater than $E(\pi|p, IS) = \max\left\{pK^* - \frac{\gamma}{(1-q)(1-p)}, 0\right\} (qA - 1) + pC$ for all $p$. As we reduce $\gamma$, $E(\pi|p, II)$ declines for values of $p$ where $\frac{\gamma}{(1-q)(1-p)} < pC$, reducing aggregate consumption since those collateral can borrow less. This implies that making the system information-insensitive for all collateral achieves a higher aggregate consumption than making it information-sensitive.

If $C < K^*$, when $\gamma = 0$, $E(\pi|p, IS) = \max\left\{pK^*, 0\right\} (qA - 1) + pC$, which is larger than $E(\pi|p, II) = 0$ for all $p$. As we increase $\gamma$, $E(\pi|p, II)$ can never increase above...
which is smaller than \( E(\pi|p, IS) \) for all \( p \). This implies that making the system information-sensitive for all collateral achieves a higher aggregate consumption than making it information-insensitive.

In the threshold, \( C = K^* \), the firm chooses the maximum between \( E(\pi|p, IS) = pC(qA - 1) + pC \) and \( E(\pi|p, II) = \min\left\{ K^*, \frac{\gamma}{1-q}(1-p), pC \right\} (qA - 1) + pC \), which cannot be greater than \( E(\pi|p, IS) \). Hence any value of \( \gamma \) gives an aggregate consumption equal to the aggregate consumption of the information-sensitive regime. Q.E.D.

The intuition for this result is the following. When \( C > K^* \), without producing information firms can borrow up to \( pC \) for sure. However, when producing information firms will borrow in expectation \( pK^* < pC \) and spend \( \gamma \). Hence, it is optimal to discourage information, even if information is free.

When \( C > K^* \), without producing information firms can borrow up to \( pC \) for sure. However, when producing information firms will borrow in expectation \( pK^* > pC \) and spend \( \gamma \). Hence, it is optimal to make information production as cheap as possible.

### 4 Aggregate Results

In this section we characterize the evolution of information about the collateral and its impact on aggregate consumption. First, we study a case without aggregate shocks to collateral and discuss the effects of endogenous information production on the dynamics of credit booms. Then, we introduce aggregate shocks and study the effects of endogenous information on the size of crises and the speed of recoveries.

#### 4.1 No Aggregate Shocks

Assume a firm lives many periods, as long as its production is successful. That is, a firm that was born in period 0 is alive at period \( t \) with probability \( q^t \). Assume also that initially (at period 0) there is perfect information about which collateral is good and which is bad. The probability each firm’s collateral becomes good when it is hit by an idiosyncratic shock is always \( \hat{p} \). In what follows we study the aggregate consumption.
in this economy as time evolves, for different values of \( \hat{p} \), and we show that, as long as \( \hat{p} \) is large enough consumption is growing with time since there is no information production about the collateral.\footnote{In this case, without aggregate shocks, the average quality of collateral in the market is given by \( \bar{p} = \hat{p} \) in all periods.}

First, for notational simplicity, we define, based on the analysis of Section 3:

\[
Z(p) = \begin{cases} 
K^*(qA - 1) & \text{if } p^H < p \\
\frac{\gamma}{(1-q)(1-p)}(qA - 1) & \text{if } p^{Ch} < p < p^H \\
pK^*(qA - 1) - \gamma & \text{if } p^{Cl} < p < p^{Ch} \\
\frac{\gamma}{(1-q)(1-p)}(qA - 1) & \text{if } P_{LI}^l < p < p^{Cl} \\
pC(qA - 1) & \text{if } p < P_{II}^l
\end{cases}
\]

From the definition of the cutoffs, we know \( Z(p) \) is monotonically decreasing in \( p \).

The aggregate consumption of the economy at period \( t \) is defined by:

\[
W_t = \int_0^1 Z(p)f(p)dp + \bar{K},
\]

where \( f(p) \) is the distribution of beliefs about all the collateral in the economy. In the simple stochastic process for idiosyncratic shocks we assume, and in the absence of aggregate shocks, this distribution has a three-point support: 0, \( \hat{p} \) and 1.

If \( \hat{p} > p^{Ch} \) or \( \hat{p} < p^{Cl} \), information is not reacquired and at period \( t \), \( f(0) = \lambda^t(1 - \hat{p}) \), \( f(\hat{p}) = (1 - \lambda^t) \) and \( f(1) = \lambda^t\hat{p} \). Since \( Z(0) = 0 \),

\[
W_{II}^t = [\lambda^t\hat{p}Z(1) + (1 - \lambda^t)Z(\hat{p})] + \bar{K}. \tag{5}
\]

If \( \hat{p} \in [p^{Cl}, p^{Ch}] \), information is reacquired in every period \( t \) for the fraction \( (1 - \lambda) \) of collateral that gets the idiosyncratic shock. Then \( f(0) = \lambda(1 - \hat{p}) \), \( f(\hat{p}) = (1 - \lambda) \) and \( f(1) = \lambda\hat{p} \). Considering \( Z(0) = 0 \),

\[
W_{IS}^t = [\lambda\hat{p}Z(1) + (1 - \lambda)Z(\hat{p})] + \bar{K}. \tag{6}
\]

Some interesting implications can be obtained from the previous equations. If the
economy is in an information-insensitive regime (this is, \( \hat{p} > p^{Ch} \) or \( \hat{p} < p^{Cl} \)), the evolution of aggregate consumption depends on \( \hat{p} \). As can be seen, \( W_0^{II} = \hat{p}Z(1) + \bar{K} \) and \( \lim_{t \to \infty} W_t^{II} = Z(\hat{p}) + \bar{K} \). If \( \hat{p}Z(1) = Z(\hat{p}) \), aggregate consumption is constant over time, which occurs when:

\[
\frac{\gamma}{(1-q)(1-\hat{p}^*)} = \hat{p}^* K^*,
\]

which is fulfilled for \( p^{Ch} < \hat{p}^* < p^H \).

For all \( \hat{p} > \hat{p}^* \), aggregate consumption grows over time. In particular the case in which the average collateral does not introduce financial restrictions (this is, \( \hat{p} > p^H \)), is characterized by an aggregate consumption increasing over time (since \( p^H > \hat{p}^* \)). This is because more and more firms are borrowing, a credit boom.

Contrarily, if the average collateral implies information production (this is \( p^{Cl} < \hat{p} < p^{Ch} \)), aggregate consumption \( W_t^{IS} \) does not depend on \( t \), being constant at the level at which information about the collateral that suffers idiosyncratic shocks is reacquired at every period.

### 4.2 Aggregate Shocks

In this section we introduce negative aggregate shocks that transform a fraction \((1-\eta)\) of good collateral into bad collateral. As with idiosyncratic shocks, the aggregate shock is observable, but which good collateral changes quality is not. This implies that when the shock hits, there is a downward revision of the perception about the quality of each unit of collateral. For example, collateral that has a \( p = 1 \), gets a new belief \( p' = \eta \) after the aggregate shock. Similarly, all collateral with \( p = \hat{p} \) get revised downwards to \( p' = \eta \hat{p} \).

We also consider positive aggregate shocks that transform a fraction \( \alpha \) of bad collateral into good collateral. In this case beliefs are revised up for all collateral. Collateral with \( p = 0 \) get revised to \( p' = \alpha \) and collateral with \( p = \hat{p} \) is revised to \( p' = \hat{p} + \alpha(1-\hat{p}) \).

We will focus on the case where prior to the negative aggregate shock, the average quality of the collateral is good enough such that there are no financial constraints (that is, \( \hat{p} > p^H \)). The next Proposition shows that the longer the economy did not face a negative aggregate shock, the larger the consumption loss when such a shock does finally occur.
**Proposition 2** Assume \( \hat{p} > p^H \) and a negative aggregate shock \( \eta \). The reduction in consumption \( \Delta(\eta, t) \) is non-increasing in \( \eta \) and non-decreasing in the time \( t \) elapsed previously without a shock.

**Proof** Assume a negative aggregate shock of size \( \eta \). Since we assume \( \hat{p} > p^H \), the average collateral does not generate information production. The aggregate consumption before the shock is given by equation 5 and after the shock aggregate consumption is:

\[
W_{t,\text{shock}}^{\text{HI}} = [\lambda^t \eta \hat{p} Z(\eta) + (1 - \lambda^t) \eta Z(\eta \hat{p})] + K.
\]

Then we can define the reduction in aggregate consumption as \( \Delta(\eta, t) = W_t - W_{t,\text{shock}} \)

\[
\Delta(\eta, t) = \lambda^t \hat{p} [Z(1) - \eta Z(\eta)] + (1 - \lambda^t) [Z(\hat{p}) - \eta Z(\eta \hat{p})].
\]

Since \( Z(p) \) is decreasing in \( p \), \( \Delta(\eta, t) \) is non-increasing in \( \eta \) and non-decreasing in \( t \).

Q.E.D.

The intuition for this Proposition is straightforward. If there is little information about collateral in the economy (that is, there is a small fraction of collateral with either \( p = 0 \) or \( p = 1 \)), a negative aggregate shock affects a high fraction of collateral in the economy, reducing borrowing and consumption a lot.

This result goes beyond the mechanical effect in which more collateral is affected. After a negative shock to collateral, either a higher amount of the numeraire good should be used to produce information or borrowing is excessively restricted to avoid such information production.

Since the fraction of collateral with information about their true quality decreases over time in an information-insensitive regime, if we define "fragility" as the probability aggregate consumption declines more than a certain value, then the following corollary follows immediately from the previous Proposition.

**Corollary 1** Given a structure of negative aggregate shocks, the fragility of an economy increases with the number of periods the debt in the economy has been informationally-insensitive, and then increases with the fraction of collateral of unknown quality.
The next Proposition shows that if there is a positive aggregate shock after a negative aggregate shock (for example the negative shock was transitory or there are policies that try to improve the quality of the collateral) that takes the average collateral $\eta \hat{p}$ to a higher new level above $p^H$, the recovery from the negative shock is faster if there was no information production as a response to the negative aggregate shocks.

**Proposition 3** Assume a positive aggregate shock with persistency $\alpha$, immediately following a negative aggregate shock $\eta$, such that $p' = \eta \hat{p} + \alpha (1 - \eta \hat{p}) > p^H$. The speed of recovery is larger if there is no information acquisition about the collateral after the negative aggregate shock. This is $W_{t,after|II} > W_{t,after|IS}$.

**Proof** As we discuss, depending on the size of the negative aggregate shock $\eta$ and the time elapsed previous to such a shock, the average collateral can enter an information-insensitive regime (if $\eta \hat{p} > p^Ch$) or an information-sensitive regime (if $\eta \hat{p} < p^Ch$). We will consider these two cases separately.

**Case 1: Information-insensitive: $\eta \hat{p} > p^Ch$**

If the negative shock happens in period $t$, the distribution in period $t$ is: $f(\eta) = \eta \lambda^t \hat{p}$, $f(\eta \hat{p}) = \eta (1 - \lambda^t)$ and $f(0) = 1 - \eta (1 - \lambda^t (1 - \hat{p}))$.

In period $t + 1$, after the idiosyncratic shocks, the distribution of beliefs is $f_{II}(\eta) = \eta \lambda^{t+1} \hat{p}$, $f_{II}(\eta \hat{p}) = \eta \lambda (1 - \lambda^t)$, $f_{II}(\hat{p}) = (1 - \lambda)$ and $f_{II}(0) = \lambda (1 - \eta (1 - \lambda^t (1 - \hat{p})))$.

After the positive aggregate shock, beliefs change from $\eta$ to $\eta + \alpha (1 - \eta)$, from $\hat{p}$ to $\hat{p} + \alpha (1 - \hat{p})$, from $\eta \hat{p}$ to $\eta \hat{p} + \alpha (1 - \eta \hat{p})$ and from 0 to $\alpha$. Since by assumption $\eta \hat{p} + \alpha (1 - \eta \hat{p}) > p^H$, for all beliefs except $\alpha$ the relative consumption is $Z(1)$, then the aggregate consumption at $t + 1$ is:

$$W_{t,after|II} = (1 - f_{II}(0)(1 - \alpha))Z(1) + (1 - \alpha) f_{II}(0) Z(\alpha) + \bar{K}.$$  

**Case 2: Information-sensitive: $\eta \hat{p} < p^Ch$**

If the negative shock happens in period $t$, the distribution in period $t$ is: $f(1) = \eta (1 - \lambda^t) \hat{p}$, $f(\eta) = \eta \lambda^t \hat{p}$ and $f(0) = 1 - \eta \hat{p}$.

In period $t + 1$, after the idiosyncratic shocks, the distribution of beliefs is $f_{IS}(1) = \lambda \eta \lambda^{t+1} \hat{p}$, $f_{IS}(\eta) = \eta \lambda^{t+1} \hat{p}$, $f_{IS}(\hat{p}) = (1 - \lambda)$ and $f_{IS}(0) = \lambda (1 - \eta \hat{p})$. 

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After the positive aggregate shock, beliefs 1 remain at 1, and other beliefs change from \( \eta \) to \( \eta + \alpha (1 - \eta) \), from \( \hat{\rho} \) to \( \hat{\rho} + \alpha (1 - \hat{\rho}) \), and from 0 to \( \alpha \). Since by assumption \( \eta \hat{\rho} + \alpha (1 - \eta \hat{\rho}) > \rho^H \), for all beliefs except \( \alpha \) the relative consumption is \( Z(1) \), then the aggregate consumption at \( t + 1 \) is:

\[
W_{t,after|IS} = (1 - f_{IS}(0)(1 - \alpha))Z(1) + (1 - \alpha)f_{IS}(0)Z(\alpha) + \bar{K}.
\]

Since \( f_{IS}(0) = \lambda (1 - \eta \hat{\rho}) \geq f_{II}(0) = \lambda (1 - \eta (1 - \lambda'(1 - \hat{\rho}))) \) for all \( t \), then \( W_{t,after|II} > W_{t,after|IS} \).

Furthermore, the larger the \( t \) at which the crisis happened, the larger is the difference \( f_{IS}(0) - f_{II}(0) \), the larger the difference \( (W_{t,after|II} > W_{t,after|IS}) \) and then the larger the speed of recovery from a negative shock if there were no information acquisition when it happened.

Q.E.D.

The intuition for this Proposition relies on the speed of recovery of information. If an aggregate negative shock does not generate information production, when in average the collateral quality recovers, borrowing can recover fast for the high fraction of collateral without information about their true quality. If an aggregate negative shock generates information production, then, even when in average the collateral quality recovers, the fact that bad collateral has been identified, restricts lending to such a collateral until they face idiosyncratic shocks.

As shown in the case without aggregate shocks, as more collateral loses information about its quality there is a credit boom and an increase in aggregate consumption. If that information is recovered, then it will take time for information to be lost again and for aggregate consumption to grow.

5 Numerical Simulations

We illustrate our main results with the following numerical exercise. We assume idiosyncratic shocks happen with probability \( (1 - \lambda) = 0.1 \), in which case the collateral becomes good with probability \( \hat{\rho} = 0.9 \). Other parameters are \( q = 0.6, A = 3 \) (these
two assumptions imply that investing in the project generates a return of 80%, $\bar{K} = 10$, $L^* = K^* = 7$ (which means the endowment is enough to invest in the optimal project size), $\gamma = 0.4$, $C = 15$ and $\beta = 0.99$.

Given these parameters we can obtain the relevant cutoffs for our analysis. Specifically, $p^H = 0.86$, $p^L_{II} = 0.08$ and the sensitive information region is in the values $p \in [0.22, 0.817]$. As discussed above, these cutoffs are obtained from comparing expected profits from taking a loan producing information with one without producing information. Figure 4 plots these functions and the respective cutoffs.

Using these cutoffs we simulate the model for 100 periods. As in the main text, we assume that at period 0 there is perfect information about the true quality of every collateral in the economy. Over time there are idiosyncratic shocks that make this information vanish unless there is costly information acquisition about the realizations after idiosyncratic shocks.

We introduce a negative aggregate shock that transforms a fraction $(1 - \eta)$ of good collateral into bad collateral in periods 5 and 50. We also introduce a positive aggregate shock that transforms a fraction $\alpha = 0.2$ of bad collateral into good collateral in periods 30 and 51. We will compute the dynamic reaction of consumption in the economy for different sizes of negative aggregate shocks, $\eta = 0.95$, $\eta = 0.91$ and $\eta = 0.90$. We will see that small differences in the size of a negative shock can have important dynamic consequences in the economy.
Figure 5 shows the average probability that collateral is good in the economy for the three possible negative aggregate shocks (this is the real collateral quality existing in the economy). While aggregate shocks have a temporary effect on quality of collateral, after aggregate shocks occur the average quality converges back to $\hat{p} = 0.9$. As can be seen, the negative aggregate shocks were constructed such that $\eta \bar{p}$ is above $p^H$ when $\eta = 0.95$, is between $p^{Ch}$ and $p^H$ when $\eta = 0.91$ and is less than $p^{Ch}$ when $\eta = 0.90$.

Figure 5: Average Quality of Collateral

Figure 6 shows the evolution of aggregate consumption for the three negative aggregate shocks. The first result to highlight is that when $\eta = 0.95$, aggregate shocks do not affect the evolution of consumption at all. The reason is that shocks do not introduce financial constraints. The second result is that positive shocks do not affect the evolution of consumption and the reason is that $\hat{p} > p^H$, and hence improvements in the belief distribution do not relax the financial constraints even more. This introduces an asymmetry on how shocks affect aggregate consumption.

The third result is that the reduction in consumption from the negative aggregate shock in period 5, when not much information has vanished yet, is much lower than the reduction in consumption from the same size negative aggregate shock in period 50. The reason is that the shock reduces financing for a larger fraction of collateral which information has vanished over time but was good enough to finance projects successfully. This is the result proved in Proposition 2.
In Figure 7 we illustrate that small differences in the size of shocks have very different consequences on the variance of beliefs about the collateral. A shock $\eta = 0.91$ does not trigger information production but a shock $\eta = 0.90$ does it. Given that after many periods without shock most collateral looks the same, these differences in information production implies that these differences have large consequences on the variance of beliefs and the information available about most collateral.

This effect of information acquisition implies that, even when the real quality of collateral is the same under the two shocks, a slightly larger shock that induces information acquisition implies that positive aggregate shocks do not have the potential to make the economy recover fast enough. This is the effect discussed in Proposition 3.

6 Conclusions

In this paper we discuss the positive and negative effects of information-insensitive debt. At the one hand, information-insensitive debt generates a credit boom because firms with bad collateral are able to borrow when information is not produced. This effect has been also highlighted in different contexts by Hirshleifer (1971) and Dang, Gorton, and Holmström (2011). On the other hand, the longer an economy is issuing information-insensitive debt and the longer the credit boom, the larger the system
fragility, such that a negative aggregate shock to collateral of a given size is more likely to create a large systemic reduction in output and consumption. Furthermore, if the crisis triggers information production, the economy takes longer to recover.

References


