Liquidity, Financial Intermediation, and Monetary Policy in a New Monetarist Model

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Abstract

A model of public and private liquidity is constructed to answer some basic questions in monetary economics, and to address some pressing monetary policy issues, in part related to the financial crisis. The model integrates financial intermediation theory with New Monetarist monetary frameworks. A one-time open market sale of nominal government bonds by the central bank can have permanent real effects. Liquidity traps can arise, even with an inflation rate greater than what the Friedman rule dictates. Costs of operating a currency system matter for optimal monetary policy. Risk shocks can produce financial crisis features.

1 Introduction

Liquidity consists of a class of assets that are useful in exchange. Some of these liquid assets are government liabilities. In the United States, for example, Federal Reserve notes serve as a medium of exchange in retail transactions, deposits with the Federal Reserve are used as a medium of exchange in interbank transactions, and Treasury bills play an important role as collateral in financial transactions. As well, there are liquid assets that are the liabilities of private financial intermediaries, or the products of these intermediaries. Banks issue deposit liabilities which can be traded using debit cards and checks, and securitized loans can be traded on financial markets or can serve as collateral in financial transactions. To further complicate matters, private financial intermediaries that issue liquid liabilities also hold liquid government liabilities as assets.

Conventional wisdom holds that the main role of a central bank is to manage public liquidity in a manner that controls inflation, and enhances the provision of private liquidity and credit. However, the mechanism by which central bank actions affect prices and quantities still appears to be poorly understood. As evidence for this, consider the dramatic actions taken by the Federal Reserve System since late 2008, and the reaction of economists to these actions. There is considerable disagreement about the implications and appropriateness of these actions, both inside and outside the Federal Reserve System. How do these dramatic interventions matter for inflation and for real activity?

The purpose of this paper is to build a model of public and private liquidity to answer some basic questions in monetary economics, and then to use this model to shed light on some current monetary policy problems and questions. What is a liquid asset, and what roles do privately-provided and publicly-provided liquid assets play in exchange? Do open market operations matter, and if so, why? Under what circumstances can a liquidity trap occur, and what is the role of monetary policy given those circumstances? How does monetary policy work when there is a positive quantity of excess reserves held by banks? What does a financial crisis do to the supply of private liquidity, and what should monetary policy do in response? This may seem like a wide-ranging (and perhaps almost all-encompassing) set of questions in contemporary monetary economics, but our model can address all of these in a straightforward way.

The model constructed here could be viewed as bringing together and building on two branches of the literature. The first branch is the literature on financial intermediation and macroeconomic credit frictions. The 1980s saw the development of models of endogenous intermediary structures, and there are essentially two subclasses of models that became widely-used in addressing macroeconomic problems. The first is a subclass of models with costly state verification and delegated monitoring, built using the insights of Townsend (1979) (basic costly state verification) and Diamond (1984) (delegating monitoring). The second builds on the risk-sharing banking model of Diamond-Dybvig (1983). Williamson (1986) melds delegated monitoring with Townsend's debt contracting framework, and this construct was further developed in the general equilibrium models of Williamson (1987) and Bernanke and Gertler (1989). This latter framework was ultimately integrated with other elements and used in "financial accelerator" models, for example Bernanke, Gertler, and Gilchrist (1999).

The second branch of the literature we build on here is the class of explicit models of money, liquidity, and asset exchange referred to in Williamson and Wright (2010a, 2010b) as "New Monetarist Economics." Important work in this literature is the "Models of Monetary Economies" conference volume (Kareken and Wallace 1980), the Kiyotaki and Wright (1989) search model of money, and particularly Lagos and Wright (2005). Key research relating to asset exchange and pricing that has a bearing on what we will do here are papers by Lagos (2008), Lester, Postlewaite, and Wright (2009), and Lagos and Rocheteau (2008).

The issues we study are also related to some recent papers which use different modeling approaches. Kiyotaki and Moore (2008) look at credit frictions arising from exogenous liquidity constraints, while Gertler and Kiradi (2009) and Gertler and Kiyotaki (2010) look at non-monetary models with limited commitment frictions, with Gertler and Kiyotaki (2010) being a melding of the Kiyotaki-Moore and Gertler-Kiradi frameworks. Curdia and Woodford (2009) extend existing New Keynesian sticky price models to include financial frictions. The value-added in our paper relative to this collection of work is the explicit use of received intermediation theory and monetary theory, an explicit treatment of monetary policy that identifies assets and liabilities in the model with the key entries in real-world central bank balance sheets, and the incorporation of retail currency transactions (which are critical to the monetary policy problem), among other things. All of these differences matter, and make our results novel.

The basic model builds on Lagos and Wright (2005) or, in some ways more closely, Rocheteau and Wright (2005). As in those models, quasilinear preferences are useful for analytical tractability. The financial intermediation sector has a costly-state-verification delegated-monitoring role for financial intermediaries, which take deposits from one set of economic agents, and lend to another set of agents who finance investment projects. As well, these financial intermediaries hold nominal government bonds and reserves, and can borrow from the central bank. Another key role played by financial intermediaries in the model is related to Diamond-Dybivg (1983) insurance. A bank depositor faces some uncertainty concerning whether he or she will be engaging in a transaction that permits the trading of claims on the bank (essentially a debit card transaction), or in a transaction in which the seller of goods will only accept currency. The bank permits liquidity to be efficiently allocated. Currency is liquidity, which is accepted everywhere, but in general will yield a low rate of return in equilibrium, and the assets held by financial intermediaries (loans, government bonds, and reserves) are also liquidity, as they are made tradeable in some types of exchanges through the intermediation process. Public liquidity consists of currency (highly liquid) and bonds (less liquid), while private liquidity consists of the loan portfolio held by financial intermediaries.

In the Diamond-Dybvig (1983) banking model, "illiquidity" is assumed as a feature of the technology, and the need for liquidity is driven purely by the random arrival of an urgent need to consume. Our model goes much beyond this, in that liquidity is determined by how, and at what prices, assets can be exchanged, as is true in reality. Thus, liquidity is an endogenous phenomenon, and the liquidity transformation and allocation performed by the banking system will be affected by monetary policy, which determines the relative supplies of different kinds of public liquidity. There are general equilibrium monetary models that include Diamond-Dybvig (1983) banks, such as Champ, Smith and Williamson (1996). However, that model has its problems.[1] What is done in this paper goes much further in many respects.

Two features of the model will be critically important for our results about monetary policy. First, in general the actions of the central bank are constrained, in reality, by what the fiscal authority does. The central bank can at best determine the composition of the supply of public liquidity; the total quantity is determined by the fiscal authority. We capture this in our model in a straightforward way. Second, optimal monetary policy depends in an important way, as in part argued in Sanches and Williamson (2010), on the costs of operating a currency system. These include direct costs, such as the costs of designing the currency to thwart counterfeiters, printing currency, and destroying worn-out notes. Perhaps more importantly, there are social costs associated with a currency system. Currency can introduce opportunities for illegal activities that are not available if exchange is carried out by other means. These illegal activities include straightforward theft of currency, trade in illegal goods and services, and tax evasion.

As a benchmark, we start with a setup where the monetary authority sets policy, and the fiscal authority responds passively. In this case the central bank is completely unconstrained. Also, in this benchmark case there are no costs associated with the currency system. We then proceed to introduce non-passive fiscal policy (with a constrained central bank) and costs associated with currency, in a way that does not affect equilibrium of quantities and prices, given policy. However, the fiscal authority constrains what monetary policy can do, while the costs associated with the currency system help determine the optimal monetary policy choice.

In the benchmark economy, an equilibrium can be one of three types: a liquidity trap equilibrium, an equilibrium with plentiful interest-bearing assets, or an equilibrium with scarce interest-bearing assets. In a liquidity trap equilibrium the nominal interest rate is zero, excess reserves are held by banks, and open market operations are irrelevant (at the margin) for equilibrium quantities and prices. A novelty here is that the liquidity trap equilibrium is not associated with the Friedman rule; indeed, it can exist for essentially any long-run money growth rate if the central bank sets the ratio of outside money to total consolidated-government liabilities appropriately. A liquidity trap arises so long as total liquid assets (public and private) are sufficiently scarce, and currency is

[1] Those problems include the policy conclusion that the central bank should intermediate all private assets, and a somewhat artificial role for currency.

sufficiently plentiful relative to other assets. This helps in understanding current observations with regard to interest rates and monetary policy in the United States.

In an equilibrium with plentiful interest-bearing assets, trading is efficient in non-currency transactions, and thus there is no liquidity premium associated with interest-bearing assets. Open market operations have standard effects in this equilibrium, in that a one-time permanent open market purchase of nominal government bonds serves to increase the price level in proportion to the money injection, and the real interest rate is unaffected. Things are quite different, though, in the equilibrium with scarce interest-bearing assets. Here, there is a liquidity premium on interest-bearing assets, reflected in a real interest rate that is less than the rate of time preference. A one-time permanent open market purchase, while it results in a proportionate increase in the price level, with no effect on the real stock of currency, acts to make public liquidity more scarce, in terms of the assets backing non-currency transactions. As a result, the real interest rate falls, and lending by banks increases, which results in a greater supply of private liquidity. This nonneutrality of money is novel, and is related to results in Lagos (2008) and Lagos and Rocheteau (2008), concerning asset scarcity and liquidity premia. Part of the novelty in our approach relative to these other papers is in how the private liquidity gets produced. In our model, the efficiency of the intermediation sector and the perceived riskiness of lending will matter for the quantity of private liquidity in existence, and we can relate this to financial crisis phenomena.

One source of much confusion concerning current monetary policy in the United States concerns how policy works when the central bank pays interest on bank reserves, in circumstances where the quantity of excess reserves held overnight is greater than zero. The Fed has now been paying interest, at 0.25%, on overnight reserves since October 2008. In our model, it is a straightforward exercise to include interest bearing reserves. What the model shows is that, if excess reserves are held in equilibrium, then open market operations are irrelevant at the margin, much like in the liquidity trap equilibrium, but with a positive nominal interest rate. Monetary policy works in this regime through changes in the interest rate on reserves, which essentially determines all short-term market interest rates.

We consider non-passive fiscal policy by looking at a regime where the fiscal authority fixes the real deficit forever, and the central bank must treat this as given. The central bank then chooses the ratio of outside money to total consolidated government liabilities (how much of the total nominal government debt to monetize), and the fiscal authority then expands the total nominal debt at a rate that finances the deficit forever. Whether or not fiscal policy is passive, and in the absence of currency system costs a Friedman rule will always be feasible and optimal, whereby monetary policy is set so that the nominal interest rate is zero. A Friedman rule, as is typically the case in most monetary models, implies that there exists an equilibrium where all transactions are executed using currency. Further, banks serve only to efficiently monitor borrowers. Financial intermediary liabilities are not traded.

This is unsatisfactory, which is where currency system costs come in. We model these costs as a proportional cost to the government of maintaining the stock of currency (in real terms), along with a fixed fraction of currency transactions that are deemed illegal and therefore socially useless. These costs, which are the direct and social costs, respectively, of operating a currency system, interact in an interesting way with the constraint implied by non-passive fiscal policy to give us interesting results. Now it will in general be desirable for the central bank to tax currency transactions through inflation. Of course, there are also costs associated with inflation. First, higher inflation implies that consumers hold less currency, and therefore socially-desirable currency transactions are less efficient. Second, given how monetary policy is constrained by fiscal policy, higher inflation tends to reduce the interest-bearing component of public liquidity, and non-currency transactions can also become less efficient. There are interesting cases where the optimal inflation rate will not be a constant (2% for example, which seems to be the implicit inflation rate target of the Fed), but will depend on all of the parameters of the model, in interesting ways.

The model can be used to address some key financial crisis issues. A nice feature of costly-state-verification constructs is that they yield optimal debt contracting, interest rate spreads due to default premia, and a mechanism by which higher risk increases spreads and reduces lending and private liquidity. These were clearly important elements of the financial crisis. In the model, we can introduce changes in risk along the lines of Williamson (1987). These are essentially the risk shocks that Christiano, Motto, and Rostagno (2009) find to be empirically important for business cycle phenomena. An increase in risk can make liquidity more scarce in general, and this acts to reduce the real interest rate, and to increase the marginal social cost of inflation. An optimal policy response is for the central bank to sell interest-bearing assets and reduce the inflation rate. The real interest rate rises due to the policy action. This is quite different from typical Keynesian financial crisis analysis, where a problem arises because of the zero lower bound on the nominal interest rate, and the real interest rate is viewed as being too high. Here, the real rate is too low in the absence of intervention, due to the shortage of liquid assets. The optimal policy response in our model is consistent, in a sense, with what the Fed actually did during the financial crisis. After the Lehman Brothers collapse in fall 2008, the Fed sold a large portion of its portfolio of Treasury securities.

Another intervention we consider is the purchase of private assets by the central bank, which corresponds to a key financial crisis intervention by the Fed: the large purchase of more than $1 trillion in mortgage-backed securities (MBS), which was part of the Fed's "quantitative easing" program. We assume in the model that the central bank is as efficient at lending as are private sector banks, and writes efficient debt contracts with lenders. This implies that, if the central bank purchases private assets on the same terms as do private sector banks, there is no effect. Private intermediaries hold reserves, which are issued by the central bank to finance its portfolio of private assets. This adds a redundant layer of intermediation, and has no effect on prices or quantities. However, if the central bank lends on better terms than do private sector banks, this will matter. The central bank can expand lending as a result, and make borrowers better off, but this comes at a cost. The central bank will suffer losses on its loan portfolio, and these have to be made up somehow, resulting in a redistribution of wealth. Further, this type of central bank intervention can reallocate credit in financial markets, just as the Fed's MBS purchases potentially reallocated credit towards the housing sector and away from other uses.

The remainder of the paper is organized as follows. The second section is a description of the model. The second and third sections deal with passive and non-passive fiscal policy, respectively. The fourth section addresses itself to the full-blown model with non-passive fiscal policy and currency system costs. The last section is a conclusion.

2 The Model

The basic model builds on Lagos-Wright (2005) and Rocheteau-Wright (2005), with an information structure related to Sanches and Williamson (2010). The financial intermediation sector shares features with Williamson (1987) and Diamond and Dybvig (1983). Time is indexed by $t = 0, 1, 2, ...$, and there are two subperiods within each period that we denote day and night. The fact that the subperiods are "day" and "night" has nothing to do with what happens over a 24-hour period in reality. These names are intended only as a reminder of which subperiod precedes which.

2.1 Private Economic Agents

The population consists of three types of economic agents: buyers, sellers, and entrepreneurs. There is a continuum of buyers with mass one, and each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

Here, $0 < \beta < 1$, H_t denotes the difference between labor supply and consumption during the day, x_t is consumption in the night, and $u(\cdot)$ is a strictly increasing, strictly concave, and twice continuously differentiable function with $u(0) = 0$, $u'(0) = \infty$, $-x\frac{u''(x)}{u'(x)} < 1$ for all $x \geq 0$, and with the property that there exists some $\hat{x} > 0$ such that $u(\hat{x}) - \hat{x} = 0$. Define x^* by $u'(x^*) = 1$. There is a continuum of sellers with unit mass, and each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where X_t is consumption in the day and h_t is labor supply in the night. The production technology potentially available to buyers and sellers allows the production of one unit of the perishable consumption good for each unit of labor supply. Buyers can produce only in the day, and sellers only in the night, so we have one of the necessary ingredients for monetary exchange - a double coincidence problem.

During the day of each period, a continuum of entrepreneurs with mass α is born, and each lives until the day of the following period. An entrepreneur has no endowment during his or her lifetime. An entrepreneur born in the day of period t consumes only in the day of period $t+1$ and is risk neutral. An entrepreneur has access to an investment project. This project is indivisible and requires one unit of the consumption good in the day of period t to operate, and yields a return of w in the day of period $t+1$, where w is distributed according to the distribution function $F(w)$, with associated density function $f(w)$, which is strictly positive on $[0, \bar{w}]$, where $\bar{w} > 0$. Assume also that $f(\cdot)$ is continuously differentiable. Investment project returns are independent across entrepreneurs. The return w is private information to the entrepreneur, but subject to costly state verification, whereby any other individual can bear a fixed cost and observe w ex post. The verification cost γ is entrepreneur-specific, and $G(\gamma)$ denotes the distribution of verification costs across entrepreneurs, with $\gamma \geq 0$.

During the night, each buyer is matched at random with a seller. The seller in a match is not able to observe the buyer's history, and the seller will never have an opportunity to signal default on a credit arrangement, so the seller will not accept a personal IOU in exchange for goods. A fraction ρ of nighttime bilateral meetings are *not monitored*, in the sense that, if the buyer wants to acquire goods from the seller, he or she must have a claim to goods in the next day, where the claim is somehow documented in an object that the buyer carries. In this model, the only physical object with these properties is currency issued by the government. We assume that it is costless for the government to issue perfectly durable and divisible currency that is not counterfeitable, and that private circulating notes are not issued, either because the government prohibits this, or because it is unprofitable.[2]. For now we will assume that there are no costs (direct costs or indirect social costs) to operating a currency system, but we will relax this later in the paper. A fraction $1-\rho$ of buyers and sellers are in *monitored* meetings at night. In these meetings, though a credit transaction cannot take place between the buyer and the seller, a communication technology is costlessly available which permits the buyer to transfer ownership of a claim on a financial intermediary to the seller. Assume that, when a buyer meets a seller, the buyer makes a take-it-or-leave-it offer of assets in exchange for goods.[3]

[2] There might be concern that we are not addressing explicitly why private circulating notes are not issued. While some countries have explicit legal restrictions prohibiting the issue of objects that look like government-issued currency (Canada banned private currency issue in 1944, for example), according to Schuler (2001), the United States no longer does. Explaining why U.S. banks do not issue close substitutes for currency is an unanswered research question, outside the scope of this paper, and potentially irrelevant for the issues addressed here.

[3] There are many ways to split the surplus from trade, including Nash bargaining (Lagos-Wright 2005), competitive search (Rocheteau-Wright 2005), or competitive pricing (Andolfatto 2009). Here, given that the seller's utility is linear in labor supply, take-it-or-leave-it offers by the buyer are equivalent to competitive pricing. Take-it-or-leave-it lends tractability to the

During the day, all sellers, buyers, and the government meet in a centralized Walrasian market, where there is lack of recordkeeping, except for records of the ownership of claims to the output of investment projects and of accounts with financial intermediaries and the government. Finally, after all production and consumption decisions are made during the day, buyers learn whether they will be in non-monitored or monitored meetings with a seller in the following night, and this information is public. This will give rise to a Diamond-Dybvig (1983) risk-sharing role for financial intermediaries, in addition to the costly-state-verification/delegated-monitoring role for intermediation that also exists here.

2.2 Government

We will first deal with the government as a consolidated entity; later we will consider issues of how we can separate monetary policy from fiscal policy. First, assume that the government has the power to levy lump-sum taxes on buyers in the centralized market during the day, with τ_t denoting the tax per buyer in units of goods. As well, the government has M_t units of currency outstanding in period t, issues B_t one-period nominal bonds held by the private sector, issues E_t nominal units of central bank reserves, and makes F_t nominal units of one-period central bank loans. All government asset transactions take place in the centralized market during the day. A government bond is an account balance held with the government which sells in the day of period t for one unit of money, and pays off q_{t+1} units of money in the day of period $t+1$. One unit of reserves acquired by a private sector agent in the daytime of period t yields s_t units of money in period $t+1$ during the day. These reserves are essentially identical to nominal government bonds; they are account balances with the consolidated government. Central bank loans are made to buyers in the day of period t, with each dollar lent in period t requiring that the borrower pay back w_{t+1} units of money in the day of period $t+1$. Assume that the government is always able to collect on its debts (tax liabilities and loans) at the beginning of the day before the Walrasian market opens. Then, letting ϕ_t denote the price of money in terms of goods in the daytime Walrasian market, the consolidated government budget constraint is

$$\phi_t(M_t + B_t + E_t - F_t) + \tau_t = \phi_t(M_{t-1} + q_t B_{t-1} + s_t E_{t-1} - w_t F_{t-1}). \qquad (1)$$

Equation (1) states that the real value of the government's net outstanding liabilities at the end of the day in period t, plus tax revenue collected, must equal the government's net outstanding liabilities at the beginning of the day, for $t = 1, 2, \ldots$. Assume that the government starts period 0 with no outstanding liabilities, so

$$\phi_0(M_0 + E_0 + B_0 - F_0) + \tau_0 = 0, \qquad (2)$$

problem, and avoids distractions associated with determining how the surplus from trade is split. One could argue that bargaining is not central to the issues we wish to address here.

or in other words private agents are endowed with no outside assets at the first date.

2.3 Financial Intermediation: Delegated Monitoring

We will assume that stochastic verification is not feasible.[4] Assume that entrepreneurs are economic agents who are subject to full commitment. Then, as in Williamson (1987), an efficient lending arrangement is for individual entrepreneurs to act as perfectly-diversified financial intermediaries. Efficient and incentive-compatible loan contracts with entrepreneurs take the form of noncontingent debt. That is, the financial intermediary observes the verification cost γ associated with the entrepreneur in the daytime of period t and offers him or her a contract that specifies a non-contingent payment $R_{t+1}(\gamma)$ that the entrepreneur must make to the intermediary in the day of period $t+1$. If the entrepreneur cannot make the loan payment, then default occurs, the intermediary incurs the verification cost γ, observes the return w, and seizes the entrepreneur's output. As shown in Williamson (1987), the expected payoff to the intermediary from the loan contract, as a function of the non-contingent payment R and the verification cost γ, is then given by

$$\pi(R,\gamma) = R - \gamma F(R) - \int_0^R F(w)dw \tag{3}$$

Then, letting $R_{t+1}(\gamma)$ denote the gross real loan interest rate on a loan to an entrepreneur of type γ, equation (3) allows us to define the default premium faced by an entrepreneur of type γ, which is

$$D_t(\gamma) = \gamma F[R_{t+1}(\gamma)] + \int_0^{R_{t+1}(\gamma)} F(w)dw. \tag{4}$$

Since the financial intermediary is perfectly diversified (this requires only that it hold a positive mass of loans to entrepreneurs), it can guarantee a certain one-period return r_{t+1} in period t per unit invested in lending to entrepreneurs. In equilibrium, the expected payoff per unit invested will be the same for each loan made by the financial intermediary, so

$$r_{t+1} = R_{t+1}(\gamma) - \gamma F[R_{t+1}(\gamma)] - \int_0^{R_{t+1}(\gamma)} F(w)dw \tag{5}$$

[4] Stochastic costly state verification has been studied by Border and Sobel (1987), and Mookherjee and Png (1989). Stochastic verification is efficient, but it is very difficult to characterize the optimal contract, particularly with a continuous distribution. There are three ways out here. First, we could argue, as in Boyd and Smith (1994), that the loss in efficiency from pure strategy contracts is quantitatively insignifiant in practice. Second, we could argue, as we do here, that stochastic verification is infeasible, as the appropriate verifiable randomization device is not available. Third, we could opt for a simple two-state probability distribution (with zero as the low state) and permit stochastic verification. The third approach would give us essentially the same results as we get here, but our approach allows a better fit to real-world contracts.

for each entrepreneur who receives a loan. Differentiating the intermediary's expected payoff function in (3), we obtain

$$\pi_1(R,\gamma) = 1 - \gamma f(R) - F(R),$$

$$\pi_{11}(R,\gamma) = -\gamma f'(R) - f(R)$$

and we assume that $-\gamma f'(w) - f(w) < 0$ for all $w \in [0,\bar{w}]$ and for all $\gamma \geq 0$.[5] Then $\pi(R,\gamma)$ is strictly concave in R for $R \in [0,\bar{w}]$ and attains a maximum for $R = \hat{R}(\gamma) < \bar{w}$, where

$$1 - \gamma f\left[\hat{R}(\gamma)\right] - F\left[\hat{R}(\gamma)\right] = 0. \tag{6}$$

In equilibrium, there is a marginal entrepreneur in each period t, with verification cost γ_t^* and facing the gross loan interest rate R_{t+1}^*, where, from (6),

$$1 - \gamma_t^* f\left(R_{t+1}^*\right) - F\left(R_{t+1}^*\right) = 0, \tag{7}$$

so that the gross loan interest rate faced by the marginal entrepreneur maximizes the expected return to the financial intermediary given the marginal entrepreneur's verification cost γ_t^*. Further, the financial intermediary earns an expected return r_{t+1} from lending to the marginal borrower, or from (5),

$$r_{t+1} = R_{t+1}^* - \gamma_t^* F\left(R_{t+1}^*\right) - \int_0^{R_{t+1}^*} F(w)dw \tag{8}$$

Then, each entrepreneur who receives a loan has $\gamma \leq \gamma_t^*$, and if $\gamma < \gamma_t^*$ then $R_{t+1}(\gamma) < \hat{R}(\gamma)$. Entrepreneurs with $\gamma > \gamma_t^*$ do not receive loans as, even if $R_{t+1}(\gamma) = \hat{R}(\gamma)$ for one of these agents, the intermediary will have an expected loss from the loan. That is, verification costs for the set of entrepreneurs with $\gamma > \gamma_t^*$ are too high for lending to be profitable.

Then, the total quantity of loans extended by financial intermediaries during the day of period t is given by

$$L_t = \alpha G(\gamma_t^*). \tag{9}$$

Therefore, given the certain return on investment r_{t+1}, (7), (8), and (9) determine the loan quantity L_t, the verification cost of the marginal borrower, γ_t^*, and the gross loan interest rate faced by the marginal borrower R_{t+1}^*. It is straightforward to show that

$$\frac{dL_t}{dr_t} = -\frac{\alpha G'(\gamma_t^*)}{F\left(R_t^*\right)} < 0 \tag{10}$$

Thus, given an increase in the payoff per unit of lending by the financial intermediary, the quantity of lending by financial intermediaries must decline. This

[5] This is a mild assumption, satsfied for example if $F(\cdot)$ is uniform. This assumption is purely technical. It gives concavity of the intermediary's payoff function. Without it, the character of the results should be the same, but we would have to work harder, and tax the reader more, to get them.

is because, from (5), the loan interest rate for each entrepreneur receiving a loan will increase, and it will be unprofitable for an intermediary to lend to a formerly marginal entrepreneur, once the payoff per unit of loans increases. Further, since the loan interest rate will be higher for each creditworthy entrepreneur when the deposit rate increases, from (4) each of these creditworthy entrepreneurs will be faced with a higher default premium.

Given (10), we can write $L_t = L(r_t)$, where $L(\cdot)$ is a decreasing function. For some of our results, all we will need is the reduced form $L(r_t)$. However, the rich detail in how entrepreneurs' investment projects are funded and the structure of loan interest rates and default premia across projects, will be particularly useful for understanding issues related to the financial crisis and central bank credit market interventions.

3 Equilibrium with Passive Fiscal Policy

The model we will ultimately use to understand recent events and policy in the United States will involve a setup where fiscal policy is treated as given, in a specific sense, by the monetary authority. However, at this point it is important to understand how the model behaves if the monetary authority determines policy, and the fiscal authority simply levies the taxes that are required to support that policy. We will also assume for now that bank reserves do not bear interest. Ultimately, including interest-bearing reserves is a minor extension.

In this model, arbitrage implies that currency is in general dominated in rate of return by all other assets, the expected rates of return on all those other assets (intermediary loans, bonds, and government loans) are equalized, and (given quasilinear utility) no expected rate of return can exceed the rate of time preference, i.e.

$$\frac{\phi_{t+1}}{\phi_t} \leq r_{t+1} = \frac{\phi_{t+1} q_{t+1}}{\phi_t} = \frac{\phi_{t+1} w_{t+1}}{\phi_t} \leq \frac{1}{\beta}. \tag{11}$$

3.1 Banks

Now, in the daytime when buyers and entrepreneurs meet, there are two reasons to form a financial intermediary. The first we have already discussed, which is the delegated-monitoring role for intermediation: a perfectly-diversified financial intermediary efficiently economizes on verification costs. Second, there is a Diamond-Dybvig (1983) role for a financial intermediary that can insure against the need for liquid assets in different types of transactions. Call this all-purpose financial intermediary a bank, and let a bank be run by an entrepreneur (who can commit). Banks form in the daytime of each period, and they dissolve in the daytime of the subsequent period, when they are replaced by a new set of banks. Note that banks form in the day before buyers know whether they will be in a non-monitored or monitored meeting in the subsequent night.

In equilibrium banks offer deposit contracts that maximize the expected utility of its depositors and earn zero profits. The depositors are essentially identical buyers at the time the bank forms. The bank acquires enough deposits from each depositor to purchase m units of currency (in real terms) and a units (in real terms) of interest-earning assets. Then, when each depositor learns his or her type, at the end of the day, each depositor who will be in a non-monitored meeting in the night withdraws $\frac{m'}{\rho}$ units of currency. Depositors in monitored meetings each receive the right to trade away deposit claims on $\frac{m-m'+a-a'}{1-\rho}$ units of the bank's original assets. After the claims (in the form of deposits and currency) of the original depositors are traded away in the night, the original depositors still have claims on $a-a'$ interest-earning assets. Without loss of generality (as this will not matter for the expected utility of the depositors), assume the bank assigns these claims to the monitored depositors, who then receive the returns to these assets in the next day. Note that we are assuming that all currency held by the bank is ultimately traded away by the depositors in equilibrium.[6] Thus, an equilibrium deposit contract (m,a,m',a') solves

$$\max_{m,a,m',a'} \left(\begin{array}{c} -m-a+\rho u\left(\beta\frac{\phi_{t+1}}{\phi_t}\frac{m'}{\rho}\right) \\ +(1-\rho)\left\{u\left[\beta r_{t+1}\left(\frac{a-a'}{1-\rho}\right)+\beta\frac{\phi_{t+1}}{\phi_t}\frac{(m-m')}{(1-\rho)}\right]+\beta r_{t+1}\left(\frac{a'}{1-\rho}\right)\right\} \end{array} \right) \tag{12}$$

subject to $m \geq 0$, $a \geq 0$, $0 \leq m' \leq m$, and $0 \leq a' \leq a$. In (12), given take-it-or-leave-it offers by buyers in nighttime meetings, each non-monitored depositor receives $\beta\frac{\phi_{t+1}}{\phi_t}\frac{m'}{\rho}$ goods from the seller they meet in exchange for their currency, while each monitored depositor receives $\beta r_{t+1}\left(\frac{m-m'+a-a'}{1-\rho}\right)$ goods in exchange for his or her deposit claims. Given the restrictions on equilibrium rates of return from the arbitrage conditions (11), the solution to problem (12) is:

1. If $\frac{\phi_{t+1}}{\phi_t} < r_{t+1} < \frac{1}{\beta}$, then $m'=m$, $a'=0$, and m and a solve, respectively,

$$\left(\frac{\beta\phi_{t+1}}{\phi_t}\right)u'\left(\frac{\beta\phi_{t+1}}{\phi_t}\frac{m}{\rho}\right)=1, \tag{13}$$

$$\beta r_{t+1}u'\left(\frac{\beta r_{t+1}a}{1-\rho}\right)=1. \tag{14}$$

2. If $\frac{\phi_{t+1}}{\phi_t} < r_{t+1} = \frac{1}{\beta}$, then $m'=m$, $a \in [(1-\rho)x^*,\infty)$, $a'=a-x^*$, and m solves (13).

3. If $\frac{\phi_{t+1}}{\phi_t} = r_{t+1} < \frac{1}{\beta}$, then $a'=0$, $m'=\rho(a+m)$, and $m \geq \frac{\rho a}{1-\rho}$, where $a+m$ satisfy

$$\beta r_{t+1}u'\left[\beta r_{t+1}(a+m)\right]=1, \tag{15}$$

4. If $\frac{\phi_{t+1}}{\phi_t} = r_{t+1} = \frac{1}{\beta}$, then $m \geq \rho x^*$, $m'=\rho x^*$, $a'=a+m-x^*$, and $a+m \geq x^*$.

[6] When the nominal interest rate is zero, there can be equilbria where currency is willingly held from one period to the next, but we do not lose anything from ignoring these equilibria.

Thus, in case 1, money is dominated in rate of return by other assets, and these other assets have a rate of return less than the rate of time preference. In this case, a bank's deposit contract stipulates that all of the currency held by the bank is withdrawn by non-monitored depositors and spent in the night, and all the remaining deposits in the bank (which are backed by assets other than currency) are traded in the night. Case 2 is the same as case 1, except that the rate of return on other assets is equal to the rate of time preference. In this case, exchange is efficient for monitored depositors (all these agents buy x^* in the night) and the bank is willing to acquire an unlimited quantity of other assets (in excess of what is required for monitored depositors to purchase x^*) so that monitored depositors can hold them until the next day and not trade them. In case 2, just as in case 1, non-monitored depositors withdraw all of the currency from the bank at the end of the current day. In case 3, the rates of return on money and other assets are equal, so monitored and non-monitored depositors consume the same amount during the day. The bank must hold enough currency to finance the consumption of non-monitored depositors, but the bank is otherwise indifferent about the composition of assets in its portfolio. Indeed, in this case the bank could hold currency on its balance sheet (effectively reserves) until the next day when the bank is liquidated. In case 4, where all rates of return are equal to the rate of time preference, monitored and nonmonitored exchange in all nighttime meetings is efficient, and the bank is willing to acquire an unlimited quantity of all assets to carry over until the next day.

What does the bank accomplish here in the way of risk sharing? Consider what would happen if buyers did not learn until the beginning of the night what type of meeting (non-monitored or monitored) they will be in. In that case, banks can still perform a delegated monitoring role, writing debt contracts with entrepreneurs, and diversifying so as to economize on verification costs. However, each buyer would leave the day with a portfolio of currency and other assets. Then, if the buyer was in a non-monitored trade, the other assets would be of no use, since they would not be acceptable in trade, and if the buyer was in a monitored trade, currency would be used inefficiently, since it would in general be dominated in return by other assets. The bank essentially permits the efficient allocation of liquidity in transactions. There is a kind of Diamond-Dybvig (1983) insurance role for banking, but this is a step beyond Diamond-Dybvig. In contrast to the Diamond-Dybvig model, liquidity and the rates of return on assets are endogenous here, and bank deposits are tradeable. Further Diamond-Dybvig (1983) is essentially a partial equilibrium approach - here we are working in general equilibrium. Our model has something of the flavor of Champ, Smith, and Williamson (1996), though the Champ-Smith-Williamson model has some undesirable features.[7]

Now, to help understand some of the transactions that are taking place in this economy, consider Figure 1. Here, we look only at the flows of physical

[7] The infinite horizon setup here is preferable to the Champ, Smith, and Williamson (1996) overlapping generations environment. As well, Champ, Smith and Williamson (1996) has the undesirable feature that it is efficient for the central bank to intermediate all of the alternative assets - this supplies the efficient quantity of liquidity.

objects in the model - currency and goods - and leave the flows of IOUs (of entrepreneurs and banks) out of the picture.

[Figure 1 here.]

3.2 Government Policy

It is typical in the monetary theory literature to treat fiscal policy as being purely passive. For example, in Lagos-Wright (2005), which is representative in this respect, the authors analyze what they consider a monetary policy experiment. This involves examining the effects of allowing the stock of fiat money to grow at different rates, assuming that the path of lump-sum taxes changes passively to support different paths for the nominal money stock. We follow a similar approach in this section, as a benchmark case to help show how the fiscal policy regime matters for monetary policy.

Now, so that we can analyze a straightforward policy, suppose that the monetary authority commits to a policy such that the total stock of nominal government liabilities grows at a constant gross rate μ, and the ratio of currency to the total nominal government debt is a constant, δ. That is,

$$M_t = \delta(M_t + B_t - F_t). \tag{16}$$

Here, B_t denotes the bonds issued by the fiscal authority but not held as assets by the central bank (where B_t could be negative), while F_t is central bank lending, and we have set $E_t = 0$ as we are assuming for now that bank reserves do not bear interest. We will want to allow for equilibria where the nominal interest rate is zero, in which case some of the stock of outside money (M_t) may be held by banks as reserves. Letting $\bar{B}_t$ denote the quantity of nominal bonds issued by the fiscal authority in the centralized market in the daytime of period t, or the total nominal outstanding debt of the consolidated government, the central bank's balance sheet constraint is

$$M_t = \bar{B}_t - B_t + F_t,$$

so from (16) we can write the ratio of currency to total nominal debt as

$$\delta = 1 + \frac{F_t - B_t}{\bar{B}_t}. \tag{17}$$

Now, in principle, $\delta \in (-\infty, \infty)$ is admissible, since $\bar{B}_t$ could be positive or negative, and the central bank could (in principle) make F_t infinitely large. Given the class of monetary policies under consideration, the arbitrage conditions (11), and the government budget constraints (1) and (2), lump sum taxes are passively determined by

$$\tau_t = -\frac{\phi_t M_t}{\delta}\left(1 - \frac{1}{\mu}\right) + \frac{\phi_t M_{t-1}(1-\delta)}{\delta}(\mu r_{t+1} - 1), \tag{18}$$

$$\tau_0 = -\frac{\phi_0 M_0}{\delta}. \tag{19}$$

Now, in equation (18), the first term on the right-hand side is the negative of the proceeds from the increase in the stock of total government liabilities in period t, which reduces taxation, and the second term is the real value of the net interest on government liabilities. Equation (19) determines the real transfer that goes to the private sector as the proceeds of the initial issue of government liabilities at $t = 0$.

3.3 Equilibrium

We will confine attention to stationary equilibria where real quantities are constant over time. This then implies that the gross real return on currency is $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$. Then, we define a stationary equilibrium as follows.

Definition 1 *Given a monetary policy (μ, δ), a stationary equilibrium with passive fiscal policy consists of real quantities of currency m and other assets a, a tax τ for periods $t = 1, 2, ...,$ an initial tax τ_0, and a gross real interest rate r, such that (i) m and a solve (12) when $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and $r_{t+1} = r$; (ii) asset markets clear*

$$a = m\left(\frac{1}{\delta} - 1\right) + L(r), \tag{20}$$

and the government budget constraints (18) and (19) hold, or

$$\tau = -\frac{m}{\delta}\left(1 - \frac{1}{\mu}\right) + m\left(\frac{1}{\delta} - 1\right)\left(r - \frac{1}{\mu}\right), \tag{21}$$

$$\tau_0 = -\frac{m}{\delta}. \tag{22}$$

For existence of this equilibrium, it is necessary that $\mu \geq \beta$, which is a necessary condition for the nominal interest rate to be non-negative. Our next step is to characterize equilibria, but how the model behaves depends critically on the relative returns on currency and other assets, as we saw already in the solution to the bank's problem. Start with the *liquidity trap case*, which occurs when $\frac{1}{\mu} = r < \frac{1}{\beta}$ so that the real rates of return on currency and other assets are equal (the nominal interest rate is zero), but assets are scarce, so that the real rate of return on assets is less than the rate of time preference (there is a liquidity premium associated with all assets). We will then analyze the *plentiful interest-bearing assets case*, which occurs when $\frac{1}{\mu} < r = \frac{1}{\beta}$. Here, the nominal interest rate is positive, but there is no liquidity premium on non-currency assets. Once we understand these two cases, the other two cases are straightforward. These latter two cases are the *scarce interest-bearing assets case* ($\frac{1}{\mu} < r < \frac{1}{\beta}$) and the *Friedman rule case* ($\frac{1}{\mu} = r = \frac{1}{\beta}$).

3.3.1 Equilibrium With $\frac{1}{\mu} = r < \frac{1}{\beta}$: Liquidity Trap

Here, from the solution to the bank's problem, (15) gives

$$\frac{\beta}{\mu}u'\left(\frac{\beta}{\mu}(a+m)\right) = 1,$$

(20) gives

$$a = m\left(\frac{1}{\delta} - 1\right) + L\left(\frac{1}{\mu}\right),$$

and the above two equations imply that the following equation solves for m.

$$\frac{\beta}{\mu}u'\left[\frac{\beta}{\mu}\left(\frac{m}{\delta} + L\left(\frac{1}{\mu}\right)\right)\right] = 1. \tag{23}$$

The condition we require for existence of the liquidity trap equilibrium is that, given the solution to (23), the real quantity of currency is sufficient to finance the purchases of non-monitored depositors, with some excess cash held by banks as reserves. This implies

$$m \geq \left(\frac{\rho\delta}{\delta - \rho}\right) L\left(\frac{1}{\mu}\right), \tag{24}$$

if $\delta > \rho$, or $\delta < 0$; otherwise the inequality in (24) is reversed. From (24), if

$$x^* - L\left(\frac{1}{\beta}\right) \geq 0, \tag{25}$$

holds, then a liquidity trap equilibrium can exist for

$$\delta \geq \frac{\rho x^*}{x^* - L\left(\frac{1}{\beta}\right)} \equiv \delta_a$$

and $\mu \in (\beta, \mu_b)$, where μ_b solves

$$\frac{\beta}{\mu_b}u'\left[\frac{\beta}{\mu_b}\left(\frac{\delta}{\delta - \rho}\right) L\left(\frac{1}{\mu_b}\right)\right] = 1. \tag{26}$$

A liquidity trap equilibrium does not exist for $0 < \delta < \delta_a$ if (25) holds. However, if $\delta < 0$ then a liquidity trap equilibrium can exist for $\mu > \mu_b$.

Now if (25) does not hold, then from (24) an equilibrium of this type cannot exist for $\delta > 0$. For $\delta_a \leq \delta < 0$, a liquidity trap equilibrium can exist for $\mu > \mu_b$, and if $\delta < \delta_a$ this equilibrium can exist for $\mu > \beta$.

This equilibrium is a liquidity-trap equilibrium, since it has the property that the nominal interest rate is zero, and from equation (23), the real stock of outside money is proportional to δ. A change in δ, essentially a one-time open market operation, is irrelevant - it leaves all prices and quantities unaffected. For example, an increase in the absolute value of δ implies that the one-time open market injection of cash is simply held as bank reserves forever, and there is no effect on the price level.

A key result here, which is new in the literature, is that this equilibrium is not a Friedman rule equilibrium, in spite of the fact that the nominal interest rate is zero. In most monetary models, if the economy is stationary with no aggregate shocks, the nominal interest rate is zero only when the central bank runs the Friedman rule, with the money supply growing at minus the rate of time preference. Here, the central bank can achieve a liquidity trap equilibrium with any money growth factor $\mu > \beta$ given judicious choice of δ. In particular, if the supply of private liquidity is plentiful when $\mu = \beta$, i.e. (25) does not hold, then to obtain a liquidity trap equilibrium, the central bank must choose $\delta < 0$. Here, $\delta < 0$ implies that the consolidated government is a net creditor, but the central bank is a debtor with liabilities m forever.

Note that the central bank is not powerless in a liquidity trap equilibrium. From equation (23), changing δ has no real (or nominal) effects, but changing μ, the gross growth rate in nominal government liabilities, matters. Indeed, an increase in μ (equal to the gross inflation rate in equilibrium) results in a decrease in m, and causes consumption to fall in the night for all buyers. Further, an increase in μ lowers the real interest rate r and increases $L(r)$, the quantity of lending to entrepreneurs.

3.3.2 Equilibrium With $\frac{1}{\mu} < r = \frac{1}{\beta}$: Plentiful Interest-Bearing Assets

In this equilibrium, currency is scarce, in that there is a liquidity premium on currency, with $\frac{1}{\mu} < \frac{1}{\beta}$, i.e. the rate of return on currency is less than the rate of time preference. However, other assets are not scarce, and so $r = \frac{1}{\beta}$ (no liquidity premium on other assets) and the nominal interest rate is greater than zero.

Now, from the bank's problem, when $\frac{1}{\mu} < r = \frac{1}{\beta}$, m solves (13), or

$$\frac{\beta}{\mu} u' \left(\frac{\beta}{\mu} \frac{m}{\rho} \right) = 1, \tag{27}$$

and (20) holds. Now, in order that assets be plentiful, we require $a \geq (1 - \rho)x^*$, i.e. banks must hold sufficient assets in their portfolios to finance efficient consumption for monitored depositors. From (20), this gives

$$m \left(\frac{1}{\delta} - 1 \right) \geq (1 - \rho)x^* - L \left(\frac{1}{\beta} \right). \tag{28}$$

Thus, given (δ, μ) with $\delta \in (-\infty, \infty)$, $\delta \neq 0$, and $\mu > \beta$, if (28) is satisfied given the m that solves (27), then this is an equilibrium of the type we are looking for. Now, first consider the case where

$$(1 - \rho)x^* - L \left(\frac{1}{\beta} \right) > 0. \tag{29}$$

Then, if $\delta < 0$ or $\delta > \delta_a$, this equilibrium does not exist; it can exist if $0 < \delta \leq \delta_a$ for $\mu \in (\beta, \mu_c]$, where μ_c solves

$$\frac{\beta}{\mu_c} u' \left(\frac{\beta}{\mu_c} \left[\frac{(1-\rho)x^* - L\left(\frac{1}{\beta}\right)}{\rho\left(\frac{1}{\delta} - 1\right)} \right] \right) = 1.$$

Next, consider the case where

$$(1-\rho)x^* < L\left(\frac{1}{\beta}\right) < x^*. \tag{30}$$

Now the equilibrium does not exist for $\delta < 0$; it exists if $0 < \delta \leq \delta_a$ for any $\mu > \beta$; and it exists for $\delta > \delta_a$ if $\mu \in [\mu_c, \infty)$. Finally, consider the case where (25) does not hold. Then, the equilibrium can exist for $\delta > 0$ or $\delta < \delta_a$ for all $\mu > \beta$. An equilibrium can also exist for $\delta_a \leq \delta < 0$ and $\mu \in [\mu_c, \infty)$.

Now, in this equilibrium, δ is irrelevant for real quantities of interest. A change in δ, interpreted as a one-time open market operation, is neutral, having no effect on the real interest rate (which is invariant at the rate of time preference) or on consumption in the nighttime in the decentralized market. From (27), the real stock of currency m is invariant to changes in δ, so the price level increases in proportion to any money injected through an open market operation.

3.3.3 Equilibrium With $\frac{1}{\mu} < r < \frac{1}{\beta}$: Scarce Interest-Bearing Assets

In this equilibrium, the nominal interest rate is positive, and currency and other assets are both scarce, in that trade is inefficient in the night in both non-monitored and monitored meetings. An equilibrium of this type consists of (m, a, r) solving the market-clearing condition (20), and the two first-order conditions (13) and (14), or

$$\frac{\beta}{\mu} u' \left(\frac{\beta}{\mu} \frac{m}{\rho} \right) = 1, \tag{31}$$

$$\beta r u' \left(\frac{\beta r a}{1-\rho} \right) = 1. \tag{32}$$

Here, equation (31) solves for the real quantity of currency, m, given μ. Then, equations (20) and (32) solve jointly for a and r, given m and δ. Equation (32) defines an upward sloping locus in (a, r) space - the upward-sloping portion of the curve D (demand for interest-bearing assets) in Figure 2. Note that the D curve is upward-sloping as $\frac{-xu''(x)}{u'(x)} < 1$, i.e. the substitution effect dominates the income effect. In Figure 2, the D curve becomes flat when $a = (1-\rho)x^*$ and $r = \frac{1}{\beta}$, which denotes the case with plentiful interest-bearing assets. In Figure 2, the downward-sloping S curve (the supply of interest-bearing assets) is a locus defined by (20) given m and δ.

[Figure 2 here.]

Now, given m (which is determined by μ) an increase in δ shifts the S curve from S_1 to S_2 in Figure 3 since, given m, the quantity of government-supplied interest-bearing assets has fallen with the one-time open-market purchase by the central bank. The price level rises in proportion to the resulting level increase in the supply of currency (m is unchanged), but the real interest rate falls and lending to entrepreneurs, $L(r)$, increases, as private assets substitute for public assets. This result is new in the literature, in that we get a permanent non-neutrality of money here. This does not work through a "real balance effect," since the real stock of currency is unaffected. Instead, the transmission mechanism involves a permanent reduction in the real value of government bonds, which increases the liquidity premium on interest-bearing assets (r falls), which then results in more lending by banks.

[Figure 3 here.]

The effect of an increase in μ, given δ, depends on the value of δ. If $\delta < 0$ or $\delta > 1$, then either the consolidated government is a net creditor, or it is a net debtor and the stock of central bank loans is positive, respectively. In those cases, an increase in μ, which reduces real currency balances m, increases the net stock of government-supplied interest-bearing assets and increases the real interest rate r through a rightward shift in the S curve in Figure 3. However, if $0 < \delta < 1$, so that the government is a net debtor and there is no central bank lending, then all of these effects work in the opposite directions.

An equilibrium with scarce interest-bearing assets exists if and only if the solution for r from equations (20), (31), and (32) satisfies $\frac{1}{\mu} < r < \frac{1}{\beta}$. Given our comparative static results above, we can show that, if (29) holds, then this equilibrium exists for $0 < \delta \leq \delta_a$ and $\mu > \mu_c$, for $\delta > \delta_a$ and $\mu > \mu_b$, and for $\delta < 0$ and $\mu > \mu_b$. However, if (30) holds, then this equilibrium exists for $\delta > \delta_a$ and $\mu \in (\mu_b, \mu_c)$, and for $\delta < 0$ and $\mu \in (\mu_b, \mu_c)$. Finally, if (25) does not hold, then this equilibrium does not exist for $\delta > 0$ or $\delta \leq \delta_a$, but if $\delta_a < \delta < 0$, then this equilibrium exists for $\mu \in (\mu_b, \mu_c)$.

3.3.4 Friedman Rule Equilibrium

A Friedman rule equilibrium, with $\frac{1}{\mu} = r = \frac{1}{\beta}$, obtains for any δ, whenever $\mu = \beta$. Thus, there are many ways to implement the Friedman rule here, and it is interesting to explore some of these possibilities. First, from (21) and (22), taxes in the Friedman rule equilibrium are given by

$$\tau = \frac{\rho x^*}{\delta}\left(\frac{1}{\beta} - 1\right), \tag{33}$$

$$\tau_0 = -\frac{\rho x^*}{\delta}. \tag{34}$$

One way to implement the Friedman rule would be to have $\delta \to \infty$, which could be achieved if the fiscal authority issues no debt and the central bank issues the entire stock of money through central bank lending. In this case, (33) and (34) give zero taxes in each period, i.e. $\tau \to 0$ and $\tau_0 \to 0$. What happens here is that the central bank lends out the entire stock of currency as a loan at $t = 0$, and then retires currency over time using the net interest on the central bank loans, while keeping the real stock of central bank loans constant over time. Alternatively, the Friedman rule could be implemented via the typical mechanism used in pure currency models, with $\delta = 1$. In this case (33) and (34) give $\tau = \rho x^* \left(\frac{1}{\beta} - 1\right)$ and $\tau_0 = -\rho x^*$, so the government issues currency at $t = 0$ as a lump sum transfer, and then retires currency over time through taxation so as to achieve the appropriate rate of deflation.

3.3.5 Existence and Optimality

Figures 4, 5, and 6 show the types of equilibria that exist for given (δ, μ) in the cases where (29) holds, (30) holds, and (25) does not hold, respectively. In Figure 4, given that (29) holds, in the region of the parameter space where $\delta < 0$ and $\mu \in (\beta, \mu_b)$, an equilibrium does not exist, as is the case in Figure 5, and in Figure 6 for $\delta_a < \delta < 0$ and $\mu \in (\beta, \mu_b)$. We get nonexistence because the nominal interest rate would have to be negative in this region of the parameter space, which of course cannot hold in equilibrium.

[Figures 4, 5, 6 here.]

When $\delta < 0$, an interesting feature is that, when an equilibrium exists, there are always two, one of which is the liquidity trap equilibrium. Why do we get this result? When $\delta < 0$, for a given (δ, μ) for which an equilibrium exists, there will be an equilibrium with a positive nominal interest rate (either an equilibrium with plentiful or with scarce interest-bearing assets) and a liquidity trap equilibrium with a zero nominal interest rate. Since $\frac{1}{\mu} < r$, the quantity of private liquid assets in the zero-nominal-interest rate equilibrium $L\left(\frac{1}{\mu}\right)$, is greater than $L(r)$, the quantity of private liquid assets in the positive-nominal-interest-rate equilibrium. Further, the real quantity of currency m in the zero-nominal-interest-rate equilibrium is by necessity larger than m in the positive-nominal-interest rate equilibrium, since the liquidity trap equilibrium requires some money be held as reserves by banks. However this implies that the total quantity of government-supplied liquid assets, $\frac{m}{\delta}$ (which is negative), is smaller in the zero-nominal-interest-rate equilibrium than in the positive-nominal-interest-rate equilibrium. Thus, we obtain two equilibria, one with plentiful currency, plentiful private assets, but a scarce quantity of potentially interest-bearing assets (the liquidity trap equilibrium), and another equilibrium with scarce currency, scarce private assets, and a relatively plentiful supply of interest-bearing assets (the positive-nominal-interest-rate equilibrium). To get multiple equilibria requires $\delta < 0$, as when $\delta > 0$ we can only obtain the liquidity trap equilibrium when currency is plentiful and other assets are scarce.

Now, what is an optimal allocation in this economy? One potential complication in determining this is the costly-state-verification delegated-monitoring financial intermediation activity in the model. However, it is straightforward to characterize an efficient allocation in this environment. Suppose first that we weight utilities equally across buyers and sellers, and treat these agents as a group. Then, just as in typical Lagos-Wright (2005) setups, welfare is proportional to the total surplus in nighttime matches, which is given by

$$W = \rho[u(x_n) - x_n] + (1-\rho)[u(x_m) - x_m], \tag{35}$$

where x_n is consumption in non-monitored meetings, and x_m is consumption in monitored meetings. In terms of daytime production and consumption, since daytime utilities of all agents are linear, the costly-state-verification delegated-monitoring intermediary structure efficiently transfers utility between buyers and sellers, as a group, and entrepreneurs, as a group, for any gross real interest rate r. Thus, our welfare criterion is given by (35). Any Friedman rule allocation is optimal, since $x_n = x_m = x^*$ in a Friedman rule equilibrium, which maximizes surplus W.

Note that, whenever two equilibria exist, the positive-nominal-interest-rate equilibrium always dominates the liquidity trap equilibrium. This is because x_n is the same in each equilibrium, but x_m is larger (but of course less than x^*) in the positive-nominal-interest-rate equilibrium.

Now, in addition to yielding an optimal allocation of resources, the Friedman rule, however implemented (through whichever value of δ), implies that there exists an equilibrium where banks serve only a delegated monitoring role, buyers in the day acquire currency and other assets (including intermediated claims on investment projects), all goods in the nighttime are purchased with currency, and other assets are held from one day until the next and not traded. What is wrong with this picture? Clearly, it does not make much sense as a description of what an optimal financial arrangement might look like in the real world. It cannot be optimal to be making all transactions using currency, and the reasons should be obvious. While currency has some very useful attributes - settlement of debts is immediate, trade under anonymity is possible, no sophisticated technology is necessary - exchange using currency is subject to some potentially severe inefficiencies. First, currency can be stolen. We all know why it is not a good idea to carry large sums of cash in our pockets, or to send it in the mail. Second, it is costly for the government to maintain the stock of currency. Worn-out notes must be shredded and replaced, armored trucks are needed to transport old currency to the local Federal Reserve Bank and distribute the new currency to financial institutions, and the currency must be designed to thwart counterfeiters in an efficient way. Third, there are social losses from the time and effort expended by counterfeiters and thieves. Fourth, the existence of currency makes illegal activities, including trade in illegal commodities and tax evasion, less costly.

The Friedman rule has always been somewhat of a puzzle in monetary economics. Most basic monetary models imply that the Friedman rule is optimal, but we never observe central banks adopting monetary policy rules that imply zero nominal interest rates forever. Our contention is that most monetary models leave out some critical elements, which are the costs associated with currency exchange, as we outlined in the previous paragraph. We will explore this further in what follows.

3.3.6 Interest on Reserves

Up to now, we have considered a setup where it is necessary for the nominal interest rate to be zero for reserves to be held by banks. Reserves are held only in the liquidity trap case. However, given that we want our model to apply generally, we need to consider what happens if outside money, held in reserve accounts with the central bank, bears interest. Some central banks in the world, including the European Central Bank and the Bank of Canada, have paid interest on reserves for some time, and the U.S. Congress recently approved the payment of interest on reserve accounts held at Federal Reserve Banks. The Fed has been paying interest on reserves since October 2008 at 0.25%.

We now allow for the possibility that reserves $E_t > 0$ in equilibrium, in (1) and (2). We continue to assume that fiscal policy is passive, but now the central bank will determine the gross rate of growth in total government liabilities, μ, and the ratio of outside money (currency and reserves) to total government debt, η, where

$$M_t + E_t = \eta(M_t + B_t + E_t - F_t) \tag{36}$$

for all t. Further, the interest rate on reserves is determined by the central bank, i.e. the central bank also sets $s_t = s$ (the nominal redemption value for reserves) for all t. Now, what happens is that, in the daytime, banks acquire outside money and other assets. Then, at the end of the day, banks determine how much of this outside money is withdrawn in the form of currency by non-monitored depositors, and how much remains with the bank as reserve balances. Then δ, defined just as before as the ratio of currency to total government debt, is endogenous, with

$$M_t = \delta(M_t + B_t + E_t - F_t). \tag{37}$$

In equilibrium, arbitrage implies that rates of return on interest-bearing assets must satisfy

$$r_{t+1} = \frac{\phi_{t+1} q_{t+1}}{\phi_t} = \frac{\phi_{t+1} w_{t+1}}{\phi_t} \geq \frac{\phi_{t+1} s_{t+1}}{\phi_t},$$

where the last weak inequality holds as an equality if $E_t > 0$. In a stationary equilibrium, we have

$$r = \frac{q}{\mu} = \frac{w}{\mu} \geq \frac{s}{\mu}.$$

Now, in a stationary equilibrium with $E_t \geq 0$ for all t, we have $\delta \leq \eta$ if $\delta > 0$, and $\delta \geq \eta$ if $\delta < 0$. We define an equilibrium in a similar manner to how this was done at the beginning of this section. First, we could have $\delta = \eta$, so that zero reserves are held, in which case it must be the case that s is set so that $\frac{s}{\mu} \leq r$ in equilibrium, but otherwise the definition of equilibrium is exactly the same as specified originally. The central bank chooses $\delta = \eta$ and μ, and then r is determined endogenously. Second, it could be the case that $\delta < \eta$ if $\delta > 0$ or $\delta > \eta$ if $\delta < 0$ in equilibrium. In this case, the central bank chooses

η, r, and μ, and then δ is determined endogenously. In this case, the central bank effectively determines all one-period interest rates by setting the interest rate on reserves, and then banks choose how to split outside money between currency and reserves.

Proposition 2 *Suppose that the interest rate $\hat{r}$ is an equilibrium interest rate given $\delta = \eta = \hat{\eta}$ and $\mu = \hat{\mu}$. Then, for any $\tilde{\eta} > \hat{\eta}$ if the central bank sets $r = \hat{r}$ and $\eta = \tilde{\eta}$, then $\delta = \hat{\eta}$ is an equilibrium value for the ratio of currency to total government liabilities.*

Proof. First, from the definition of equilibrium at that beginning of this section, if the choices of $m = \hat{m}$ and $a = \hat{a}$ are optimal given $r = \hat{r}$ when $\delta = \eta = \hat{\eta}$ and $\mu = \hat{\mu}$, then these choices are also optimal when the central bank sets $r = \hat{r}$ and $\mu = \hat{\mu}$. Next, from (20), (21), and (22), $\delta = \hat{\eta}$ must be an equilibrium since it yields the same solutions to all of these equations. ■

What this proposition says is that, if the central bank pays interest on reserves, then in an equilibrium where banks are holding a positive quantity of reserves bearing the market interest rate, an open market operation is irrelevant. Changing η in these circumstances has no effect on any prices or quantities, as effectively the government is just swapping one interest-bearing asset for another, much as in the liquidity trap equilibrium. What does matter in general, however, is the interest rate on reserves r. Reducing r, which is always possible, will have the effect, given our analysis of the equilibrium with scarce interest-bearing assets, of increasing δ, from equations (20), (31), and (32). Since m is unchanged and we are holding η constant, the price level increases, i.e. the real quantity of currency is unchanged, but the nominal quantity of currency has risen, and the nominal quantity of reserves has fallen, so the price level has to have risen.

There is an important lesson here that relates to the current predicaments of central banks in the world, including the Fed and the European Central Bank Both of these central banks currently have issued large quantities of reserve balances, well in excess of reserve requirements. To traditional monetarists, it might appear puzzling that inflation is low in the United States and Europe in spite of large increases in outside money. However, our model tells us that large increases in outside money accomplished by increasing interest-bearing reserves can have no effect on prices, so this is not puzzling. Further, our model tells us that the existence of a large quantity of excess reserves is not a problem for inflation control. The central bank has all the control it needs by using the interest rate on reserves as a policy instrument, in spite of the fact that open market operations have no effect with a positive quantity of excess reserves in the system.

4 Non-Passive Fiscal Policy

Now, there are two respects in which our model needs some work. First, as was discussed above, the model's predictions for optimal policy, like those of most mainstream monetary models, are problematic. The Friedman rule is optimal in this model, but running the Friedman rule supports equilibria where currency is the dominant means of payment, which does not seem to be too helpful. Second, given the relationship in typical developed economies between the central bank and the fiscal authority, there are better ways to think about monetary policy than to have fiscal policy be purely passive. Our approach in the rest of this paper will be to add to our basic model by first dealing with the second problem, and then dealing with the first. In this section we will consider an alternative approach to modeling policy decisions, which will be a straightforward extension of what we have done thus far. Then, in the next section, we will add costs of currency exchange to the model, which will allow us to approach the monetary policy problem in an interesting and novel way.

Now, we want to have a plausible notion of the dimension or dimensions of fiscal policy that are treated as given by the central bank. First, suppose that there is a fixed level of lump sum taxation that funds transfers and spending on goods and services. Given the nature of preferences in this model, there is more than one way for transfers and spending to enter into the framework so that this fiscal activity is irrelevant for the problems under consideration. This then justifies leaving this element of government activity out of the model.[8]

What is critical for the central bank is the deficit that the consolidated government needs to finance each period. Assume that the fiscal authority fixes the deficit at a constant level σ forever, in real terms. To do this, the fiscal authority also commits to levying lump-sum taxes in each period to pay the net interest on the outstanding government debt. As before, the central bank sets δ, the ratio of currency to total outstanding government debt according to (16). Now, however, instead of (18), lump sum taxes to pay the interest on the government's debt are determined passively, i.e.

$$\tau_t = \frac{\phi_t M_{t-1}(1-\delta)}{\delta}(\mu r_{t+1} - 1), \tag{38}$$

for $t = 1, 2, 3, ...$, and we also assume that τ_0 is determined as in equation (19). Also, most importantly, from (18) the deficit is financed each period by the issue of interest-bearing debt and currency, according to

$$\sigma = \frac{\phi_t M_t}{\delta}\left(1 - \frac{1}{\mu}\right), \tag{39}$$

for $t = 1, 2, 3, ...$. This implies that δ and μ cannot be set independently. An interpretation is that the fiscal authority fixes the deficit and issues whatever nominal debt is necessary to finance it given the setting of the monetary policy instrument δ by the central bank. An important assumption here is that the

[8] For example, we could assume that all government transfers and spending are in the centralized market during the day, and that government-provided goods and services are perfect substitutes for private goods. It seems sensible for what we want to accomplish in this paper to leave aside questions related to the optimal provision of public goods and optimal taxation.

central bank can choose $\delta > 0$ and the fiscal authority will follow the central bank's lead and issue nominal debt each period, while if $\delta < 0$ then the fiscal authority lends in nominal terms to the private sector.

Now, in a stationary equilibrium, $m = \phi_t M_t$ (currency demand equals currency supply in the daytime), so (39) gives

$$\sigma = \frac{m}{\delta}\left(1 - \frac{1}{\mu}\right). \tag{40}$$

The types of equilibria we are dealing with are the same ones as in the previous section. First, in a liquidity trap equilibrium, where $\frac{1}{\mu} = r < \frac{1}{\beta}$, substituting for m in equation (23) we get

$$\frac{\beta}{\mu}u'\left[\beta\left(\frac{\sigma}{\mu-1} + \frac{1}{\mu}L\left(\frac{1}{\mu}\right)\right)\right] = 1, \tag{41}$$

which determines μ. Recall that in a liquidity trap equilibrium, monetary policy is irrelevant, i.e. changing δ (at the margin) has no effect on quantities and prices. In a liquidity trap, μ is the gross rate at which total government liabilities have to grow to finance the deficit, and monetary policy does not matter, as open market operations are just swaps of identical assets.

Second, in an equilibrium with plentiful interest-bearing assets, where $\frac{1}{\mu} < r = \frac{1}{\beta}$, or in an equilibrium with scarce interest-bearing assets, where $\frac{1}{\mu} < r < \frac{1}{\beta}$, substituting for m from (40) in (27) we get

$$u'\left(\frac{\beta\sigma\delta}{(\mu-1)\rho}\right) = \frac{\mu}{\beta}, \tag{42}$$

and equation (42) determines the set of policies (δ, μ) that permit the fiscal authority to just finance its deficit.

Finally, in a Friedman rule equilibrium, $\mu = \beta$ and, given any δ, there exists an equilibrium where agents hold a sufficient quantity of consolidated-government liabilities that the deficit is financed each period, for any $\sigma > 0$. Just as in the passive-fiscal-policy case, a Friedman rule yields an efficient equilibrium allocation.

4.1 Example

At this point, it is useful to consider an example. Suppose that $u(x) = 2x^{\frac{1}{2}}$, and $\alpha = 0$ so that $L(r) = 0$ and there are no entrepreneurial investment projects and no private liquidity. Thus, banks serve only to intermediate government bonds. From our analysis in the previous section, first consider equilibria with passive fiscal policy. A liquidity trap equilibrium exists for $\delta \geq \rho$ and $\mu > \beta$, while an equilibrium with plentiful interest-bearing assets exists for $\delta \in (0, \rho)$ and $\mu \in (\beta, \mu_c]$, where

$$\mu_c = \frac{\rho\beta\left(\frac{1}{\delta} - 1\right)}{1 - \rho}.$$

An equilibrium with scarce interest-bearing assets exists for $\delta \in (0, \rho)$ and $\mu \in [\mu_c, \infty)$. A Friedman rule equilibrium exists for $\mu = \beta$ and any $\delta \in (-\infty, \infty)$. An equilibrium does not exist if $\delta < 0$ and $\mu > \beta$. In the equilibrium with scarce interest-bearing assets, the real interest rate is determined, using (20), (31), and (32), by

$$r = \frac{\rho\left(\frac{1}{\delta} - 1\right)}{(1-\rho)\,\mu},$$

which is decreasing in δ and in μ.

Now, when fiscal policy is not passive, then in a liquidity trap equilibrium, from (41), μ solves

$$\sigma\mu^2 - \beta\mu + \beta = 0, \tag{43}$$

and in an equilibrium with plentiful or scarce interest-bearing assets, from (42), δ and μ must satisfy

$$\sigma\delta\mu^2 - \rho\beta\mu + \rho\beta = 0. \tag{44}$$

We already know from the general case that a Friedman rule, with $\mu = \beta$ and any δ yields an efficient equilibrium allocation such that the government's deficit is financed in each period. Further, our characterization of equilibria and equations (43) and (44) determine the set of equilibria. If $\sigma < 0$, an equilibrium does not exist, since this would require $\delta < 0$, i.e. with no source of private liquidity the government cannot be a net creditor in equilibrium. However, if $0 < \sigma < \frac{\beta}{4}$, then if $\delta < \rho$ there is an equilibrium with either scarce or plentiful interest-bearing assets, and there are two solutions for μ, which from (44) are

$$\mu = \frac{\rho\beta \pm \left(\rho^2\beta^2 - 4\rho\beta\sigma\delta\right)^{\frac{1}{2}}}{2\sigma\delta} \tag{45}$$

If $0 < \sigma < \frac{\beta}{4}$ and $\delta \geq \rho$, then we will have a liquidity trap equilibrium, and from (43) there are two solutions for μ, which are

$$\mu = \frac{\beta \pm \left(\beta^2 - 4\beta\sigma\right)^{\frac{1}{2}}}{2\sigma}$$

If $\sigma \geq \frac{\beta}{4}$, then the liquidity trap equilibrium will not exist. However, in this case if $\delta \in \left(0, \frac{\rho\beta}{4\sigma}\right)$ then an equilibrium with scarce or plentiful interest-bearing assets exists, and there are two solutions for μ, given by (45).

A key feature of the model that this example illustrates is that, with non-passive fiscal policy, monetary policy need not uniquely determine the inflation rate. If we think of the central bank determining δ, the ratio of currency to total consolidated-government liabilities, and the fiscal authority determining μ so as to finance its real deficit, in this example there are essentially always two values of μ that finance the deficit for every feasible δ. We get this result because of a type of Laffer-curve phenomenon. Given δ, the gross inflation rate μ determines the tax rate on the total stock of nominal government liabilities (currency and bonds), and the size of the tax base falls with μ since real currency balances fall with the inflation rate. For μ sufficiently high, the marginal revenue from the inflation tax for the consolidated government is negative, so in general there is always a high inflation rate and a low inflation rate that finance the deficit for any feasible δ.

5 Non-Passive Fiscal Policy and Costs Associated With Currency

Now, to complete our model, we will incorporate some of the direct costs and social costs associated with exchange using currency. First, assume that the costs of maintaining the stock of currency, for the government, are proportional to the stock of currency in existence at the beginning of each day, in real terms, before government actions take place in the centralized market. In particular, it requires $\omega\phi_t M_{t-1}$ in units of goods for the government to replace worn-out currency and to make the currency non-counterfeitable, where $\omega > 0$.[9] Second, suppose that a fraction ν of non-monitored exchanges in the nighttime decentralized market, i.e. a fraction $\rho\upsilon$ of total meetings in the night, are deemed by society to be of no social value. This is a simple approach to capturing the fact that a significant fraction of the stock of currency is being used as a medium of exchange in illegal trades.

Assume that, when banks hold outside money as reserves (in a liquidity trap equilibrium), that these reserves are held as a non-interest-bearing account with the central bank, which can then be sold directly in the next centralized market to another bank, or converted into currency (though conversion to currency will not occur in the stationary equilibria we are studying). Further, assume that there are no costs to maintaining electronic reserves, which then implies that, in any equilibrium, our measure of welfare will be, rather than (35),

$$W = \rho(1-\upsilon)u(x_n) - \rho x_n + (1-\rho)[u(x_m) - x_m] - \frac{\rho\omega x_n}{\beta}, \tag{46}$$

where, relative to (35), we have subtracted the utility from illegal consumption, $\rho\upsilon u(x_n)$, and subtracted the cost of maintaining the currency $\frac{\rho\omega x_n}{\beta}$.

[9] One might argue that the costs associated with maintaining the currency stock are proportional to the nominal stock of currency, not the real stock. For example, there is a particular cost of maintaining each $1 note (it circulates at a high rate, and wears out quickly, but it is typically not counterfeited), another cost for each $20 note (it doesn't wear out so quickly, but it is more often counterfeited), etc. However, if the structure of denominations were determined optimally, we would periodically want to have a currency reform, or reset the size of denominations, given inflation. For example, if a $1 note is convenient because the average small transaction involves the purchase of about $1 in goods and services, and if prices double, we should either have a currency reform under which a new $1 note trades for two old $1 notes, or we should issue new denominations, with a $2 note replacing the $1 note. Under this system, the costs of maintaining the currency will be roughly proportional to the real money stock over time. In our model, it is too complicated to deal with indivisibilities of money, denominations, and making change. Modeling a perfectly divisible currency stock with costs proportional to the real money stock seems a good way to go.

Now, the beauty of this approach to modeling the costs of a currency system is that quantities traded in the nighttime decentralized market, prices, and interest rates, are all invariant to ν and ω, assuming that the costs of maintaining the currency stock are financed with lump-sum taxation. These parameters then only matter for welfare and for the determination of an optimal monetary policy.

At this point, we have all the elements of the model that we need to analyze optimal monetary policy in a sensible fashion. It will be useful to proceed with an example, in the next subsection.

5.1 Optimal Monetary Policy: An Example

Assume for convenience that $u(x) = \log(x)$, and that $L(r) = \alpha - \psi r$,[10] with $\psi > 0$ a parameter and $\alpha - \frac{\psi}{\beta} > 0$, which guarantees that there will always be some lending to entrepreneurs in equilibrium. First, consider the case with passive fiscal policy. Then, in a liquidity trap equilibrium, solving for m from (23), we obtain

$$m = \delta\left(1 - \alpha + \frac{\psi}{\mu}\right).$$

For a liquidity trap equilibrium to exist, we require $m > \rho$. Therefore, if

$$1 - \alpha < -\frac{\psi}{\beta},$$

then a liquidity trap equilibrium exists for

$$\delta < \delta_a \equiv \frac{\rho}{1 - \alpha + \frac{\psi}{\beta}},$$

and $\mu > \beta$, and for $\delta_a \leq \delta < \frac{\rho}{1-\alpha}$ and $\mu \in (\beta, \mu_b)$, where

$$\mu_b = \frac{\psi}{\frac{\rho}{\delta} + \alpha - 1}.$$

However, if

$$-\frac{\psi}{\beta} < 1 - \alpha < 0$$

then a liquidity trap equilibrium exists for $\delta > \delta_a$ and $\mu \in (\beta, \mu_b)$, and for $\delta < 0$ and $\mu > \mu_b$. Finally, if $1 - \alpha > 0$, then an equilibrium exists for $\delta_a < \delta < \frac{\rho}{1-\alpha}$ and $\mu \in (\beta, \mu_b)$, and for $\delta \geq \frac{\rho}{1-\alpha}$ and $\mu > \beta$.

In an equilibrium with plentiful interest-bearing assets, (27) gives $m = \rho$ and then (28) implies that

$$\frac{\rho}{\delta} \geq 1 - \alpha + \frac{\psi}{\beta} \tag{47}$$

[10] I have not worked out distribution functions $F(\cdot)$ and $G(\cdot)$ which will imply a linear $L(\cdot)$, but the setup is sufficiently flexible that this should not be a problem.

must be satisfied for this equilibrium to exist. Therefore, if

$$1-\alpha+\frac{\psi}{\beta}>0, \tag{48}$$

then an equilibrium of this type exists if and only if $\delta \leq \delta_a$, but if the inequality in (48) goes the other way, then this equilibrium exists if $\delta > 0$ and $\mu > \beta$ or if $\delta \leq \delta_a$.

In an equilibrium with scarce interest-bearing assets, (31) and (32) give $m = \rho$ and $a = 1 - \rho$, respectively, and then r can be determined from (20) to give

$$r=\frac{\alpha-1+\frac{\rho}{\delta}}{\psi}, \tag{49}$$

and for existence of the equilibrium we require that the solution satisfy $\frac{1}{\mu} \leq r \leq \frac{1}{\beta}$. Therefore, if

$$1-\alpha<-\frac{\psi}{\beta},$$

then this equilibrium exists for $\delta_a \leq \delta < \frac{\rho}{1-\alpha}$ and $\mu \geq \mu_b$, while if

$$-\frac{\psi}{\beta}<1-\alpha<0,$$

the equilibrium with scarce interest-bearing assets exists for $\delta < 0$ and $\mu \geq \mu_b$, and for $\delta \geq \delta_a$ and $\mu \geq \mu_b$. Finally, if $1-\alpha > 0$, then this equilibrium exists for $\delta_a \leq \delta < \frac{\rho}{1-\alpha}$ and $\mu \geq \mu_b$.

Now, with passive fiscal policy, determining an optimal monetary policy is straightforward. There always exists an optimal policy (δ^*, μ^*), with $\delta^* > 0$, such that an equilibrium with plentiful interest-bearing assets exists, and

$$\mu^*=\frac{\beta+\omega}{1-\upsilon}. \tag{50}$$

To see this, note that $\delta > 0$ with δ sufficiently small guarantees that (47) is satisfied, which implies $x_m = x^*$ in (46), maximizing surplus in monitored exchange at night. Then, note that setting $\mu = \mu^*$ will yield an equilibrium with plentiful interest-bearing assets, so long as δ is sufficiently small, and this thus serves to maximize W, since from (46), in an equilibrium with plentiful interest-bearing assets,

$$\frac{\partial W}{\partial \mu}=\frac{\rho^2}{\mu^2}\left[-(1-\upsilon)\mu+\beta+\omega\right].$$

Thus, a monetary policy that is unconstrained by the fiscal authority serves to generate sufficient public liquidity that there is efficient trading in monitored exchange, and sufficient growth in nominal government liabilities (and therefore sufficient inflation) that currency is taxed at the optimal rate. Note that the efficient gross inflation rate rises with υ, the fraction of socially undesirable currency transactions and with ω, the marginal cost of maintaining the stock of currency, from (50).

Next, consider what happens when fiscal policy is not passive, so that the constraint (40) must hold. This need not reduce the level of welfare that the monetary authority can achieve at the optimum, since it is possible that (δ^*, μ^*) satisfies (40) in equilibrium. Indeed, suppose that this is so. Then, since $m = \rho$ in an equilibrium with plentiful interest-bearing assets, (40) gives

$$\delta = \frac{\rho}{\sigma}\left(1 - \frac{1}{\mu}\right), \tag{51}$$

so if we can achieve the unconstrained optimal policy given the constraint (51), then from (51) and (50), we have

$$\delta = \frac{\rho}{\sigma}\left(\frac{\beta + \omega - 1 + \upsilon}{\beta + \omega}\right),$$

Therefore, from (47) we obtain

$$\frac{\sigma(\beta + \omega)}{\beta + \omega - 1 + \upsilon} \geq 1 - \alpha + \frac{\psi}{\beta}, \tag{52}$$

and this is the condition that is satisfied if and only if the fiscal constraint (51) does not bind at the optimum. Note that, for example, (52) is satisfied if $\sigma > 0$ (the government runs a perpetual deficit), $\beta + \omega + \upsilon > 1$ (the unconstrained optimal inflation rate is greater than zero), and $1 - \alpha + \frac{\psi}{\beta} < 0$ (there is a large supply of private liquid assets when the interest rate is equal to the rate of time preference).

To illustrate the possibilities, next construct a case such that an optimal monetary policy will imply an equilibrium either with scarce interest-bearing assets or a liquidity trap. Suppose that $0 < \sigma < 1 - \alpha$, so that the fiscal authority runs a perpetual deficit, financing needs are not too great, and the potential supply of private liquid assets is small. Further, assume that

$$\frac{1 - \alpha + \frac{\psi}{\beta}}{1 - \alpha + \frac{\psi}{\beta} - \sigma} < \frac{\beta + \omega}{1 - \upsilon}, \tag{53}$$

which implies that an unconstrained optimal policy is not feasible. The optimal policy must then lie outside the region of the parameter space where an equilibrium with plentiful interest-bearing assets exists.

Now, in an equilibrium with scarce interest-bearing assets, we have $m = \rho$, $a = 1 - \rho$, and r is determined by (49) and (40), which give

$$r = \frac{1 - \alpha + \mu(\sigma + \alpha - 1)}{\psi(\mu - 1)} \tag{54}$$

We can then use (46) to determine the derivative of welfare with respect to μ in the scarce-interest-bearing-assets region of the parameter space, given the constraint on policy. This derivative is

$$\frac{\partial W}{\partial \mu} = \chi(\mu) = \frac{\rho}{\mu^2}\left[-(1-\upsilon)\mu + \beta + \omega\right] - \frac{(1-\rho)\beta\sigma}{\psi(\mu-1)^2}\left[\frac{\frac{\psi}{\beta}(\mu-1)}{1-\alpha+\mu(\sigma+\alpha-1)} - 1\right]. \tag{55}$$

Now, an equilibrium with scarce interest-bearing assets exists so long as we get a solution for r such that $\frac{1}{\mu} \leq r \leq \frac{1}{\beta}$, which from (54) implies that $\mu \in [\underline{\mu}, \bar{\mu}]$, where

$$\underline{\mu} = \frac{1-\alpha+\frac{\psi}{\beta}}{1-\alpha+\frac{\psi}{\beta}-\sigma},$$

and $\bar{\mu}$ solves

$$\frac{\sigma\bar{\mu}}{\bar{\mu}-1} = 1-\alpha+\frac{\psi}{\bar{\mu}}$$

Now, our assumption (53) implies that $\chi(\underline{\mu}) > 0$, so an increase in the growth rate in nominal government liabilities (and therefore in inflation as well) that results in a scarcity of liquid assets is welfare-improving in this case. Thus, inflation is costly, as it reduces the quantity of trade in socially-desirable currency transactions, and also reduces the real quantity of total government debt, which reduces the quantity of liquid assets backing monitored transactions. However, it is also socially beneficial to tax currency transactions, so at the margin it may be beneficial to have more inflation, even though it reduces trade in all nighttime transactions.

Further, note that $\chi(\mu) < 0$ for $\mu \geq \mu^*$, so that the optimal gross inflation rate must be less than μ^* in this case. Thus, if it is possible to achieve an optimal monetary policy without making liquid interest-bearing assets scarce, the inflation rate will be higher than if policy has to make tradeoffs in terms of asset scarcity.

It is possible that the optimal monetary policy will imply a liquidity trap equilibrium at the optimum. That is, when $\mu = \bar{\mu}$, the nominal interest rate goes to zero ($r = \frac{1}{\mu}$), and we could have $\chi(\bar{\mu}) > 0$, for example if σ is sufficiently small. A sufficient condition for a scarce-interest-bearing-assets equilibrium to be optimal is $\bar{\mu} > \mu^*$, or

$$\sigma > \frac{(\beta+\omega+\upsilon-1)\left[1-\alpha+\frac{\psi(1-\upsilon)}{\beta+\omega}\right]}{\beta+\omega}.$$

Now, if a scarce-interest-bearing-assets equilibrium is optimal, then the optimal gross inflation rate $\hat{\mu}$ is determined by $\chi(\hat{\mu}) = 0$. If this is the case then each of the parameters in the model (all of which enter the expression on the right-hand side of (55)) matter for the determination of $\hat{\mu}$. In particular, note that $\chi(\hat{\mu})$ is decreasing in ψ, so that an increase in ψ will reduce the optimal gross inflation rate $\hat{\mu}$, and will also reduce δ, the ratio of currency to total government liabilities, from (51). A reduction in ψ could result from adverse shifts in the distribution of entrepreneurial project returns, $F(\cdot)$, or increased verification costs, i.e. changes in the payoffs to investment projects or in the costs of lending which reduce the "demand" for loans. These effects will be important in the next section.

5.2 The Financial Crisis

A feature of the financial crisis that this model can capture in a nice way is the effect of an increase in risk on the supply of liquid assets, and the resulting general equilibrium implications. It is well-known that increases in risk can result in a reduction in lending, an increase in interest rate spreads, and an increase in defaults, in models with costly state verification. For an early analysis of these effects, see Williamson (1987). Christiano, Motto, and Rostagno (2009) measure the contribution of these "risk shocks" to business cycles, and find that they are very important.

A risk shock works in the following fashion. To consider the simplest case, suppose that the economy is in a stationary equilibrium with an optimal monetary policy, constrained by the fiscal authority's real deficit σ, and the equilibrium has plentiful interest-bearing assets, so the gross real interest rate is $r = \frac{1}{\beta}$. Then, from (7) and (8), entrepreneurs with verification costs $\gamma \in [0, \gamma^*]$ receive loans with gross loan interest rates in the range $R \in [\underline{R}, R^*]$. Entrepreneurs with $\gamma > \gamma^*$ do not receive loans. Here, R^* and γ^* solve, from (7) and (8),

$$1 - \gamma^* f(R^*) - F(R^*) = 0, \tag{56}$$

$$\frac{1}{\beta} = R^* - \gamma^* F(R^*) - \int_0^{R^*} F(w)dw, \tag{57}$$

and $\underline{R}$ solves the zero-profit condition

$$\frac{1}{\beta} = \underline{R} - \int_0^{\underline{R}} F(w)dw, \tag{58}$$

Now, consider the following mean-preserving spread in the probability distribution of payoffs to investment projects $F(w)$. Suppose that the distribution is now $H(w) = F(w) + J(w)$, and the probability density function is $h(w) = f(w) + j(w)$, where $J(0) = J(\underline{R}) = J(\bar{w}) = 0$, $j(w) = 0$ for $w \in [\underline{R}, R^*]$, $\int_0^{\bar{w}} wj(w)dw = 0$, and $\int_0^R J(w)dw > 0$ for $R \in [\underline{R}, R^*]$. This increase in risk is set up in such a way that it affects all funded investment projects in exactly the same way. There is no effect on the probability of default, $F(R)$, for any funded investment project, but the term $\int_0^R F(w)dw$ increases for all funded investment projects. Effectively, moving probability mass into the tails of the the distribution of investment returns reduces the expected return to the bank in the event that an entrepreneur defaults, and this reduces the expected payoff on the loan given the loan interest rate.

Now, we can show, using (56) and (57), that R^* and γ^* both fall as a result of the increase in risk. However, from (58), $\underline{R}$ remains unchanged. Thus $L(\frac{1}{\beta})$, the quantity of lending at the interest rate $r = \frac{1}{\beta}$, must also decrease. If this decrease in lending is not too large, then the equilibrium is still one with plentiful interest-bearing assets, and there is no change in the optimal monetary policy. However, for each entrepreneur who receives a loan, the default premium, from (4), must increase, i.e. there is an increase in the spread between the default-free gross interest rate on government debt, r, and the gross loan interest rate R for any funded investment project. There is no change in nighttime production and consumption or in the efficiency of nighttime exchange, but output falls in the day because of the reduction in output from investment projects.

More interestingly, for purposes of understanding the recent financial crisis and recession, suppose that the risk shock is large enough that it is no longer optimal for the central bank to choose a monetary policy that yields an equilibrium with plentiful interest-bearing assets. Now, the effects on interest rate spreads are complicated somewhat by general equilibrium effects, since the gross real interest rate r is now endogenous. Williamson (1987) accounts for these effects in a different model, but the results here should be similar, with spreads still increasing, in general. Here, we are more interested in how monetary policy should respond to a large risk shock.

From our last subsection, if we consider the case where $\sigma > 0$, so that the fiscal authority is running a perpetual deficit, the risk shock will work much like an increase in ψ. Then, we can have cases in the example in the previous subsection where the economy can initially be in a plentiful-interest-bearing-assets equilibrium under optimal monetary policy, but then an increase in ψ implies that optimal policy will be such that interest-bearing assets are scarce. Then, the risk shock will imply that δ and μ will both decrease, at the optimum. Thus, since the risk shock reduces the supply of private liquid assets, the marginal loss from higher inflation becomes greater. Inflation falls, and δ falls (given the constraint on monetary policy given by fiscal policy) at the optimum, which acts to increase the gross real interest rate r and increase the total real quantity of public debt. Thus, the risk shock in some sense makes the real interest rate too low, which reflects an increase in the scarcity of liquid interest-bearing assets. The optimal monetary policy response is to conduct open market sales of government bonds (reduce δ) so as to increase the public's holdings of government debt, and this has the effect of reducing inflation and increasing r.

The policy prescription here is quite different from conventional prescriptions. For example a common notion is that a "liquidity shortage" is remedied by having the central bank conduct open market purchases and make central bank loans, which in our model has the effect of increasing δ. Here, the liquidity shortage produced by a risk shock reduces the stock of private assets that can be intermediated and used in exchange, and the remedy for this is to mitigate the liquidity shortage by having the central bank sell government debt. A low real interest rate is not a good thing here, but a symptom of the liquidity shortage. Further, while inflation-targeting central banks appear to take seriously the idea that the optimal inflation rate can be constant for long periods of time, the optimal inflation rate in this model is certainly not constant. In a version of our model with aggregate shocks, the marginal costs and benefits of inflation could be expected to fluctuate, particularly due to fluctuations in the supplies of liquid assets, and these fluctuations could be substantial.

5.3 Private Asset Purchases by the Central Bank

As has become apparent, particularly given recent US experience, some central banks have substantial powers to intermediate wide classes of assets. In our model, suppose that the central bank has access to the same verification technology as the private sector, and avails itself of the same efficient debt contracts as do private banks when lending to entrepreneurs. Thus, we will assume that central bankers are no more (and no less) capable than private sector bankers.

First, suppose that the central bank lends to a mass k of private entrepreneurs each period, and makes these loans on the same terms as would the private sector, i.e. if the gross real interest rate is r, each private sector loan made by the central bank earns an expected gross return of r. The central bank finances its lending by issuing $\phi_t E_t$ units of reserve balances (in real terms) in period t, where $\phi_t E_t = k$ for each t. In each period, the central bank uses the returns on its loan portfolio to pay interest on reserves at the real gross rate r. Provided $k \leq L(r)$ (lending by the central bank does not exceed the quantity of lending in the absence of this policy intervention), this policy will have no effect on prices or quantities. The quantity of assets a in a stationary equilibrium will be

$$a = m\left(\frac{1}{\delta} - 1\right) + L(r) - k + \phi_t E_t = m\left(\frac{1}{\delta} - 1\right) + L(r)$$

Therefore, given our preceding analysis, particularly that of the case with interest on reserves, if the real interest rate r is an equilibrium interest rate in the absence of central bank lending, then if $0 < k \leq L(r)$ and the central bank sets an interest rate on reserves equal to r, the equilibrium allocation will be identical to what was achieved without central bank lending. The central bank simply adds another layer of redundant intermediation, there is an expansion in the stock of outside money, and there is no effect on prices.

Things are different, however, if the central bank lends on better terms than does the private sector. Suppose, for example, that the private sector makes loans to $L(r)$ entrepreneurs which each yield an expected gross return r to private banks. The central bank then lends $L(\bar{r}) - L(r)$ to marginal borrowers, offering efficient loan contracts that each yield an expected gross return $\bar{r}$ to the central bank. This loan portfolio is financed with reserves that earn a gross interest rate r, which is set by the central bank. Now, the quantity of assets held by banks in equilibrium is given by

$$a = m\left(\frac{1}{\delta} - 1\right) + L(\bar{r}),$$

and $\bar{r}$ is another policy instrument for the central bank. The central bank will now suffer a loss each period on its lending activities, equal to $(r - \bar{r})\left[L(\bar{r}) - L(r)\right]$, which we will assume is financed by lump-sum taxation on buyers in the centralized daytime market.

What is the effect of this policy? First, if parameters are such that the economy is initially in a plentiful-interest-bearing-assets equilibrium, and if (δ, μ)

remains the same, then there is no effect, except for a redistribution from buyers to the group of entrepreneurs who would otherwise not be funded. With plentiful assets, the government lending program just adds to the stock of liquid intermediated assets, and does not affect exchange. Credit is allocated in a different way, however, and buyers suffer as a result (because they are taxed to make up for the central bank losses). Second, suppose that the economy is initially in an equilibrium with scarce liquid assets. Then, for a given (δ, μ), the real interest rate will be higher. There is a beneficial effect, in that there is a larger supply of liquid assets, and exchange is more efficient in monitored transactions during the night. However, entrepreneurs who borrow privately suffer relative to those who borrow cheaply from the central bank, and buyers also pick up the tab for the losses on the central bank portfolio.

6 Conclusion

This model can do a lot. We have incorporated some elements of received banking theory into a model of asset trading and liquidity related to existing search and matching models of money, in the spirit of New Monetarist economics. The model yields some novel results: a one-time open market purchase can have permanent real effects; a liquidity trap can exist under a wide range of circumstances; the central bank, constrained by the actions of the fiscal authority, and motivated by the costs of operating a currency system, will typically choose an inflation rate greater than the Friedman-rule rate, trading off the benefits of taxing currency transactions with the costs of reducing trading efficiency. Further, the model was addressed to some current issues in monetary policy. It shows how the key instrument of monetary policy under a regime with positive excess reserves is the interest rate on reserves, that increases in the riskiness of underlying investment projects can give rise to financial-crisis phenomena and liquid-asset shortages, and that central bank purchases of private assets can be irrelevant, but in general will reallocate credit and wealth.

There is much this model does not do, of course, but the basic framework is very adaptable. In principle there is no problem dealing with aggregate uncertainty and fluctuations, or with choices by consumers among alternative means of payment, among other things. Quantitative applications are certainly feasible. An important issue, typically ignored, is that a typical central bank's asset portfolio is normally financed mainly by currency. The use of currency in the world is remarkably persistent (see for example Alvarez and Lippi 2007), in spite of the rapid emergence of alternative payment technologies, including cell phone technologies that permit one-on-one decentralized transactions. One has to ask how much of the transactions activity supported by currency is socially desirable, and if doing away with currency entirely could be feasible, or optimal. In principle, a modified version of this model could answer these questions.

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Figure 1: Flows of Physical Objects in the Model

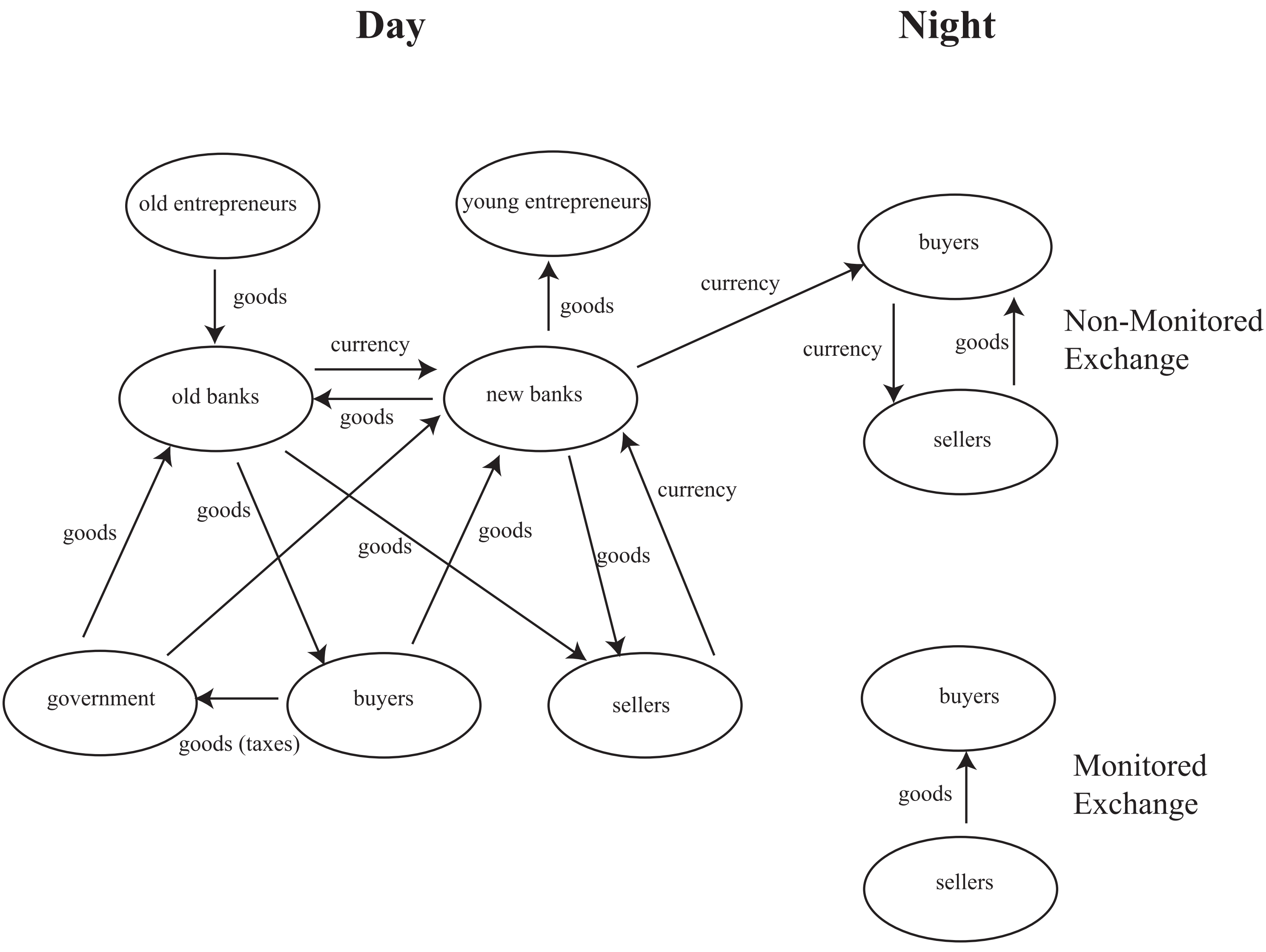

Figure 2: Asset Demand and Supply: Scarce and Plentiful Assets

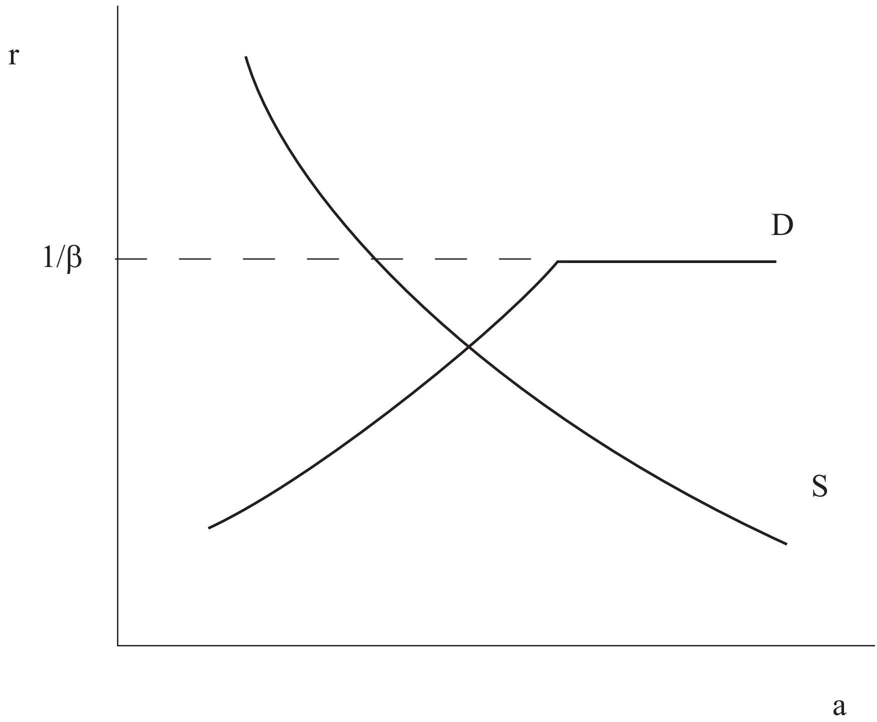

Figure 3: An Increase in δ With Scarce Assets

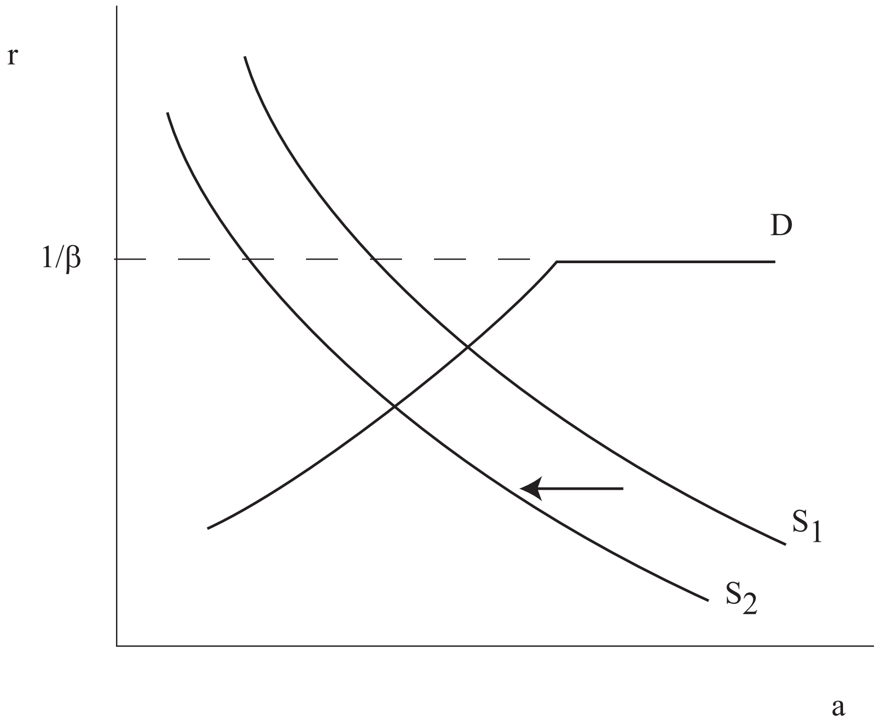

Figure 4: Equilibria when $L(1/\beta) < (1-\rho)x^*$

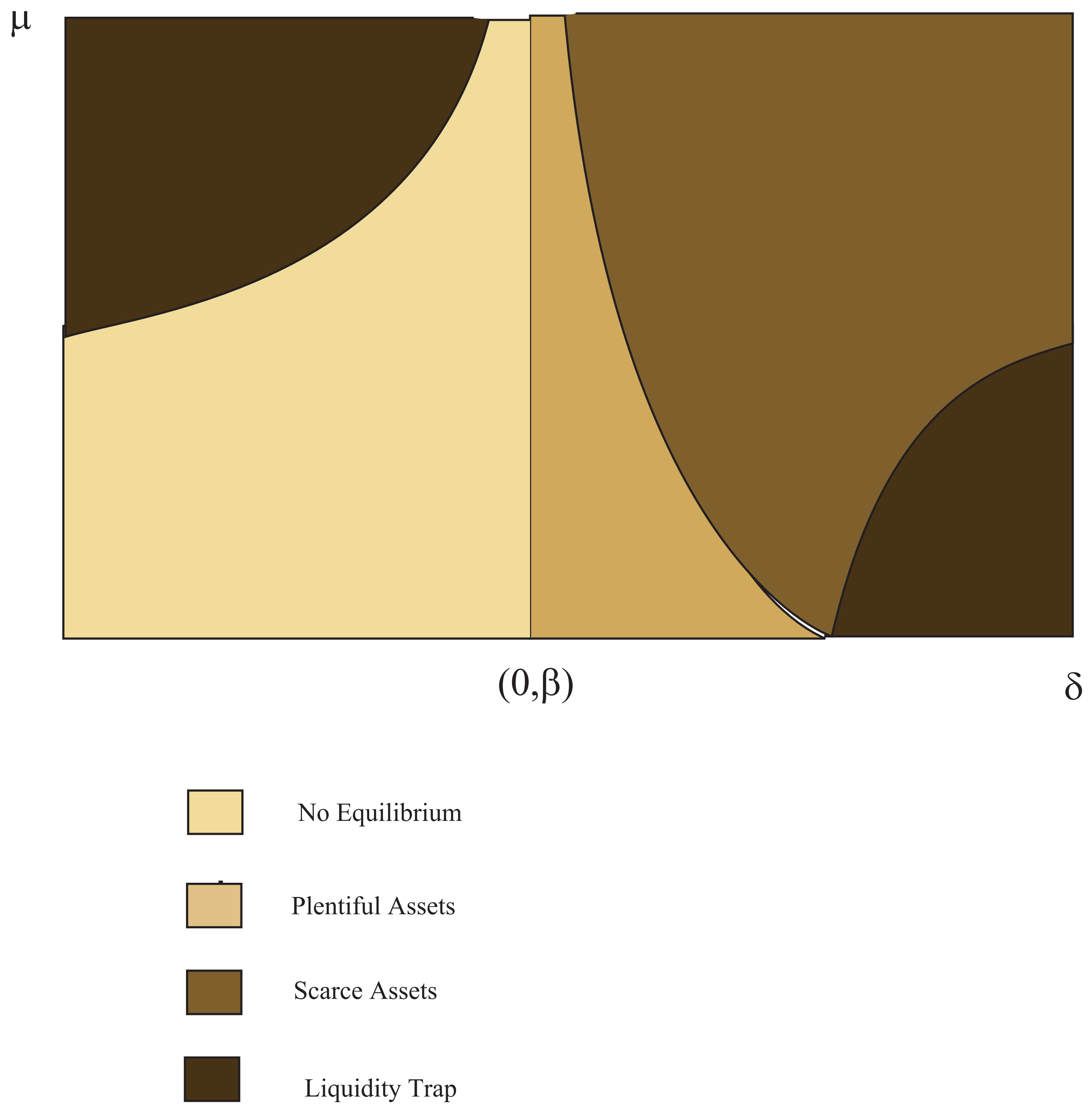

Figure 5: Equilibria when $(1-\rho)x^*<L(1/\beta)<x^*$

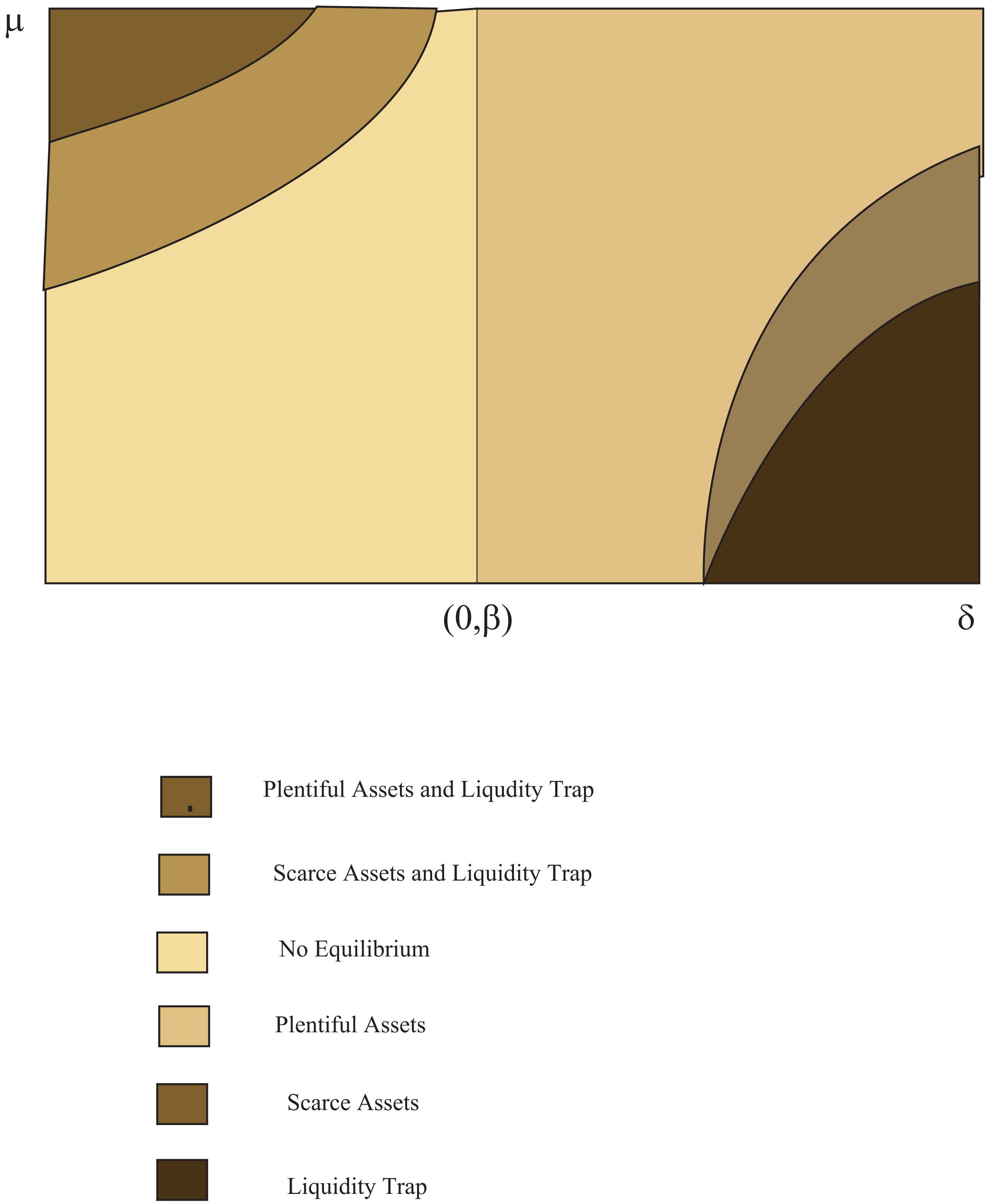

Figure 6: Equilibria when $x^* < L(1/\beta)$