

Corporate Credit Spreads and Business Cycles*

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Abstract

This paper studies credit spreads between the high and low default-risk corporate bonds, labeled as corporate credit spreads, and business cycles in a general equilibrium model. The mechanism of and conditions for the negative impact of corporate credit spreads on output and TFP are presented. In the presence of corporate credit spreads, a high-risk firm has more difficulty in raising fund relative to a low-risk firm does, resulting in the difference in the (expected) marginal product of capital between the two firms, i.e., misallocation of capital. As a result, output and TFP are lower than they would be. The higher the corporate credit spreads, the larger the extent of misallocation of capital, the lower the output and TFP. This key mechanism is embedded into an otherwise standard growth model, which is then calibrated to data for the U.S. economy during the period 1964-2009, default rates of corporate bonds by credit ratings in particular. Simulated results show that fluctuations in the corporate credit spread, essentially driven by shocks to the default risk for high-risk firms, can account for about 60% of output fluctuations and 70% of TFP fluctuations. The firm-level investment data supports the model's prediction on the comovement between the corporate credit spread and allocation of capital between high-risk and low-risk firms.

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1 Introduction

This paper studies the credit spread between the high and low default-risk corporate bonds, labeled as *corporate credit spread*, and business cycles in a general equilibrium model. As documented well in the literature studying the relationship between the financial market outcome and real economy, corporate credit spreads fluctuate countercyclically: for instance, the (detrended) yield spread between the Moody's Baa and Aaa corporate bonds is negatively correlated with (detrended) output as well as TFP for the U.S. during the period 1964-2009¹. Less attention has been paid, however, to the mechanism of and conditions for the negative effects of corporate credit spreads on output and TFP, which this paper mainly focuses on. Moreover, such effects are quantified for the U.S. economy where fluctuations in corporate credit spreads are calibrated to data on default and recovery rates of corporate bonds by credit ratings.

For the mechanism of the effect of corporate credit spreads on TFP, this paper focuses on the channel of cross-sectional resource misallocation. In the presence of corporate credit spreads, a high-risk firm has more difficulty in raising fund for its investment relative to a low-risk firm does, resulting in the difference in the (expected) marginal product of capital between the two firms, which measures the extent of capital misallocation across firms (see, e.g., Restuccia and Rogerson (2008) and Hsieh and Klenow (2009))². In other words, capital is allocated too little for high-risk firms and too much for low-risk firms compared to the optimal allocation, which is mirrored to the higher (equilibrium) expected marginal product of capital for a high-risk firm relative to a low-risk firm. As a result, an increase in the corporate credit spread implies that the extent of resource misallocation becomes larger, and thereby output and TFP drop. Put differently, an increase in corporate credit spreads is likely to induce that capital is reallocated further away from high-risk firms toward low-risk firms.

The mechanism discussed above is embedded into an otherwise standard growth model. In the model, firms borrow their capital in the one-period non-contingent bond market at the beginning of every period prior to the realization of their idiosyncratic productivity shocks. The key ingredient of the model is that firms differ ex-ante essentially in the risk (i.e., the variance) of their idiosyncratic productivity shocks, tightly linked to the default probability; low-risk firms, labeled as *safe*, have a (normalized) zero default probability while high-risk firms, labeled as *risky*, have a positive default probability. Default event is essentially exogenous, i.e., *jump* shock: a firm hit by the jump shock loses the competitive edge in its business and is liquidated,

¹It is based on my calculation, which extends Gertler and Lown (1999). See also Gilchrist and Zakrajsek (2011).

²Therefore, I argue that corporate credit spreads indicate the extent of capital misallocation. Importantly, difference in the cost of capital across firms, resulting in resource-misallocation, is an equilibrium outcome in this paper while it is exogenous institutional arrangement in those two papers.

incurring the liquidation cost. Importantly, the bond market is imperfect, represented by the default-cost distortion measuring a bondholder's loss rate in the event of default, i.e., one minus the recovery rate, which includes the liquidation cost, bankruptcy cost and so on.

In the model economy, with the default-cost distortion, the higher default probability for a risky firm relative to a safe firm results in the misallocation of capital: capital is allocated too little for a risky firm and too much for a safe firm compared to the optimal allocation. As a result, output and TFP are lower than they would be to the extent of the credit loss rate, which is the product of the risky firm's default probability and default-cost distortion.

In the model, as the default probability for risky firms increases, the credit spread increases, capital is reallocated away from risky firms toward safe firms, and output and TFP decrease. The induced decrease in TFP in turn leads to lower labor supply, consumption and investment. The key to the negative response of TFP to an increase in the default probability for risky firms is that the extent of misallocation of capital becomes larger: the ratio of capital allocated to a risky firm relative to a safe firm decreases further below the optimal level because of the higher spread in the cost of capital between risky and safe firms.

The model is calibrated to data for the U.S. economy and default rates of corporate bonds by credit ratings during the period 1964-2009. The key in my calibration is that (annual) default rates for risky firms are high on average, about 1.9%, and quite volatile, standard deviation of 1.8%, while default rates for safe firms are almost constant and close to zero as in the data. Properties of the calibrated dynamic model are analyzed numerically, and standard statistics studied in business cycle research are compared to the data.

Simulated results show that fluctuations in the corporate credit spread, essentially driven by shocks to the default probability for risky firms, can account for about 60% of output fluctuations and 70% of TFP fluctuations. As predicted, the default-cost distortion, again measured as one minus the recovery rate, plays an important role as an amplification device: almost no fluctuations in output and TFP for the counterfactual case of the zero default-cost distortion. For the same reason, fluctuations in key variables are slightly larger for the case of the default-cost distortion positively correlated with default rates as in the data, i.e., the lower recovery rate during periods of the higher default rate.

The sensitivity of simulated results with respect to the capital structure is examined because of the importance of the capital structure³ in affecting a firm's cost of capital³. Because of the dominance of internal and debt financing for corporate investments in data, the internal-financing case is considered in a simple way: internal financing is operative for a firm's investment at the

³Faced by the higher cost of the new debt than the usual, a high-risk firm could substitute a cheaper financial instrument for an expensive debt, which might mitigate the effects of an increase in the corporate credit spread on the capital misallocation and TFP relative to the benchmark setup of the 100% (short-term) debt financing.

margin while a constant size of debt is rolled over every period as long as the firm does not default⁴. In this case, the qualitative results are the same as in the benchmark case of 100% debt financing while the quantitative results are slightly larger than the benchmark⁵.

An empirical evidence is, in turn, provided to support the key mechanism, i.e., the credit-spread driven capital reallocation between high-risk and low-risk firms. More specifically, using the COMPUSTAT dataset on the firm-level investment during the period 1980-2007, I empirically study how the allocation of capital between firms differing in risk changes over business cycles, its comovement with the corporate credit spread in particular. The regression results show that a high-risk firm indeed cuts its investment more than a low-risk firm does in response to an increase in the aggregate level of corporate credit spreads: the negative sensitivity of the firm-level investment to the aggregate level of corporate credit spreads is stronger for a firm of the higher risk⁶.

Related Literature This paper contributes mainly to the literature studying the effects of financial market imperfections on the real economy over business cycles. There are a number of papers studying comovements in aggregate output and the credit market outcomes; Gertler and Lown (1999) and Gilchrist and Zakrajsek (2011) studied the empirical relationship between credit-spread cycles and business cycles, which is studied and quantified in a general equilibrium model in this paper. Up to my knowledge, this paper is the first to quantify the effects of default-rate dynamics by credit ratings of corporate bonds on output and TFP in a framework of the standard growth model. Gomes and Schmid (2009) study the implications of aggregate productivity shocks for credit-spread cycles and business cycles in a general equilibrium model with the restriction that firm-level capital is fixed over time while endogenous fluctuations in the firm-level capital are the key mechanism studied in this paper. Lastly, pioneered by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), the research program studying the implications of credit market imperfections for business cycles has been developed for the recent decades. This paper differs from many papers in this literature essentially in the sense that in my model, both TFP and output endogenously fluctuates, while many papers in this literature typically focus on the effects of exogenous changes in aggregate productivity.

This paper is also related with a recently growing body of literature studying the importance of misallocation for business cycles. For instance, Khan and Thomas (2011) and this paper

⁴See Myers (2001) for discussion of facts for corporate capital structure.

⁵For an increased (exogenous) default probability, in terms of the shareholder value, internal financing is more costly than debt is, and hence, a risky firm's investment is cut more when financed by internal cash flows than financed by debt. A more detailed discussion is provided in the later section of simulated results for this case.

⁶The firm-level risk is measured by the firm's own stock return volatility as in the literature (see, e.g., Campbell and Taksler (2003)). For the aggregate level of corporate credit spreads, the Moody's Baa-Aaa corporate credit spread is used. A number of other control variables discussed in the literature are also included to the regression equation.

share that TFP and output would fluctuate channelled by the reallocation of resources across firms. This paper differs from Khan and Thomas (2011) in the shocks and mechanism studied, which results in that the larger part of fluctuations in output and TFP are accounted for in this paper than in Khan and Thomas (2011)⁷. Gourio (2011) studies the implications of shocks to the disaster risk, which are common to every firm, for credit spreads and output and focuses on the channel of aggregate investment dynamics.

In addition, this paper also contributes to the literature studying the sources and mechanism of business cycles, too. This literature can be classified to two categories: one is the amplification mechanism of aggregate productivity shocks while the other considers impulses other than aggregate productivity shocks, for instance, shocks to the second moment of aggregate or idiosyncratic productivity as in Bloom (2009). Compared to the first category, fluctuations in TFP and output are obtained in this paper without shocks to the aggregate productivity: shocks to the default risk for risky firms, the impulse imposed to the model in this paper, generate almost no fluctuations in TFP and output when the endogenous misallocation channel is shut down. For the second category, this paper focuses on the *dispersion*, or the extent of *heterogeneity*, in the firm-level risk across firms, while many papers in this literature focus on the *common level* of risk for every firm (see, e.g., Bloom (2009), Bloom et al. (2009) and Arellano et al. (2010)).

This paper is organized as follows. Section 2 provides an empirical evidence on the hypothesis of capital-reallocation driven by corporate credit spreads. Section 3 develops and studies the baseline model which embeds the heterogeneity in default risk between low-risk and high-risk firms into an otherwise standard growth model. Section 4 calibrates the dynamic model and discusses the simulation results. Section 5 extends the baseline model to the case of the internal-financing combined with roll-over debt, calibrates it and discusses the simulation results. Section 6 concludes.

2 Corporate Credit Spreads and Cross-Section of Capital

This section empirically documents how the firm-level investment is correlated with corporate credit spreads. Key here is to characterize the heterogeneity between high-risk and low-risk firms in the response of investment to changes in the aggregate level of corporate credit spreads. Re-

⁷This paper focuses on credit spread fluctuations, essentially driven by shocks to the default risk for risky firms but not for safe firms, propagated by the mechanism of capital reallocation between firms of different *risk* while Khan and Thomas (2011) consider shocks to the collateral value, applying to every firm, propagated by the mechanism of capital reallocation between firms of different *size*. In a nutshell, (calibrated) default rate dynamics, more volatile for the higher-risk firms, generate the larger fluctuations in the extent of misallocation than the (calibrated) collateral-value process does.

sults of this empirical analysis will provide an evidence supporting the main hypothesis examined in this paper that the allocation of capital between high-risk and low-risk firms is significantly affected by the aggregate level of corporate credit spreads.

2.1 Augmented Two-Factor Model

Below I analyze the firm-level investment by using a two-factor model given by:

$$E\left[i_t(j)\middle|(F_{1,t}, F_{2,t})\right] = \alpha_0 + \lambda_1(j) \cdot F_{1,t} + \lambda_2(j) \cdot F_{2,t}$$

where the subscript t indexes time, $i_t(j)$ denotes firm j 's investment in period t , $(F_{1,t}, F_{2,t})$ denotes the two common factors, α_0 refers to the constant, and $(\lambda_1(j), \lambda_2(j))$ refers to firm j 's two factor loadings. The above two-factor model of investment assumes that firms respond in their investment policies to the two common factors with differing sensitivity. The two common factors are aggregate efficiency, proxied by real GDP per capita $RGDP_t$, and aggregate risk, proxied by the Moody's Baa-Aaa corporate credit spreads CS_t .

The main objective of this empirical analysis is to characterize the structure of the heterogeneity in factor loadings $(\lambda_1(j), \lambda_2(j))$. Naturally, factor loadings are modeled as functions of the firm-level characteristics. More specifically, two firm-level characteristics are considered: firm size and firm-level risk. The firm size is likely to represent the firm's efficiency while the firm-level risk is intended to measure the firm's vulnerability, including default probability, to changes in the business environment. Factor loadings are linear in each of those two characteristics:

$$E\left[\lambda_h(j)\middle|x_t(j)\right] = \alpha_h + \beta_h x_t(j), \quad h \in \{1, 2\}$$

where $x_t(j) \equiv (size_t(j), risk_t(j))$ is the vector of firm j 's (time-varying) two characteristics, size and risk. In the above expression of the h -th factor loading for firm j , α_h refers to the component common to every firm, and β_h represents the impact of firm j 's characteristics, again size and risk, on the firm's own h -th factor loading.

Moreover, the above basic two-factor model is modified so that other important variables are also controlled for; several firm characteristics are added as independent variables as in the literature studying the cross-section of the firm-level investment. Let the vector $z_t(j)$ denote such additional control variables, e.g., cash flows and leverage ratio, affecting firm j 's investment. The fully specified, augmented two-factor model of the firm-level investment is given by:

$$i_t(j) = \alpha_0 + v(j) + [\alpha_1 + \beta_1 x_t(j)] \cdot RGDP_t + [\alpha_2 + \beta_2 x_t(j)] \cdot CS_t + \gamma z_t(j) + \delta_1 \cdot t + \delta_2 \cdot t^2 + \epsilon_t(j)$$

where $v(j)$ refers to the fixed-effect term, $RGDP_t$ refers to the per-capita real GDP, CS_t refers to the Baa-Aaa corporate credit spread, γ refers to the sensitivity vector to the firm characteristic vector $z_t(j)$, and $\epsilon_t(j)$ is an i.i.d. Gaussian random variable. Lastly, as in Covas and Haan (2011), linear and quadratic time-trend terms are also included so that trend in a dependent variable, if any, should be controlled for.

2.2 Data Sources

One period is one calendar year. The first factor, per capita real GDP, is measured as the chain-weighted real GDP per capita provided by NIPA and then logged and detrended by the HP-Filter with the smoothing parameter equal to 100. For the second factor, the monthly Moody's Baa-Aaa corporate credit spread is aggregated to the annual frequency and then logged and demeaned.

For the firm-level investment and characteristics, annual industrial COMPUSTAT files from 1980 to 2007 for publicly listed non-financial firms are used⁸. For the firm-level risk, CRSP daily stock returns for NYSE, AMEX and NASDAQ stocks between 1979 and 2007 are used. Below I basically follow Covas and Haan (2011) in selecting and measuring key variables. For detailed discussion, see 'COMPUSTAT Variables' in the Data Sources appendix.

As in the literature, the ratio of investment to one-period lagged capital stock is used for the dependent variable $i_t(j)$ so that it is stationary:

$$i_t(j) = \text{INVESTMENT}_t(j) / \text{CAPITAL}_{t-1}(j).$$

where $\text{INVESTMENT}_t(j)$ refers to firm j 's investment at t and $\text{CAPITAL}_{t-1}(j)$ refers to firm j 's capital stock in the previous period. For the firm size, the lagged book value of assets is used as in the literature, which is then expressed as the differential from the median firm size so that it should be stationary:

$$\text{size}_t(j) = \frac{\text{ASSETS}_{t-1}(j) - \overline{\text{ASSETS}}_{t-1}}{\overline{\text{ASSETS}}_{t-1}}$$

where $\text{ASSETS}_{t-1}(j)$ refers to firm j 's lagged total assets and $\overline{\text{ASSETS}}_{t-1}$ denotes the median value of $\text{ASSETS}_{t-1}(j)$ across j . And the firm-level risk is measured as the volatility of the firm's stock returns, lagged by 180 days, for one year as in Campbell and Taksler (2003)⁹. To control

⁸Financial firms (SIC codes 6021, 6022, 6029, 6035, 6036) are excluded from the sample. In addition, I also exclude from the sample firms with a missing value for the book value of assets and firms most affected by the accounting change in 1988, see Covas and Haan (2011).

⁹The volatility of the firm's stock returns are calculated for the period 6-month earlier than the investment is

for the time-varying component of stock-return volatility common to every firm, I express the firm-level risk as the difference of the firm's stock-return volatility from the (cross-sectional) median volatility of stock returns:

$$risk_t(j) = Var(\text{RETURN}_{t-\Delta}(j)) - \overline{Var(\text{RETURN}_{t-\Delta}(\cdot))}$$

where $Var(\text{RETURN}_{t-\Delta}(j))$ refers to firm j 's stock-return volatility and $\overline{Var(\text{RETURN}_{t-\Delta}(\cdot))}$ refers to the median value of $Var(\text{RETURN}_{t-\Delta}(j))$ across j , and subscript $t - \Delta$ indicates that the volatility is calculated for the stock returns lagged by 180 days.

Next, for additional firm-characteristics explanatory variables, the following lagged variables are included as in the literature: marginal product of capital $mpk_{t-1}(j)$, leverage ratio $lvg_{t-1}(j)$, cash flows $cf_{t-1}(j)$, and Tobin's Q denoted by $q_{t-1}(j)$:

$$z_t(j) \equiv \left(mpk_{t-1}(j), lvg_{t-1}(j), cf_{t-1}(j), q_{t-1}(j) \right).$$

Lastly, all variables are winsorized as in the literature so that statistical results are not severely influenced by outliers. For each variable, observations higher than the 99th percentile is replaced by the the 99th percentile and observations lower than the first percentile is replaced by the first percentile.

2.3 Estimation Results and Discussion

Below estimation results are discussed. The estimated investment policy for firm j is given by:

$$\begin{aligned} i_t(j) = & \frac{0.832^{***}}{(0.064)} + \left[\frac{1.07^{***}}{(0.14)} - \frac{0.007^*}{(0.004)} size_t(j) + \frac{1.57}{(2.25)} risk_t(j) \right] RGDP_t \\ & + \left[- \frac{0.12^{***}}{(0.019)} - \frac{0.0002}{(0.0003)} size_t(j) - \frac{0.33^{**}}{(0.16)} risk_t(j) \right] CS_t + \frac{0.03^{***}}{(0.0005)} mpk_{t-1}(j) \\ & - \frac{0.61^{***}}{(0.03)} lvg_{t-1}(j) + \frac{0.09^{***}}{(0.014)} cf_{t-1}(j) + \frac{0.07^{***}}{(0.003)} q_{t-1}(j) - \frac{0.03^{***}}{(0.005)} t + \frac{0.0005^{***}}{(0.0001)} t^2 + \epsilon_t(j) \quad (1) \end{aligned}$$

where numbers in the parenthesis refer to the standard errors¹⁰. For a firm of median size and median risk ($size_t(j) = 0, risk_t(j) = 0$), the investment is highly positively correlated with real GDP, dubbed as *output*, and highly negatively correlated with the Baa-Aaa corporate credit spread, dubbed as *credit spread*. Moreover, all of the four characteristics variables (mpk, lvg, cf, q) are also significantly correlated with the firm-level investment.

measured so that the investors' ex-ante expectations are captured well.

¹⁰The superscript '*' of an estimate indicates that the estimate is significant at 10% level, '**' at 5% level and '***' at 1% level.

For the heterogeneity in the two factor loadings, let's focus on terms inside brackets for $RGDP_t$ and CS_t (recall that factor loadings for those two factors are linear in the firm size and firm-level risk). On the one hand, the impact of the firm size on the factor loading neither for output nor for the credit spread is significant at 5% level. On the other hand, the negative impact of the firm-level risk on the factor loading for the credit spread is significant at 5% level: coefficient of -0.33 with standard error of 0.16. As the credit spread increases, a high-risk firm reduces its investment more than a low-risk firm does. Put differently, as it is more costly for a high-risk firm to raise capital relative to a low-risk firm, a high-risk firm contracts more than a low-risk firm does¹¹.

Note that the finding above is opposed to the hypothesis in the literature that small firms are more prone to the financial shocks than large firms are. For instance, Khan and Thomas (2011) discuss the relationship between firm size and tightness in the borrowing constraint. The key is that small firms are on average riskier than large firms are in the data. Once firm-level risk and firm size are controlled for at the same time, firm size loses, however, its significance in determining the sensitivity of the firm-level investment to the Baa-Aaa corporate credit spread while firm-level risk is still significant. Put differently, firm size is less informative than firm-level risk is in measuring the extent to which the firm is exposed to the credit market risk.

In this section, it is empirically studied how the firm-level investment is correlated with corporate credit spreads. Results of the empirical analysis of the COMPUSTAT firm-level investment data show that a high-risk firm cuts its investment more than a low-risk firm does as the aggregate level of corporate credit spreads increases. Naturally, it follows what are causes and consequences of such a relationship between corporate credit spreads and the allocation of capital between high-risk and low-risk firms. For this purpose, a general equilibrium model will be developed and studied in the next section.

3 Model

This section develops and studies a general equilibrium model of corporate credit spreads, which embeds the cross-sectional difference in the default risk of corporate bonds into an otherwise standard growth model. The model will be used to assess the quantitative implications of stochastic fluctuations in the default risk for high-risk firms relative to low-risk firms for the corporate credit spread, allocation of capital, TFP and output. In addition, the role of financial frictions as an amplification device will be also highlighted.

¹¹Similarly, Campello et al. (2010) find that for the recent episode of the financial crisis of 2008, financially constrained firms planned to reduce investment and employment more than unconstrained firms did in the U.S. as well as in the Europe and Asia.

3.1 Environment

Technology There is a single final good, which is produced by a continuum of measure one of firms. The stochastic production function of firm $i \in [0, 1]$ is given by:

$$y_t(i) = z_t(i) \left[[k_t(i)]^\theta [h_t(i)]^{1-\theta} \right]^\alpha, \quad \alpha \in (0, 1), \quad \theta \in (0, 1) \quad (2)$$

where $y_t(i)$ is firm i 's output at t , $k_t(i)$ is capital services employed by firm i and $h_t(i)$ is labor services hired by firm i , $\alpha \in (0, 1)$ refers to the returns-to-scale parameter and $z_t(i)$ refers to firm i 's idiosyncratic productivity shock at t .

A key feature of the model is that there is ex-ante heterogeneity across firms in the extent of riskiness (measured as the variance of productivity shocks $z_t(i)$). For simplicity, I assume that there are two types of firms: low-risk firms, labeled as *safe*, and high-risk firms, labeled as *risky*. Firm i is safe for $i \in [0, \lambda]$, $\lambda \in (0, 1)$, and is risky for $i \in (\lambda, 1]$, which is a public information. For the simplicity of exposition, I abstract from changes in the measure of firms¹². Note that two types of firms differ only in terms of the variance, but not the expected value, of the firm-level productivity as will be shown soon.

Safe firm i 's riskiness is normalized to zero and its productivity is constant equal to one every period:

$$z_t(i) = 1, \quad \forall t = 0, 1, 2, \dots, \quad \forall i \in [0, \lambda]. \quad (3)$$

By contrast, risky firm i 's riskiness is positive; risky firm i 's productivity shocks $z_t(i)$ are discrete random variables drawn independently from an identical distribution such as:

$$z_t(i) = \begin{cases} 0 & \text{w.p. } \nu_t \\ 1/[1 - \nu_t] & \text{w.p. } 1 - \nu_t \end{cases}, \quad \forall i \in (\lambda, 1] \quad (4)$$

where $\nu_t \in (0, 1)$ refers to the time-varying probability of drawing $z_t(i) = 0$, labeled as the *jump* shock, for a risky firm at t , which is intended to capture bad shocks to a firm's long-term value, resulting in the firm's (costly) liquidation/bankruptcy as studied in the corporate bond pricing literature¹³. The key here is that as will be discussed in detail later, risky firms are exposed to the higher probability of such a jump shock than safe firms are.

As in the firm dynamics literature (see, e.g., Cooley and Quadrini (2001)), $z_t(i) = 0$ is an

¹² This assumption seems reasonable for the purpose of assessing the quantitative implications for TFP and output of shocks to the default-risk distribution between risky and safe firms because it is well known in the literature that entering and/or exiting firms are small and thereby their effects on output are small in the data.

¹³ See, e.g., Leland (2006)). If firm i is hit by the jump shock, then a substantial part of the firm value is lost, e.g., losing competitiveness in its business, liquidation costs and bankruptcy costs, resulting in that firm i 's investors, either shareholders or debt-holders, lose a large part of their capital invested to firm i .

absorbing state; once risky firm i draws the jump shock $z_t(i) = 0$ in period t , then firm i 's productivity thereafter is permanently equal to zero:

$$\text{if } z_t(i) = 0, \text{ then } z_{t+s}(i) = 0, \forall s > 0. \quad (5)$$

Moreover, firm i hit by the jump shock $z_t(i) = 0$ is immediately liquidated and replaced by a new risky firm, which is then indexed by i from the period $t + 1$ and thereafter until it is liquidated. Importantly, liquidation is costly: $\tilde{\tau} \geq 0$ fraction of the undepreciated capital held by the liquidated firm is lost, labeled as the *liquidation cost*.

The jump-shock probability for a risky firm ν_t follows the first order Markov process. Let $f(\nu_{t+1}|\nu_t)$ denote the density of ν_{t+1} conditional on ν_t and Λ denote the support of ν_{t+1} : $\Lambda \subset [0, 1]$.

Discussion of the specification of the productivity shock process is in order. Note that two types of (operating) firms have the same expected productivity constant equal to one while (operating) risky firms have the positive and time-varying variance of productivity:

$$\text{Var}(z_t(i)) = \frac{\nu_t}{1 - \nu_t}, \quad \forall i \in (\lambda, 1] \quad (6)$$

which is increasing in ν_t . Given the allocation of capital fixed, ν_t is related with the mean preserving spread for a risky firm's productivity but has no implications for the aggregate productivity.

Lastly, the parameter ν_t , again representing the probability of a risky firm's jump shock, is a risky firm's default probability for the debt financing case as will be discussed soon and will play a key role in this paper. Given the importance of a risky firm's default probability ν_t for the corporate credit spread, the subsequent analysis will focus on consequences of an increase in ν_t for allocation and prices of capital in this economy.

Household Preferences of the infinitely-lived representative household is given by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \psi \frac{h_t^{1+\omega}}{1+\omega} \right] \right], \quad \beta \in (0, 1)$$

where $E_0[\cdot]$ is the expectation operator at $t = 0$, c_t is consumption in period t , h_t is labor supply, and β refers to the discount factor.

The household is endowed with one unit of time every period and k_0 units of capital in the initial period $t = 0$. The household owns all of the firms.

Timing In each period t , there are two subperiods, initial and final. In the initial subpe-

riod, ν_t is realized and observed by everyone, the household supplies to firms labor services via (defaultable) wage contracts and capital via (defaultable) debt contracts. In the final subperiod, productivities for risky firms $\{z_t(i)\}$ are realized, production takes place, firms make default decisions, profits are transferred to the household, confiscated outputs of the defaulting firms are distributed to the suppliers of labor and capital, and the household makes its consumption/investment decision. Lastly, the time interval between the final subperiod of t and initial subperiod of $t + 1$ is infinitesimal.

Debt and Wage Contracts Firms finance their capital entirely issuing non-contingent bonds. Moreover, as standard in the literature, only one-period bond is available, which promises to deliver one unit of the final good per unit of bond at the final subperiod. Implications of the simplifying assumption of 100% debt financing for properties of the model will be discussed later. The price of firm i 's bond is $q_t(i) \equiv 1/[1 - \delta + r_t(i)]$:

$$r_t(i) = \begin{cases} r_t^S & , \forall i \in [0, \lambda] \\ r_t^R & , \forall i \in (\lambda, 1] \end{cases} \quad (7)$$

where r_t^S refers to the interest rate for a safe firm and r_t^R does the same for a risky firm.

Similarly to debt contracts, wage contracts are also defaultable and one-period contracts¹⁴. Firm i promises to pay $w_t(i)$ units of the final good per unit of labor services hired by the firm:

$$w_t(i) = \begin{cases} w_t^S & , \forall i \in [0, \lambda] \\ w_t^R & , \forall i \in (\lambda, 1] \end{cases} \quad (8)$$

where w_t^S is the wage rate for a safe firm and w_t^R is the wage rate for a risky firm.

The arrangement for the event of default is as follows. For simplicity, I focus on the case in which a firm defaults on both its debt and wage contracts, which suffices for my analysis because in this model a firm either defaults on both of these contracts or does not default on either. If a firm defaults on both its debt and wage contracts, then the suppliers of labor and suppliers of capital split the confiscated output of the defaulting firm i in the proportions $(1 - \theta : \theta)$, which is then distributed to individual suppliers of labor and/or capital in proportion to their shares in labor and/or capital supplied to the defaulting firm.

In addition, the undepreciated capital employed by the defaulting firm is taken back by the suppliers of capital to the firm. Importantly, lenders lose a fraction $\tau \geq 0$ of their undepreciated capital taken back from the defaulting firm, labeled as *default-cost distortion*, e.g., bankruptcy

¹⁴It does not make no difference to my results whether wage contracts are defaultable or not.

costs, liquidation costs and so on; the default-cost distortion τ includes and hence is larger than the liquidation cost:

$$\tau \geq \tilde{\tau}. \quad (9)$$

The larger the default-cost distortion τ , the smaller the recovery rate of a defaulted bond as will be discussed later.

Lastly, the bond market is competitive in the sense that both individual lenders and individual firms take the market interest rates as given. Similarly, the wage contract market is also competitive.

Resource Constraint The resource constraint for this economy is given by:

$$c_t + k_{t+1} = y_t + [1 - \delta] \left[\lambda k_t^S + [1 - \lambda] [1 - \nu_t \tau] k_t^R \right] \quad (10)$$

where y_t is aggregate output in period t given by:

$$y_t = \lambda \left[[k_t^S]^\theta [h_t^S]^{1-\theta} \right]^\alpha + [1 - \lambda] \left[[k_t^R]^\theta [h_t^R]^{1-\theta} \right]^\alpha. \quad (11)$$

A Firm's Problem A firm's problem is essentially static. First, safe firm i never defaults and equates its marginal product of capital and labor to its interest and wage rates, respectively:

$$\alpha \theta [k^S(\nu, K)]^{\alpha\theta-1} [h^S(\nu, K)]^{\alpha[1-\theta]} = r^S(\nu, K), \quad (12)$$

$$\alpha [1 - \theta] [k^S(\nu, K)]^{\alpha\theta} [h^S(\nu, K)]^{\alpha[1-\theta]-1} = w^S(\nu, K) \quad (13)$$

which are first order conditions, in recursive form, for a safe firm.

Second, risky firm i defaults and is liquidated if and only if hit by the jump shock $z_t(i) = 0$, and hence ν_t fraction of risky firms default in period t . As the default event is essentially exogenous, risky firm i 's problem is essentially static every period as long as it is operative; it chooses capital and labor to maximize its profit conditional on non-default $z_t(i) \neq 0$, characterized in recursive form as:

$$\alpha \theta \frac{1}{1 - \nu} [k^R(\nu, K)]^{\alpha\theta-1} [h^R(\nu, K)]^{\alpha[1-\theta]} = r^R(\nu, K), \quad (14)$$

$$\alpha [1 - \theta] \frac{1}{1 - \nu} [k^R(\nu, K)]^{\alpha\theta} [h^R(\nu, K)]^{\alpha[1-\theta]-1} = w^R(\nu, K). \quad (15)$$

The Household's Problem Given no aggregate-level risk at an initial subperiod, the

expected bond-returns and expected wage rates should be the same between risky and safe firms¹⁵:

$$1 - \delta + r^S(\nu, K) = [1 - \nu] \left[1 - \delta + r^R(\nu, K) \right] + \nu[1 - \delta][1 - \tau], \quad (16)$$

$$w^S(\nu, K) = [1 - \nu]w^R(\nu, K). \quad (17)$$

3.2 Equilibrium

I study a recursive competitive equilibrium, which is a list consisting of the value function $v(k, \nu, K)$, policy functions $c(k, \nu, K)$, $k'(k, \nu, K)$, $h^S(k, \nu, K)$, $h^R(k, \nu, K)$, $k^S(k, \nu, K)$, $k^R(k, \nu, K)$, interest rate functions $r^S(\nu, K)$ and $r^R(\nu, K)$, wage rate functions $w^S(\nu, K)$ and $w^R(\nu, K)$, the expected default probability for a risky firm equal to ν , the expected confiscated output in the event of default for a risky firm equal to zero, total profit function $\pi(\nu, K)$ and aggregate capital transition function $K'(\nu, K)$ that satisfy:

1. $v(k, \nu, K)$ solves the Bellman equation for the household's problem, and policy functions $c(k, \nu, K)$, $k'(k, \nu, K)$, $h^S(k, \nu, K)$, $h^R(k, \nu, K)$, $k^S(k, \nu, K)$ and $k^R(k, \nu, K)$ are optimal decision rules of such a problem.
2. $\forall(\nu, K) \in (0, 1) \times [0, \infty)$, $h^S(K, \nu, K)$ and $k^S(K, \nu, K)$ are the optimal quantity of labor and capital for safe firm $i \in [0, \lambda]$, and $h^R(K, \nu, K)$ and $k^R(K, \nu, K)$ are the optimal quantity of labor and capital for risky firm $i \in (\lambda, 1]$.
3. Markets clear: $\forall(\nu, K) \in (0, 1) \times [0, \infty)$,

$$c(K, \nu, K) + k'(K, \nu, K) = y(\nu, K) + [1 - \delta] \left[\lambda k^S(K, \nu, K) + [1 - \lambda][1 - \nu\tau]k^R(K, \nu, K) \right] \quad (18)$$

where output function $y(\nu, K)$ is given by:

$$y(\nu, K) = \lambda \left[[k^S(K, \nu, K)]^\theta [h^S(K, \nu, K)]^{1-\theta} \right]^\alpha + [1 - \lambda] \left[[k^R(K, \nu, K)]^\theta [h^R(K, \nu, K)]^{1-\theta} \right]^\alpha. \quad (19)$$

4. $\pi(\nu, K)$ is consistent with decision rules of individual firms:

$$\pi(\nu, K) = [1 - \alpha]y(\nu, K), \quad \forall(\nu, K) \in (0, 1) \times [0, \infty). \quad (20)$$

¹⁵ Note that any firm-level idiosyncratic risk is diversified away, and thus only the expected returns and expected wage rates should matter for the household for the case in which the household's supply of capital and labor are homogeneous across firms of the same type, which should be the equilibrium outcome because of the demand side.

5. $K'(\nu, K)$ is consistent with $k'(k, \nu, K)$:

$$K'(\nu, K) = k'(K, \nu, K), \quad \forall (\nu, K) \in (0, 1) \times [0, \infty). \quad (21)$$

3.3 Results

This section characterizes the equilibrium allocation and prices of capital and then analyzes the effects of an increase in a risky firm's default risk ν on aggregate output and productivity.

LEMMA 1. *The equilibrium allocation of capital and labor is characterized by:*

$$\frac{k^R}{k^S} = \left[1 + \frac{\tau \nu [1 - \delta]}{r^S} \right]^{-\frac{1 - \alpha(1 - \theta)}{1 - \alpha}}, \quad \frac{h^R}{h^S} = \left[\frac{k^R}{k^S} \right]^{\frac{\alpha \theta}{1 - \alpha(1 - \theta)}}. \quad (22)$$

It follows that $k^R/k^S < 1, h^R/h^S < 1$ for $\tau \in (0, 1]$ and that $k^R/k^S = h^R/h^S = 1$ for $\tau = 0$.

Proof. Given in the appendix, see the Mathematical Appendix. □

Results of Lemma 1 highlight that with the default-cost distortion $\tau > 0$, capital is allocated less to a risky firm relative to a safe firm even though the same expected productivity between the two types of firms. For the planner's problem maximizing the household's welfare, capital and labor should be allocated the same between the two types of firms given the same expected productivity between the types of firms. This optimal allocation turns out an equilibrium outcome for the special case of no default-cost distortion $\tau = 0$. More generally, the default-cost distortion is, however, positive $\tau > 0$ and the equilibrium allocation of capital and labor are distorted: capital is allocated too little for a risky firm and too much for a safe firm relative to the optimal allocation. The equilibrium misallocation of capital is mirrored to the higher equilibrium expected marginal product of capital for a risky firm relative to a safe firm, which measures the extent of capital misallocation in this economy. Moreover, the capital misallocation incurs, in turn even without direct distortions in the labor-market, the labor misallocation because the smaller amount of capital allocated for a risky firm shifts down a risky firm's marginal-product-labor curve relative to a safe firm's.

For the case of the planner's planning problem, allocation of capital is not distorted by the default-cost distortion, and hence, the expected marginal product of capital should be equalized between the two types of firms. By contrast, the equilibrium allocation of capital via defaultable debt contracts is affected by the default-cost distortion, for the case of the difference in default-risk between risky and safe firms in particular. More specifically, default event is costly for investors: capital losses by the proportion of τ on top of the loss of the promised interest.

The positive default probability for a risky firm results in that the promised interest rate for a risky bond should be high enough to compensate such capital losses as well as interest losses. Offered such a high interest rate, a risky firm uses a small amount of capital, and thus, a risky firm's expected marginal product of capital is larger than a safe firm's, i.e., the equilibrium misallocation of capital arises.

Next, the default risk for a safe firm should be of a first order importance for the determination of the equilibrium credit spread, see Lemma 2.

LEMMA 2. *Given the safe-bond interest rate r^S , the equilibrium corporate credit spread $r^R - r^S$ is increasing in both ν and τ is given by:*

$$r^R - r^S = \frac{\nu}{1 - \nu} [r^S + \tau[1 - \delta]] > 0, \forall \tau \in [0, 1]. \quad (23)$$

Proof. It immediately follows from the earlier no arbitrage condition for the household-investor. \square

From Lemma 2, we can see that given the safe-bond interest rate r^S , both ν and τ are important for the credit spread $r^R - r^S$. Given the importance of ν and τ for both the equilibrium allocation of capital and credit spread, results of increases in either ν or τ will be paid more attention to in the subsequent analysis.

Comparative Statics The primary objective of this paper is to assess the consequences of an increase in the corporate credit spread on aggregate output and productivity. As discussed earlier, ν and τ are important in determining the equilibrium corporate credit spread. Below comparative statics results for ν and τ are presented.

It will be shown that for the case of $\tau > 0$, the credit spread increases, capital is reallocated away from risky firms toward safe firms, and aggregate output decreases in response to either an increase in ν or an increase in τ given r^S and aggregate capital and labor. Because inputs are constant in the equilibrium, it follows that TFP will also be decreasing in both ν and τ . In response to increases in either ν or τ , the expected gross returns to risky bonds relative to safe bonds shift downward because the expected losses of capital increases. To equalize the expected gross returns between safe and risky bonds, capital should be reallocated away from risky firms toward safe firms. Aggregate output decreases because more capital is allocated to safe firms, which at the margin have lower (expected) productivity since k^S is already larger than k^R . Proposition 1 presents formally the effects of an increase in ν .

PROPOSITION 1. *Assume $\tau > 0$. In this case, given r^S and aggregate capital and labor ($K = \lambda k^S + [1 - \lambda]k^R$, $H = \lambda h^S + [1 - \lambda]h^R$), the credit spread is increasing in ν , but k^R/k^S , h^R/h^S*

and aggregate output y are decreasing in ν :

$$\begin{aligned} \text{Given } (r^S, K = \lambda k^S + [1 - \lambda]k^R, H = \lambda h^S + [1 - \lambda]h^R), \quad \frac{d[r^R - r^S]}{d\nu} > 0, \\ \text{and } \frac{dk^R/k^S}{d\nu} < 0, \frac{dh^R/h^S}{d\nu} < 0, \frac{dy}{d\nu} < 0 \text{ if } \tau > 0. \end{aligned} \quad (24)$$

Proof. Given in the appendix, see the Mathematical Appendix. \square

Results of Proposition 1 essentially says that with $\tau > 0$, the higher the default risk for a risky firm ν , the larger the extent of the capital misallocation, and hence the lower TFP and the smaller output. The higher default probability for a risky firm results in the higher credit spread, and hence, a risky firm cuts its investment much more, resulting in the lower aggregate productivity and output. Here $\tau > 0$ does the role of amplification device in the sense that with the larger $\tau > 0$, both the credit spread and extent of capital misallocation increase more in response to a unit increase in ν . For the special case of $\tau = 0$, allocation of capital is efficient $k^R/k^S = 1$ for any ν as implied from the earlier discussion of Lemma 1.

Next, Proposition 2 presents the results concerning the effects of an increase in τ on the equilibrium capital allocation and credit spread holding ν constant.

PROPOSITION 2. *Assume $\tau > 0$. In this case, given r^S and aggregate capital and labor ($K = \lambda k^S + [1 - \lambda]k^R, H = \lambda h^S + [1 - \lambda]h^R$), the credit spread is increasing in τ , but $k^R/k^S, h^R/h^S$ and aggregate output y are decreasing in τ :*

$$\begin{aligned} \text{Given } (r^S, K = \lambda k^S + [1 - \lambda]k^R, H = \lambda h^S + [1 - \lambda]h^R), \quad \frac{d[r^R - r^S]}{d\tau} > 0, \\ \text{and } \frac{dk^R/k^S}{d\tau} < 0, \frac{dh^R/h^S}{d\tau} < 0, \frac{dy}{d\tau} < 0 \text{ if } \tau > 0. \end{aligned} \quad (25)$$

Proof. Given in the appendix, see the Mathematical Appendix. \square

The main message of Proposition 2 is that given a risky firm's default risk $\nu > 0$, an increase in the default-cost distortion $\tau > 0$ cause both the credit spread and extent of capital misallocation to increase. Key here is that given $\nu > 0$ constant, an increase in τ leads to an increase in the expected losses of capital for risky bonds, which is the same as for the effect of an increase in ν . Note that an increase in the default cost τ represents a decrease in the recovery rate on defaulted debt, which could be interpreted as a negative financial shock to the collateral value of a firm as studied by Khan and Thomas (2011) and Gourio (2011). I do not explore further to compare my result to the literature because I do not focus on studying the effects of financial shocks (see, e.g., Khan and Thomas (2011) and references therein for this issue).

This section illustrated the key economic mechanism which links an increase in the default risk for a risky firm ν to decreases in output and productivity: reallocation of capital away from risky firms toward safe firms for an increase in ν , resulting in the larger extent of capital misallocation. It was also emphasized how the default-cost distortion $\tau > 0$ does the role of an amplification device in linking the increase in default risk for risky firms to the credit spread and extent of misallocation. In particular, in the case of $\tau = 0$, i.e., 100% recovery rate, the extent of misallocation is zero and an increase in default risk for risky firms has no effect on reallocation of capital and thereby no effects on output and TFP. Lastly, an increase in τ itself behaves as if an increase in ν , which implies that the effects of an increase in ν are larger for the case of ν positively correlated with τ compared to the case of constant τ .

3.4 Discussion: Capital Structure and Reallocation of Capital

This subsection discusses how the capital structure other than 100% debt financing, assumed in this section of the baseline model, would affect the properties of the model. In particular, I focus on how the capital structure would alter the response of the capital allocation between risky and safe firms to an increase in the default risk for a risky firm.

On the one hand, the capital structure does not affect qualitative properties of the model. On the other hand, for quantitative properties of the model, the capital structure may be important because the availability of financial instruments other than debt, e.g., internal cash flows, would affect the cost of capital for a risky firm relative to a safe firm, the crux in determining the extent of misallocation of resources.

For alternative means of financing, the internal financing is considered because of the dominance of internal and debt financing for corporate investment in the data (see Myers (2001)). The capital structure indeed affects quantitative properties of the model as will be discussed in the later section of simulation results. More specifically, in the later section 5, I consider an extreme case in which firms are restricted to use internal financing and to roll over the sufficiently large (constant) debt so that every firm's marginal capital should be always financed by internal cash flows and that risky firms are still exposed to default risk. Simulated results show that in this case, fluctuations in the extent of misallocation, TFP and output are slightly *larger* relative to the benchmark case of 100% debt financing.

Lastly, there is another issue of debt financing: the availability of long-term bonds. It is well known that long-term bonds provides the advantage of hedging against the interest-rate risk compared to the one-period bond studied in this paper. If all of the risky firms use long-term bonds instead of one-period bonds, then not all of but only a fraction of risky firms, whose bonds are matured, would be exposed to the risk of the stochastic cost of capital (see, e.g., Leland and

Toft (1996)). Put differently, in this case, a smaller *extensive* margin would be operative for the capital reallocation. At the same time, as the longer-maturity induces the stronger sensitivity of the interest rate to the per-period default probability (because of the *cumulative* default probability for a longer-maturity), a larger *intensive* margin would be operative for the capital reallocation. It would be interesting, although beyond the scope of this paper, to study which of the two opposing effects above would be dominant.

4 Quantitative Analysis

This section quantifies effects of fluctuations in default probabilities of risky and safe corporate bonds on the U.S. economy during the period 1964-2009. The model is calibrated to NIPA data for the U.S. economy and Moody's data for corporate bond market. The key to the calibration is that (annual) default rates for risky firms are high on average, about 1.9%, and quite volatile, standard deviation of 1.8%, while default rates for safe firms are almost constant and close to zero as in the data. Then properties of the calibrated dynamic model are analyzed numerically, and standard statistics studied in the business cycle research are compared to the data.

4.1 U.S. Economy And Corporate Bond Market

This subsection documents facts about fluctuations in real economic aggregates and fluctuations in the yields and default rates of corporate bonds over the period 1964-2009 as well as recovery rates for the period 1982-2009.

Data For the U.S. economy, I use NIPA to measure output, consumption and investment, and the BLS to measure labor. Based on the NIPA investment, capital stock is constructed by the perpetual inventory method as in the literature. TFP is measured as the Solow residual. Both NIPA and BLS series are for the period 1964-2009 at quarterly frequency. As in the literature, all series are detrended; they are logged and then HP-Filtered with the smoothing parameter of 1600.

Below it is discussed in more detail how to measure NIPA and BLS variables in the data. Output is measured as real GDP per capita. For consumption, real personal consumption expenditures on non-durables and services are used. For investment, real private investment is used¹⁶. All of the above NIPA variables are measured by the chain-weighted method and seasonally adjusted¹⁷. For labor, seasonally adjusted BLS aggregate hours index is used.

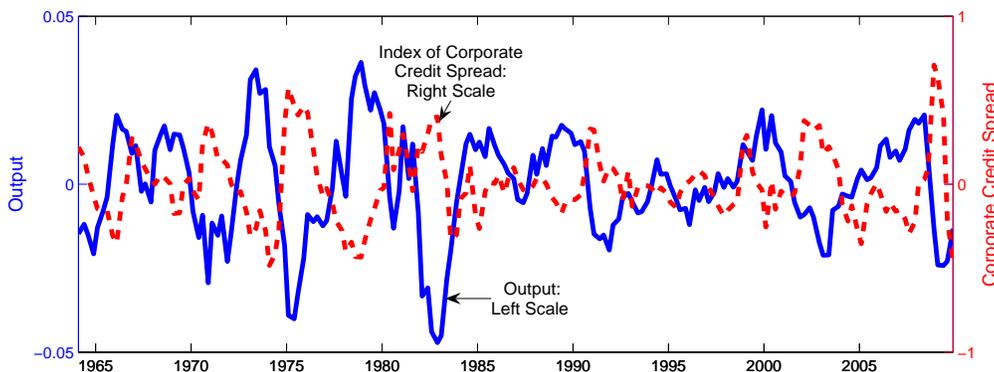
¹⁶NIPA item #8, headed 'Gross private domestic investment: Fixed Investment'.

¹⁷See the Data Sources appendix for detailed discussion.

For the corporate bond market, Moody’s global corporate bond dataset is used (see Moody’s (2010)). Moody’s uses letter grades to classify corporate bonds based on creditworthiness. This dataset provides annualized nominal yields for bonds of only two different letter grades, Aaa and Baa, at monthly frequency, which I convert to real returns at quarterly frequency¹⁸. The difference in nominal yields between Baa and Aaa grade corporate bonds is taken as an index for overall corporate credit spreads. Moody’s dataset also provides annual default and recovery rates for seven letter grades¹⁹. Recovery rate refers to one minus the loss rate in the event of default²⁰.

U.S. Business Cycles And Credit Spread Cycles Below comovements of business cycles and corporate credit spreads for the U.S. economy are discussed. I begin by discussing fluctuations in real GDP and the Baa-Aaa corporate credit spread, see Figure 1.

Figure 1: Output And an Index of Corporate Credit Spreads



Note: Both of the two series are quarterly, logged and HP-filtered. The solid line refers to output while the dashed line refers to the Baa-Aaa corporate credit spread.

From Figure 1, we can see that detrended output is negatively correlated with the detrended Baa-Aaa corporate credit spread during the period 1964-2009 where the correlation coefficient is about -0.64 . The relationships between key aggregates and the credit spread are summarized in table 1.

Table 1 highlights that the corporate credit spread is highly negatively correlated with output, TFP and labor during the period 1964-2009²¹.

¹⁸For inflation rate, the growth rate of CPI is used. Then yields at monthly frequently are aggregated to quarterly series by the simple average method. Note that yields are always reported as returns per year.

¹⁹Seven letter grades are Aaa, Aa, A, Baa, Ba, B and Caa-C. See the Data Sources appendix.

²⁰See the Data Sources appendix for detailed discussion.

²¹See Gertler and Lown (1999) for more detailed discussion of empirical relationships between the ‘High-Yield/Aaa’ corporate credit spread and output over the U.S. business cycles since 1980s.

Table 1: Statistics of Key Aggregates and the Corporate Credit Spread

x	y	h	k	TFP	c	i	Baa-Aaa
$\sigma(x)/\sigma(y)$	1.00	1.27	0.34	0.53	0.54	3.35	13.77
$corr(x, y)$	1.00	0.88	0.15	0.45	0.84	0.91	-0.58
$corr(x, \text{Baa-Aaa})$	-0.58	-0.49	0.25	-0.37	-0.52	-0.65	1.00

Note: The percentage standard deviation of output $\sigma(y)$ is 1.56. $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable x to that of output $\sigma(y)$, $corr(x, y)$ refers to the correlation coefficient of a variable x and output while $corr(x, \text{Baa-Aaa})$ does the same for the Baa-Aaa credit spread. The column headed ‘Baa-Aaa’ refers the Baa-Aaa corporate credit spread. All variables are log deviations from trend

Next, fluctuations in default rates of corporate bonds are discussed²². I begin by discussing how to classify corporate bonds of different letter grades into safe and risky bonds, and then present statistics of the default rates for safe and risky corporate bonds. As mentioned earlier, Moody’s provides yields for only two letter grade bonds, Aaa and Baa. Naturally, Baa letter grade is used as the threshold in classification of bonds into the categories of safe vs. risky: Aaa, Aa and A grade bonds are counted as safe and Baa, Ba and B grade bonds as risky. Then default rates of safe and risky bonds are calculated as the cross-sectional averages of corporate bonds belonging to each of the two categories²³.

Table 2: Annual Default Rates of Corporate Bonds During the Period 1964-2009

Grade	Safe	Risky
Mean	0.02%	1.93%
Std.	0.07%	1.76%

Note: The line headed ‘Mean’ presents sample means of historical default rates of safe and risky bonds at annual frequency and the line headed ‘Std.’ does the same for sample standard deviations.

As shown in table 2, default rates for risky bonds are high on average and quite volatile while default rates for safe bonds are almost constant and close to zero. For simplicity, from now on, default rates for safe bonds are assumed zero.

²²Note that through the lens of the model, changes in the difference in the default risk between risky and safe bonds are the main source of changes in the credit spread between risky and safe bonds. In the model, the ex-ante default probability for a risky bond equals for sure the ex-post default rate and is known to investors every period and thereby no risk-premium exists in equilibrium.

²³The Moody’s Caa-C grade bonds are excluded from the consideration here because default rates of Caa-C grade bonds are starkly higher, by factor larger than 4, than those of Baa, Ba and B grade bonds while the share of the Caa-C grade bonds issuers in the corporate bond market is small, about 4% in the data. See the Data Sources appendix for more detailed discussion.

4.2 Calibration

This subsection discusses how to calibrate parameter values of the model to data for the U.S. economy during the period 1964-2009. Parameter values are chosen so that the model economy in the steady state, essentially ν_t being constant, is consistent with key statistics.

One period in the model is one quarter in the data. The benchmark parameter values are listed in table 3.

Table 3: Benchmark Parameter Values

	PARAMETER	VALUE	TARGET
ν :	Prob. of $z(i) = 0$ for Risky Firm i	0.0048	Default Rate of Risky Bonds
δ :	Depreciation Rate	0.015	Benchmark
τ :	Default Losses of Undepreciated Capital	0.599	Recovery Rate of Risky Bonds
λ :	Measure of safe firms	0.389	Volume-Share of Safe Bonds
β :	Discount Factor	0.9909	Real Return to Aaa Bonds
α :	Returns to Scale	0.87	Khan and Thomas (2011)
θ :	Capital Share	0.33	Investment/GDP
ω :	Curvature of Labor-Disutility	0.30	Benchmark
ψ :	Level of Labor-Disutility	3.32	$h_{ss} = 0.33$

The steady state ν is set to the average quarterly default rates for risky corporate bonds, 1.93% at annual frequency. As in the literature, depreciation rate of capital δ is set to .015, i.e., 6% at annual frequency, and the default-cost distortion τ is set to 0.599 by targeting the recovery rate for risky corporate bonds, about 40% on average during the period 1982-2009²⁴:

Second, the measure of safe firms λ is set to 0.389 based on the relative size of safe bonds to risky bonds:

$$\frac{\lambda k^S}{\lambda k^S + [1 - \lambda]k^R} = \frac{\text{size of newly issued safe bonds}}{\text{total size of newly issued bonds}} \quad (26)$$

where λk^S refers to the size of newly issued safe bonds and $[1 - \lambda]k^R$ does the same for risky bonds. For bond size, data availability lets me use the flow measure, i.e., volume of new issues, rather than the stock measure, i.e., the outstanding volume: the relative size of safe bonds to the total is about 47.7% during the period 1993-2009²⁵.

Third, discount factor β is set to 0.9909 to match quarterly real-returns to the Moody's Aaa grade corporate bonds, about .92% on average during the period 1964-2009²⁶. For the returns-to-scale α , 0.87 is chosen as benchmark as in Khan and Thomas (2011). In turn, capital share

²⁴In the model, the recovery rate becomes $[1 - \tau]$ for the case of $\delta = 0$. In more general case of $\delta > 0$, the recovery rate is not exactly the same with, but close to, $[1 - \tau]$ (see the Calibration appendix).

²⁵Given interest rates and the values of α and θ , I can back out k^S and k^R and then solve for λ the above equation. For data on the size of safe vs. risky corporate bonds, see the Calibration appendix.

²⁶The Euler equation is given by: $1/\beta = 1 + \text{real return to a safe bond}$.

parameter $\theta = 0.33$ is chosen as benchmark as is common in the literature. This implies, in the model, about 29% non-proprietary capital income share in GDP and 58% non-proprietary labor income share in GDP, investment to GDP ratio equal to about 18% and capital to annual GDP ratio equal to 2.8, which is broadly consistent with the business cycle literature.

Lastly, two preferences parameters (ω, ψ) are calibrated. The curvature parameter of disutility from labor ω is set to 0.30 as my benchmark, resulting in the volatility of labor about 70% relative to the output volatility in my simulation results where the counterpart in the data is about 127%. Then I choose $\psi = 3.32$ by targeting steady state (normalized) labor supply of 0.33.

Shock Process for ν Next, calibration of the shock process for ν is discussed. Key here is to match the distribution of historical defaults rates for risky corporate bonds, clustered around zero, which leads me to posit that the density of ν' conditional on ν is given by a mixture of uniform and truncated normal distribution:

$$f(\nu'|\nu) = \phi(\nu) \cdot f^U(\nu'|\nu) + [1 - \phi(\nu)] \cdot f^{TN}(\nu'|\nu) \quad (27)$$

where $f^U(\nu'|\nu)$ refers to the pdf for a uniform distribution over $[0, \underline{\nu}]$, with $\underline{\nu} \in (0, 1)$, and $f^{TN}(\nu'|\nu)$ refers to the pdf of a normal distribution truncated at 0 and 1. And $\phi(\nu)$ refers to the ν -dependent weight on the uniform distribution in drawing ν' .

For $\underline{\nu}$ very close to zero, the uniform distribution $f^U(\nu'|\nu)$ over $[0, \underline{\nu}]$ is essentially intended to capture that ν' is clustered around zero for the invariant distribution of ν' as in the data. Meanwhile, the truncated normal distribution $f^{TN}(\nu'|\nu)$ mainly governs, together with the state-dependent weight $\phi(\nu)$, the usual AR(1) mean-reverting process and is given by :

$$\nu' = (1 - \rho)\bar{\nu} + \rho\nu + \sigma \cdot \epsilon' \quad (28)$$

where ϵ' is a standard normal random variable truncated so that $\nu' \in (0, 1)$.

Space of ν is discretized into 41 points: in an ascending order, $\nu \in \{\nu_1, \nu_2, \dots, \nu_{41}\}$. I approximate the continuous process of ν by using the method of Tauchen (1986) and imposing simplifying restrictions²⁷. The resulting transition matrix of discretized ν' is given by:

$$\pi(\nu', \nu) \equiv Prob(\nu'|\nu) = \begin{cases} \phi_1 \cdot \chi_{(\nu'=\nu_1)} + (1 - \phi_1) \cdot \pi^{TN}(\nu', \nu) & \text{if } \nu = \nu_1 \\ \phi_2 \cdot \chi_{(\nu'=\nu_1)} + (1 - \phi_2) \cdot \pi^{TN}(\nu', \nu) & \text{if } \nu > \nu_1 \end{cases} \quad (29)$$

where (ϕ_1, ϕ_2) refers to the ν -dependent weight on the uniform distribution in drawing ν' , and $\chi_{(\nu'=\nu_1)}$ refers to the indicator function such that $\chi_{(\cdot)} = 1$ if $\nu' = \nu_1$ and $\chi_{(\cdot)} = 0$ if $\nu' > \nu_1$, which

²⁷See the Calibration appendix for more detailed discussion.

represents the uniform distribution $f^U(\nu'|\nu)$ over the narrow interval around ν_1 . And $\pi^{TN}(\nu', \nu)$ refers to the transition matrix of discretized ν' implied from $f^{TN}(\nu'|\nu)$, governed by $(\rho, \bar{\nu}, \sigma)$. For $\nu_1 = 0.00025$, (ϕ_1, ϕ_2) essentially captures the clustering of ν' around zero while ϕ_2 and $(\rho, \bar{\nu}, \sigma)$ essentially determine the mean-reverting process for $\nu' > \nu_1$.

Five parameters $(\phi_1, \phi_2, \rho, \bar{\nu}, \sigma)$ are jointly estimated to match five key statistics of the historical default rates for risky bonds: sample mean, sample standard deviation, serial correlation, skewness and the frequency of annual default rates less than 0.2%, see table 4 for the estimation results and table 5 for the default-rate statistics²⁸.

Table 4: Estimation Results for the Shock Process for ν

ϕ_1	ϕ_2	ρ	$\bar{\nu}$	σ
0.5290	0.1500	0.8365	0.0048	0.0039

Table 5: Statistics of Default Rates for Risky Bonds at Annual Frequency

Statistic	Mean	Std.	$Corr(\nu', \nu)$	$Prob(\nu < 0.2\%)$	Skewness	Kurtosis
Data	1.93%	1.76%	0.317	15.22%	1.35	2.12
Model	1.97%	1.75%	0.316	15.18%	1.35	2.11

Note: ‘Std.’ refers to the sample standard deviation of the risky bond’s default rate ν , ‘ $Corr(\nu', \nu)$ ’ refers to the serial correlation of ν , and ‘ $Prob(\nu < 0.2\%)$ ’ refers to the relative frequency of ν less than 0.2%. All statistics are at annual frequency.

The mean and standard deviation of the model-generated default rates ν_t are, as desired, close to the data, and the model-generated results are also close to the data for the other three statistics ($Corr(\nu', \nu)$, $Prob(\nu < 0.2\%)$, Skewness), too. Moreover, the kurtosis is, even though not targeted, also almost the same between the model and data. As the calibrated model economy mimics the data for risky-bond default rate dynamics well, we are ready to analyze the quantitative properties of my dynamic model.

4.3 Results

This section presents and discusses simulated results. Benchmark results are presented, and then results of alternative cases for the default-cost distortion τ are discussed. Moreover, results of the case of the risk-neutral measure of default probability and sensitivity analysis are also provided. Below I discuss the simulation method and then proceed to discussing simulation

²⁸I use the method of weighted minimum distance between the model and the data with weights of (20:10:1:1:1) on the above four statistics; larger weights are put to the first two statistics, the sample mean and standard deviation, because of their importance. For the model-generated statistics at annual frequency, the simulated quarterly series are aggregated to annual frequency.

results.

Simulation Method I feed a series of shocks to ν for 183 periods, which corresponds to the period 1964Q2-2009Q4, to the calibrated model. For the initial state, the model economy is assumed to be in the deterministic steady state. Statistics of model-generated series are reported based on simulations by 1,000 times.

Equilibrium policy functions are numerically solved for by the first order finite element method as illustrated by McGrattan (1996). The aggregate state vector is the pair (ν, K) where K refers to the aggregate capital. As ν is already discretized, the finite element method is applied with respect to continuous K ²⁹.

Benchmark Results Model-generated statistics standard in the business cycle literature are presented, see table 6.

Table 6: Statistics of Key Aggregates: Benchmark

x	$\sigma(y)$	$\sigma(x)/\sigma(y)$					$corr(x, y)$				
	y	h	k	TFP	c	i	h	k	TFP	c	i
Data	1.56	1.27	0.34	0.53	0.54	3.35	0.88	0.15	0.45	0.84	0.91
Model	0.93	0.74	0.74	0.66	0.59	5.32	0.81	0.18	0.86	0.37	0.88

Note: $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable x to that of output $\sigma(y)$ and $corr(x, y)$ refers to the correlation coefficient of x and output. All variables refer to their log deviations.

As shown in table 6, shocks to risky-bond default rate ν account for a substantial part of the fluctuations in output and TFP, about 60% for output and 70% for TFP, which supports my claim that fluctuations in corporate credit spreads, essentially driven by shocks to the default risk for risky firms, are an important source of fluctuations in output and productivity in the U.S. economy³⁰. The model-generated correlations between key aggregates with output are also consistent with the data.

²⁹Four policy functions are solved for: consumption, interest and wage rates for safe firms, and aggregate labor supply. Then other allocation and price functions are immediately calculated. I limit the space of K to be sufficiently large, upper bound of 120% and lower bound of 80% relative to the steady state K ; the solved transition function of aggregate capital $K'(\nu, K)$ never binds at those two boundaries. Numerical error is about 10^{-6} percentage point uniformly over the domain between the guessed and updated policy functions.

³⁰TFP is measured as: $TFP_t \equiv y_t / [\tilde{K}_t^\theta h_t^{1-\theta}]$ where \tilde{K}_t refers to capital stock constructed by the perpetual inventory method as in the data and given by: $\tilde{K}_{t+1} = [1 - \tilde{\delta}]\tilde{K}_t + I_t$, $\tilde{K}_0 = K_0$. K_t refers to the correctly measured capital stock based on the time-varying depreciation rate owing to default losses. The constant (quarterly) depreciation rate $\tilde{\delta}$ is calibrated such that \tilde{K}_t is the same with the correctly measured capital stock K_t in the deterministic steady state, resulting in $\tilde{\delta} = .0165$, i.e., 6.6% depreciation rate per year on average: (annual) depreciation of 0.6 percentage points for default losses and the remaining 6.0 percentage points for physical and economical depreciation and amortization.

As clarified in the previous analysis, the key mechanism of the impact of an increase in the risky-bond default probability ν on TFP and output is the reallocation of capital away from risky firms toward safe firms tightly linked to the credit spread. Results for capital allocation between risky and safe firms together with the credit spread are presented in table 7.

Table 7: Statistics of Resources Allocation and Credit Spread: Benchmark

x	$r^R - r^S$	k^S	k^R	h^S	h^R	k^R/k^S
$\sigma(x)/\sigma(y)$	0.37	20.38	23.08	14.76	15.07	43.16
$corr(x, y)$	-0.95	-0.88	0.97	-0.91	0.96	0.94
$corr(x, r^R - r^S)$	1.00	0.99	-1.00	0.99	-1.00	-1.00

Note: $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable x to that of output y , $corr(x, y)$ refers to the correlation coefficient of x and y , and $corr(x, r^R - r^S)$ does the same for the credit spread $r^R - r^S$. All variables refer to their log deviations except that $r^R - r^S$ is non-logged deviation.

From table 7, we can see that as expected, the corporate credit spread $r^R - r^S$ is countercyclical and highly correlated with resource allocation between risky and safe firms. As predicted in the earlier analysis, capital allocated for a risky firm relative to a safe firm k^R/k^S decrease as the credit spread increases, which further induces labor reallocation because capital shifts the marginal-product-of-labor curve.

Next, fluctuations in the model-generated corporate credit spread, in terms of the quarterly yields, are about 0.34 percentage points, comparable to 0.36 percentage points in the data during the period 1997-2009³¹. Recall that the credit spread plays a role of multiplier of the risky-bond default risk to capital allocation, TFP and output. Therefore, my results are not exaggerated by spuriously large fluctuations in the corporate credit spread.

Below I discuss how a number of extensions of my model would affect simulated results and then proceed to discuss the results of the two alternative cases for τ . The model predicts sizable effects on output and TFP of one unit increase in the default probability for risky firms via a substantial extent of reallocation of capital between risky and safe firms, for which the following three ingredients, omitted in the model, could be important: investment opportunities other than lending to the private-domestic-production sector, sophisticated financing, and adjustment costs of resource-reallocation.

First, consider the case in which the household can invest to securities issued by government

³¹Source: the BofA Merrill Lynch, available at <http://research.stlouisfed.org/fred2/categories/32347>. I calculated the quarterly yield spread between the risky and safe bonds where I take the Merrill Lynch's AAA, AA and A grade bonds as the safe bond and the other bonds (BBB, BB, B, CCC and below) as the risky bond. The Moody's number-of-issuers shares were used in calculation of weighted yields for safe and risky bonds as was for the weighted default rates. For the case in which CCC and below grade bonds are excluded, the standard deviation of the quarterly yield spread between risky and safe bonds for the same period is 0.30 percentage points (comparable to 0.36 percentage points when CCC and below grade bonds are included).

or foreign firms on top of the lending to the private-domestic-production sector. In this case, the impact of one unit increase in the risky-firm’s default risk ν on output via the endogenous TFP channel would be mitigated to some extent while the impact of ν on output via the aggregate investment dynamics would be larger. Consequently, results for output would be still sizable.

Second, we could assume that firms are allowed to use financing vehicles more sophisticated than the 100% debt financing. In this case, the earlier quantitative results would be affected because the response of a risky firm’s relative cost of capital to a safe firm would be also sensitive to the availability of financial instruments. For instance, a risky firm can substitute its internal cash flows for debts in financing its capital or insure against the interest rate shocks by issuing long-term bonds rather than one-period bonds. The case of the internal financing will be discussed later in section 5. The case of long-term bond is, although interesting, left for future work.

Third, with adjustment costs of resource-reallocation across firms, the extent of resource-reallocation would be smaller relative to the benchmark case of no adjustment costs. In this case, fluctuations in key aggregates would be also smaller than the benchmark results. Key here is how large, if any, adjustment costs of resource-reallocation would be, which calls for an empirical investigation.

Importance of τ as an Amplification Device The importance of the default-cost distortion τ is examined where τ captures a key friction in the corporate bond market. I consider the case of $\tau = 0$ keeping other parameters the same as for the benchmark case. For simulated results in this case, see table 8.

Table 8: Statistics of Key Aggregates: Case of $\tau = 0$

x	$\sigma(y)$	$\sigma(x)/\sigma(y)$					$corr(x, y)$				
	y	h	k	TFP	c	i	h	k	TFP	c	i
Data	1.56	1.27	0.34	0.53	0.54	3.35	0.88	0.15	0.45	0.84	0.91
Bench	0.93	0.74	0.74	0.66	0.59	5.32	0.81	0.18	0.86	0.37	0.88
$\tau = 0$	0.00	2.71	9.09	0.18	4.55	15.41	-1.00	1.00	-1.00	1.00	-1.00

Note: The line headed $\tau = 0$ refers to the case of $\tau = 0$ and the line headed ‘Bench’ does the same for the benchmark case of $\tau = 0.599$. $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable x to that of output $\sigma(y)$ and $corr(x, y)$ refers to the correlation coefficient of x and output. All variables refer to their log deviations.

Table 8 highlights that for the alternative case of $\tau = 0$, shocks to ν generate almost no fluctuations in key variables, consistent with the earlier analysis. Recall that for the case of $\tau = 0$, there is no misallocation of resources and hence reallocation of capital, if any, would have no first order effect on TFP and output. In fact, there is no reallocation of capital because the

capital allocated for a risky firm relative to a safe firm is constant equal to one. This result confirms that shocks to ν studied in this paper differs from the aggregate productivity shocks that directly affect output even for the case of resource allocation kept constant, which is not the case, as shown above, for shocks to ν considered in this paper.

Lastly, I also examine the second alternative case in which τ is time-varying and positively correlated with ν as is in the data (see, e.g., (Altman and Karlin 2010)). For an extreme case of τ perfectly positively correlated with ν and calibrated to data for time series of recovery and default rates, effects of shocks to ν on key variables are slightly larger compared to the benchmark case of the constant τ ³².

Risk-Neutral Measure of Default Risk This section discusses simulated results for the case of the risk-neutral measure of default risk. In this case, the shock process for ν is estimated to match statistics of the credit spread between risky and safe corporate bonds rather than actual default rates³³. In this case, the newly estimated results of the shock process for ν are as follows: $\phi_1 = 0.0070$, $\phi_2 = 0.0004$, $\rho = 0.9320$, $\bar{\nu} = 0.0025$, and $\sigma = 0.0030$. Statistics of the corporate credit spread between risky and safe bonds are close between the data and model: average credit spread is 0.75% for data and 0.59% for model, and standard deviation of credit spread is 0.33% for both data and model³⁴. Next, statistics of risky-bonds' default rates are presented in table 9. As shown by table 9, the estimated default-risk for a risky firm is, as

Table 9: Risky-Bonds' Default Rates at Annual Frequency: Risk-Neutral Measure of ν

Statistic	Mean	Std.	$Corr(\nu', \nu)$	$Prob(\nu < 0.2\%)$	Skewness	Kurtosis
Data	1.93%	1.76%	0.32	15.22%	1.35	2.12
Model: Risk-Neutral	3.76%	1.85%	0.64	0%	1.09	1.02

Note: 'Std.' refers to the sample standard deviation of the risky bond's default rate ν , ' $Corr(\nu', \nu)$ ' refers to the serial correlation of ν , and ' $Prob(\nu < 0.2\%)$ ' refers to the relative frequency of ν less than 0.2%. All statistics are at annual frequency. Given the average return to the safe bond, the shock process for ν is estimated to match statistics of the credit spread at quarterly frequency. Then the model-generated ν is aggregated over time.

expected, larger on average for the case of risk-neutral measure relative to the benchmark case of the physical measure while the standard deviation is similar between the two cases.

Parameters other than the shock process for ν are held the same as for the benchmark case. Simulated results in this case are presented in table 10.

³²In this case, output fluctuations are 1.16% and TFP fluctuations are 0.70%. The reason is that fluctuations in the corporate credit spread are larger for the case of τ positive correlated with ν , about 0.39% points, than they are for the benchmark case of the constant τ , about 0.34% points (see the online appendix).

³³ The source of data on the credit spread is the BofA Merrill Lynch corporate-bond yields during the period 1997-2009 at quarterly frequency as discussed earlier.

³⁴Results for other targeted statistics of the corporate credit spread are available upon request.

Table 10: Statistics of Key Aggregates: Case of Risk-Neutral Measure of ν_t

x	$\sigma(y)$	$\sigma(x)/\sigma(y)$				
	y	h	k	TFP	c	i
Data	1.56	1.27	0.34	0.53	0.54	3.35
Bench	0.93	0.74	0.74	0.66	0.59	5.32
Risk-Neutral	1.04	0.75	1.93	1.08	0.80	5.08

Note: $\sigma(x)$ refers to the variable x 's percentage standard deviation. 'Bench' refers to results for the benchmark case of the physical measure of default risk while 'Risk-Neutral' does for the case of the risk-neutral measure.

Results of table 10 highlight that fluctuations in output and TFP are sizable (and highly negatively correlated with the credit spread even though not reported here), consistent with the earlier benchmark case of the physical measure of default risk. More specially, fluctuations in output and TFP are larger for the case of risk-neutral measure of default risk relative to the benchmark case of the physical measure. Key here is that the average default risk is larger for the risk-neutral measure than the physical measure due to risk premium and so on, and so is the average extent of capital misallocation. Therefore, first-order effects of ν on output and TFP are also larger in the risk-neutral measure case than in the physical measure case.

Sensitivity Analysis For robustness check, results for different values of returns-to-scale α are also provided, see table 11³⁵.

Table 11: Results for Sensitivity Analysis w.r.t. Returns-To-Scale α

	y	TFP	k^R/k^S	$r^R - r^S$
Data	1.56	0.83	-	0.36
Bench: $\alpha = .87$	0.91	0.59	38.8	0.34
$\alpha = .90$	1.17	0.72	48.9	0.35
$\alpha = .80$	0.63	0.46	28.6	0.34
$\alpha = .70$	0.44	0.38	22.4	0.35

Note: numbers refer to the percentage standard deviation for log deviations of variables except that $r^R - r^S$ refers to the standard deviation, in terms of percentage points, of non-logged spread.

As shown in table 11, returns-to-scale α is quantitatively important for simulated results: effects of shocks to ν on TFP and output become larger for higher α . Nevertheless, even for a low value of α , the model-generated fluctuations in key variables are still sizable: for the case of $\alpha = 0.70$, the model-generated output fluctuations are about 28% relative to the data.

Returns-to-scale α affects fluctuations in TFP and output greatly because α determines the

³⁵For different values of α , the measure of safe firms λ is recalibrated so as to match the safe-bond size relative to the total while other parameters are kept the same as for the benchmark.

size of fluctuations in the capital allocation between risky and safe firms k^R/k^S given a unit increase in the corporate credit spread. For higher α , a risky firm needs to cut its capital to a larger extent to adjust its expected marginal product of capital for one unit increase in its interest rate r^R . As a result, the higher α , the larger fluctuations in k^R/k^S , and hence, the larger fluctuations in TFP and output.

5 Extension: Internal Financing and Rolled-Over Debt

This section discusses the sensitivity of the earlier simulated results of the baseline model of 100% debt financing to the availability of alternative methods of financing. As discussed already, I focus on the availability of internal financing. More specifically, the investment decision made by a risky firm under the time-varying default risk will be studied for the case in which the internal cash-flow is operative in financing capital at the margin. For the purpose of exposition, a simple scenario is considered such that firms use internal financing and roll over constant and sufficiently large size of one-period bonds so that the same default-rate dynamics are still present and internal financing is always operative in financing the marginal capital.

5.1 Environment

Preferences of the representative household are the same as in baseline model. For capital structure and technology, two features are essentially added in this model compared to the previous baseline model of 100% debt financing. First, non-defaulting firms roll over their debts of constant size and finance their (marginal) investments via their internal cash flows. Second, constant mass of risky firms are created and enter the market immediately every period.

Technology The technology for this economy is essentially the same as for the baseline economy except that (i) measure of risky firms changes over time by the simplifying assumption of the constant entry and (ii) every firm receives a small amount of endowment income.

There is a continuum of safe firms of measure $\lambda \in (0, 1)$ and a continuum of risky firms of measure $N_t > 0$: firm i is safe for $i \in [0, \lambda]$ and is risky for $i \in (\lambda, N_t + \lambda]$ at period t . Note that N_t can change over time t whereas N_t is constant equal to $1 - \lambda$ in the baseline model.

For simplicity, I assume that constant mass M of risky firms are created and enter the market at the beginning of every period incurring expenses χ , labeled as the entry cost, per such an entrant risky firm. As ν_t mass of risky firms will be liquidated at the final subperiod of t , the

mass of operative risky firms at $t + 1$ is given by³⁶:

$$N_{t+1} = (1 - \nu_t)N_t + M, \quad \forall t = 0, 1, 2, \dots \quad (30)$$

Next, firm i 's production function is the same as for the baseline model except that the firm receives an endowment at the final subperiod on top of its income generated from production $y_t(i)$. Safe firm i 's production function is given by:

$$y_t(i) = \left\{ \left[[k_t(i)]^\theta [h_t(i)]^{1-\theta} \right]^\alpha + \kappa \right\}, \quad \forall i \in [0, \lambda], \quad \alpha \in (0, 1), \quad \theta \in (0, 1), \quad \kappa > 0 \quad (31)$$

where the constant term $\kappa > 0$ represents firm i 's endowment. $\kappa > 0$ is essentially intended to make the firm value high enough to match the observed leverage ratio in the data: $\kappa > 0$ is interpretable as the growth option in the valuation of a firm or the locally increasing-returns-to-scale for the initial employment of resources.

Risky firm i 's production function is stochastic and given by:

$$y_t(i) = z_t(i) \left\{ \left[[k_t(i)]^\theta [h_t(i)]^{1-\theta} \right]^\alpha + \kappa \right\}, \quad \forall i \in (\lambda, N_t] \quad (32)$$

where $z_t(i)$ is the stochastic idiosyncratic productivity; The probability distribution of $z_t(i)$ and the shock process for $\nu_t = \text{Prob}(z_t(i) = 0 | i \in (\lambda, N_t])$ are the same as for the baseline model³⁷. Note that the expected endowment for a risky firm equals κ , the same as for a safe firm.

Timing Timing of events are essentially the same as for the baseline model and only the new features are discussed here. The time interval between the final subperiod of t and initial subperiod of $t + 1$ is zero. At the final subperiod of t , firm i makes its decision on whether to revolve/default their current debt after observing its current productivity $z_t(i)$ as well as ν_{t+1} , which is essentially intended that the next period allocation of resources, $(k_{t+1}(i), h_{t+1}(i))$, is a function of ν_{t+1} as in the baseline model.

Capital Structure First, survived and non-defaulting firms roll over their one-period bonds and finance their investments via their internal cash flows. Let $B_t(i)$ denote firm i 's outstanding

³⁶If the mass of newly created risky-firms were assumed to equal $(1 - \nu_t)$, then N_{t+1} would be constant equal to $1 - \lambda$ as in the baseline model. In this case, aggregate entry-costs $\chi \cdot [1 - \nu_t]$ would be cyclical and affect a firm's investment decision largely by the general equilibrium effect via the household's consumption. Therefore, I purposefully assume that constant mass of new risky-firms enter for every period.

³⁷I keep notations ν_t , the probability of a jump shock $z_t(i) = 0$ for risky firm i in period t , and $f(\nu_{t+1}|\nu_t)$, the density of ν_{t+1} conditional on ν_t , as in the baseline model.

one-period bonds at t :

$$B_t(i) = B > 0. \quad (33)$$

Note that every non-defaulted/operative firm carries constant size of debt, which captures the debt used in financing the initial entry costs and so on. As the size of debt does not matter, as long as it is not too large, for a safe firm's problem because of no default risk, without loss of generality, the size of the rolled-over one-period bonds is assumed the same between risky and safe firms. Let $q_t(i)$ denote the price of one-period (non-contingent) bond issued by firm i at the initial subperiod of t . Firm i 's interest expenses at the final subperiod of t is given by: $[1 - q_t(i)]B$.

Let $x_t(i) \equiv k_{t+1}(i) - [1 - \delta]k_t(i)$ denote firm i 's investment at the final subperiod of t . Firm i 's dividend $d_t(i)$, restricted to be non-negative, is written as:

$$d_t(i) = \max \left\{ 0, \quad y_t(i) - x_t(i) - w_t(i)h_t(i) - [1 - q_t(i)]B \right\} \quad (34)$$

which states that firm i 's dividend is equal to its output net expenses on investment, labor and interest and that its dividend should be non-negative. Given the non-negativity restriction imposed on the dividend policy, the option of external equity financing is already ruled out, which was again motivated by the empirical fact reported by Myers (2001).

Second, new risky firm i entering at the initial subperiod of $t + 1$ also issues one-period (non-contingent) bond, of the size equal to B same as for survived firms, at the given price of $q_{t+1}(i)$. Entrant risky firm i uses the proceeds of debt $q_{t+1}(i) \cdot B$ for its capital in use for the production and immediate, at the initial subperiod of $t+1$, dividend-payouts to its equity holders.

Costs of Default As in the baseline model, bondholders of defaulting firm i lose $\tilde{\tau}$ fraction of capital held by the defaulting firm in the liquidation process. Let $(1 - \delta)k_t(i)$ denote the undepreciated capital owned by the defaulting firm i at the final subperiod of t . The ex-post return, again at the final subperiod of t , to the defaulted bond issued by the firm i is written as:

$$[1 - \tau_t(i)](1 - \delta) \equiv \frac{(1 - \tilde{\tau})(1 - \delta)k_t(i)}{B} \quad (35)$$

where $\tau_t(i) \equiv 1 - (1 - \tilde{\tau}) \cdot [k_t(i)/B]$ refers to the (endogenous) default-cost distortion per unit of undepreciated capital supplied to the defaulting firm i at t . The default-cost distortion $\tau_t(i)$ is determined by two components: per-capital liquidation cost $\tilde{\tau}$ and the ratio of capital to debt. That is, default-cost distortion τ is larger than the liquidation cost $\tilde{\tau}$ if, and to the extent to that, debt is larger than the capital held by firm i . As shown above, $[1 - \tau_t(i)]$ is decreasing in both $\tilde{\tau}$ and debt-to-capital ratio $B/k_t(i)$, which will play a key role in determining the recovery

rate for defaulted risky bonds and hence the equilibrium credit spread.

Resource Constraint The resource constraint is given by:

$$c_t + \lambda k_{t+1}^S + N_{t+1} k_{t+1}^R + \chi M = y_t + (1 - \delta) \left[\lambda k_t^S + N_t [1 - \nu_t \tilde{\tau}] k_t^R \right] \quad (36)$$

which states that output plus undepreciated capital net liquidation costs is used to consumption, next-period capital and entry-costs of new risky firms where output y_t is given by:

$$y_t = \lambda \left[[k_t^S]^\theta [h_t^S]^{1-\theta} \right]^\alpha + N_t \left[[k_t^R]^\theta [h_t^R]^{1-\theta} \right]^\alpha + \kappa \left[\lambda + N_t \right]. \quad (37)$$

Note that output y_t is independent of ν_t holding allocation of resources and the mass of risky firms N_t to constant.

A Firm's Problem Problems for safe and risky firms are analyzed in recursive form. First, safe firm i 's problem is represented by the following value functions. At the *final subperiod* indicated by the subscript '+', safe firm i chooses whether to default or not:

$$V_+^S(k, h, K^S, K^R, N, \nu, \nu') = \max \left\{ 0, V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu') \right\} \quad (38)$$

where (k, h) is the vector of firm i 's individual state variables, chosen at the initial subperiod, (K^S, K^R, N, ν, ν') is the vector of aggregate state variables, the value for default is zero, and $V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu')$ refers to the firm's value function conditional on non-default. Let k' denote safe firm i 's choice of the next-period capital and $V_-^S(k, K^S, K^R, N, \nu)$ denote the firm's value function at the *initial subperiod* indicated by the subscript '-'. Safe firm i 's value function conditional on non-default $V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu')$ is given by:

$$V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu') = \max_{k' \geq 0} \left\{ \left[[k]^\theta [h]^{1-\theta} \right]^\alpha + \kappa + (1 - \delta)k - w^S h - [1 - q^S]B \right. \\ \left. - k' + V_-^S(k', K_+^S, K_+^R, N', \nu') \right\} \quad (39)$$

where (K_+^S, K_+^R, N') denotes, aggregate (endogenous) state variables at the next period, capital owned by an individual safe firm and an individual risky firm and the number of (operative) risky firms, respectively. Next, safe firm i 's initial-subperiod value function $V_-^S(k, K^S, K^R, N, \nu)$

is written as:

$$V_-^S(k, K^S, K^R, N, \nu) = \max_{h \geq 0} \left\{ q^S \cdot \int_0^1 V_+^S(k, h, K^S, K^R, N, \nu, \nu') \cdot f(\nu', \nu) d\nu' \right\} \quad (40)$$

which means that the safe firm optimally chooses, at the initial subperiod, its current-period labor to maximize the present value of its final-subperiod value where every firm is assumed to discount its future cash flows by the risk-free rate.

Second, risky firm i 's problem is also represented by the following value functions. The final-subperiod value function for the risky firm of the productivity z is given by:

$$V_+^R(k, h, K^S, K^R, N, \nu, \nu', z) = \max \left\{ 0, V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu', z) \right\} \quad (41)$$

where the risky firm's non-default value function $V_+^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z)$ is given by:

$$V_+^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z) = \max_{k' \geq 0} \left\{ z \left(\left[[k]^\theta [h]^{1-\theta} \right]^\alpha + \kappa \right) + (1 - \delta)k - w^R h - [1 - q^R]B - k' + V_-^R(k', K_+^S, K_+^R, N', \nu') \right\} \quad (42)$$

and the risky firm's initial-subperiod value function $V_-^R(k, K^S, K^R, N, \nu)$ is given by:

$$V_-^R(k, K^S, K^R, N, \nu) = \max_{h \geq 0} \left\{ q^S \cdot \int_0^1 \left[\nu \cdot V_+^R(k, h, K^S, K^R, N, \nu, \nu', 0) + (1 - \nu) \cdot V_+^R\left(k, h, K^S, K^R, N, \nu, \nu', \frac{1}{1 - \nu}\right) \right] \cdot f(\nu', \nu) d\nu' \right\}. \quad (43)$$

Below the subsequent analysis is focused on the case in which (i) firm i 's current capital $k(i)$ is smaller than debt B so that the firm should default if hit by the jump-default shock $z(i) = 0$ and (ii) $k(i)$ is not too small relative to B such that the firm should not default in the event of drawing positive productivity $z(i) > 0$. That is, more attention is paid to a particular set of equilibrium outcomes for a range of the firm-level debt-to-capital ratio so that given fluctuations in ν_t , (equilibrium) default-rate dynamics for risky bonds in this economy should be the same as in the previous baseline economy. In this case, safe firm i never defaults, and risky firm i defaults if and only if hit by the jump-default shock $z_t(i) = 0$ as was the case in the baseline model.

First, decision rules for hiring labor for safe and risky firms are analyzed. Given the current-period capital holdings k^S , safe firm i 's optimal choice of labor h^S is given by:

$$\alpha(1 - \theta)[k^S]^{\alpha\theta}[h^S]^{\alpha(1-\theta)-1} = w^S. \quad (44)$$

Similarly, given the current-period capital k^R , risky firm i 's optimal choice of labor h^R is given by:

$$\frac{1}{1 - \nu}\alpha(1 - \theta)[k^R]^{\alpha\theta}[h^R]^{\alpha(1-\theta)-1} = w^R \quad (45)$$

which is simplified, when combined with the equilibrium condition $w^S = [1 - \nu] \cdot w^R$, to:

$$\alpha(1 - \theta)[k^R]^{\alpha\theta}[h^R]^{\alpha(1-\theta)-1} = w^S. \quad (46)$$

As shown above, J -type firm i 's optimal choice of labor h^J is a function of its current-period capital k^J and the wage rate for a safe firm w^S :

$$h^J = \left[\frac{\alpha(1 - \theta)}{w^S} \right]^{\frac{1}{1-\alpha(1-\theta)}} [k^J]^{\frac{\alpha\theta}{1-\alpha(1-\theta)}}, \quad \forall J \in \{S, R\}.$$

Next, a firm's investment decision is analyzed. Safe firm i 's optimal decision for its next-period capital k_+^S is given by:

$$1 = q^S \cdot \left[1 - \delta + \alpha\theta[\alpha(1 - \theta)]^{\frac{\alpha(1-\theta)}{1-\alpha(1-\theta)}} [k_+^S]^{\frac{-(1-\alpha)}{1-\alpha(1-\theta)}} [w_+^S]^{\frac{-\alpha(1-\theta)}{1-\alpha(1-\theta)}} \right] \quad (47)$$

which corresponds to the usual Euler equation for a firm's problem where the firm discounts future cash flows by the risk-free rate. Similarly, risky firm i 's optimal choice for its next-period capital k_+^R is given by:

$$1 = q^S \cdot \left[[1 - \nu'][1 - \delta] + [1 - \nu'] \frac{1}{1 - \nu'} \alpha\theta[\alpha(1 - \theta)]^{\frac{\alpha(1-\theta)}{1-\alpha(1-\theta)}} [k_+^R]^{\frac{-(1-\alpha)}{1-\alpha(1-\theta)}} [w_+^S]^{\frac{-\alpha(1-\theta)}{1-\alpha(1-\theta)}} \right]. \quad (48)$$

The above equation implies that a risky firm's decision rule for its next-period capital k_+^R is independent of its current-period capital k^R , from which it follows that the above optimality condition for investment is applied to a survived risky firm as well as to an entrant risky firm. Therefore, the initial-subperiod value of an entrant risky firm is given by:

$$V^E(k^R, K^S, K^R, N, \nu) \equiv \underbrace{q^R B - k^R}_{\text{instant dividend payout}} + \underbrace{V_-^R(k^R, K^S, K^R, N, \nu)}_{\text{continuation value for a risky firm}} \quad (49)$$

which will be used, in the later section of quantitative analysis, to calibrate χ , the costs of entry for a risky firm, so that the entry cost equals the value of an entrant risky firm in the steady state: $\chi = V^E(K^{R*}, K^{S*}, K^{R*}, N^*, \nu^*)$ where ‘*’ in the superscript denotes the steady state.

Combining the above Euler equations for safe and risky firms, I write the condition for capital allocation, one of the key factors in determining aggregate productivity in this economy, as:

$$\left[\frac{k_+^R}{k_+^S} \right]^{\frac{-(1-\alpha)}{1-\alpha(1-\theta)}} = \frac{\nu'[1-\delta]}{[1/q^S] - [1-\delta]} + 1. \quad (50)$$

As shown by the above condition for capital allocation, it is obvious that given $0 < q^S < 1$, k_+^R/k_+^S is less than one and decreasing in ν' holding q^S to constant. That is, capital is reallocated away from risky firms and toward safe firms as the risky-firm’s default probability ν' increases holding the price of safe bonds to constant.

The Household’s Problem The household’s problem is essentially the same as for the baseline model. As discussed in the previous baseline model, expected returns should be equalized between safe and risky bonds. As discussed earlier, the ex-post (gross) return to the risky bond equals $(1-\tau)(1-\delta)K^R/B$ in the event of default and $1/q^R$ in the event of non-default, respectively. The equalization of expected returns between risky and safe bonds results in the equilibrium risky-bond-pricing formula given the the safe-bond price as follows:

$$\frac{1}{q^S} = [1-\nu] \frac{1}{q^R} + \nu \frac{(1-\tilde{\tau})(1-\delta)K^R}{B}. \quad (51)$$

Lastly, the safe-bond price q^S will be determined by the usual Household’s Euler equation.

5.2 Equilibrium

I study a recursive equilibrium for this economy, which is a list of value functions $(V_+^S(k, h, K^S, K^R, N, \nu, \nu'), V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu'), V_-^S(k, K^S, K^R, N, \nu), V_+^R(k, h, K^S, K^R, N, \nu, \nu', z), V_+^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z), V_-^R(k, K^S, K^R, N, \nu))$; policy functions $(h^S(k, K^S, K^R, N, \nu), h^R(k, K^S, K^R, N, \nu), k_+^S(k, h, K^S, K^R, N, \nu, \nu'), k_+^R(k, h, K^S, K^R, N, \nu, \nu', z), c(b, h, K^S, K^R, N, \nu, \nu'))$; price functions $(q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu), w^R(K^S, K^R, N, \nu))$; law-of-motion functions $(K_+^S(K^S, K^R, N, \nu, \nu'), K_+^R(K^S, K^R, N, \nu, \nu'))$; total transfer function

$\Pi(K^S, K^R, N, \nu, \nu')$ that satisfy the following conditions:

1. Optimality:

- (a) Safe firm: the value functions $\left(V_+^S(k, h, K^S, K^R, N, \nu, \nu'), V_+^{S,n}(k, h, K^S, K^R, N, \nu, \nu'), V_-^S(k, K^S, K^R, N, \nu) \right)$ solve the Bellman equations for a safe firm's problem, and policy functions $\left(h^S(k, K^S, K^R, N, \nu), k_+^S(k, h, K^S, K^R, N, \nu, \nu') \right)$ are the optimal decision rules for the safe firm's problem where the safe firm takes as given the price functions $\left(q^S(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu) \right)$.
- (b) Risky firm: the value functions $\left(V_+^R(k, h, K^S, K^R, N, \nu, \nu', z), V_+^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z), V_-^R(k, K^S, K^R, N, \nu) \right)$ solve the Bellman equations for a risky firm's problem, and policy functions $\left(h^R(k, K^S, K^R, N, \nu), k_+^R(k, h, K^S, K^R, N, \nu, \nu', z) \right)$ are the optimal decision rules for the risky firm's problem where the risky firm takes as given the price functions $\left(q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^R(K^S, K^R, N, \nu) \right)$.
- (c) Household: $h = \lambda \cdot h^S(K^S, K^S, K^R, N, \nu) + N \cdot h^R(K^R, K^S, K^R, N, \nu)$ is the household's optimal decision rule, at the initial supperiod, on labor supply, $c(b, h, K^S, K^R, N, \nu, \nu')$ on consumption, and $b' = [\lambda + N']B > 0$ on savings where the household takes as given the total transfer function $\Pi(K^S, K^R, N, \nu, \nu')$ and price functions $\left(q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu), w^R(K^S, K^R, N, \nu) \right)$.

2. Markets clear: for all (K^S, K^R, N, ν, ν') , the following condition holds:

$$\begin{aligned} c(b, h, K^S, K^R, N, \nu, \nu') + \lambda K_+^S(K^S, K^R, N, \nu, \nu') + N' K_+^R(K^S, K^R, N, \nu, \nu') + \chi M \\ = y(K^S, K^R, N, \nu, \nu') + (1 - \delta) \left[\lambda K^S + N[1 - \nu\tilde{\tau}]K^R \right] \end{aligned} \quad (52)$$

where $b \equiv [\lambda + N]B$ and $h \equiv \lambda \cdot h^S(K^S, K^S, K^R, N, \nu) + N \cdot h^R(K^R, K^S, K^R, N, \nu)$.

3. Consistency:

(a) Law-of-motion:

$$k_+^S(K^S, K^S, K^R, N, \nu, \nu') = K_+^S(K^S, K^R, N, \nu, \nu'), \quad \forall (K^S, K^R, N, \nu, \nu'), \quad (53)$$

$$k_+^R(K^R, K^S, K^R, N, \nu, \nu', z = \frac{1}{1 - \nu}) = K_+^R(K^S, K^R, N, \nu, \nu'), \quad \forall (K^S, K^R, N, \nu, \nu'). \quad (54)$$

- (b) Total transfer function: $\Pi(K^S, K^R, N, \nu, \nu')$ is consistent with decision rules of safe and risky firms for all (K^S, K^R, N, ν, ν') .

Capital Structure And Reallocation of Capital This subsection analyzes the impact of capital structure on the change in the allocation of capital between safe and risky firms, and thereby the change in TFP, in response to an increase in ν' . More attention will be paid to the issue of how the response of k_+^R/k_+^S to an increase in ν' differs between this economy of the internal financing plus rolled-over debt and the baseline economy of 100% debt financing. For this purpose, the derivative of k_+^R/k_+^S w.r.t. ν' is derived, and its absolute value is computed; it is given, for this economy, by:

$$\left| d\left(\frac{k_+^R}{k_+^S}\right) / d\nu' \right| = \left[\frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \right] \frac{1 - \delta}{[1/q^S] - [1 - \delta]} \quad (55)$$

and the counterpart in the baseline economy is given by:

$$\left| d\left(\frac{k_+^R}{k_+^S}\right)^{BE} / d\nu' \right| = \left[\frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \right] \frac{\tau[1 - \delta] + \delta}{[1/q^S]^{BE} - 1} \quad (56)$$

where the superscript ‘BE’ denotes variables in the baseline economy³⁸.

PROPOSITION 3. *Consider the case in which δ is sufficiently small and the default-cost distortion for the baseline economy is less than one $\tau^{BE} \in [0, 1)$, i.e., the positive recovery rate. In this case, the magnitude of the derivative of k_+^R/k_+^S w.r.t. ν' is larger in this economy of internal financing combined with rolled-over debt than in the baseline economy of 100% debt financing if risk-free rates are the same between the two economies:*

If $\tau^{BE} \in [0, 1)$ and $1/q^S = [1/q^S]^{BE}$, then $\exists \bar{\delta} > 0$ s.t.

$$\left[\forall \delta \in [0, \bar{\delta}), \quad \left| d\left(\frac{k_+^R}{k_+^S}\right) / d\nu' \right| > \left| d\left(\frac{k_+^R}{k_+^S}\right)^{BE} / d\nu' \right| \right]. \quad (57)$$

Proof. Assume that $\tau^{BE} \in [0, 1)$ and that $1/q^S = [1/q^S]^{BE}$. Consider the case of $\delta = 0$. It immediately follows that:

$$\left| d\left(\frac{k_+^R}{k_+^S}\right) / d\nu' \right| = \frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \frac{1}{[1/q^S] - 1} > \frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \frac{\tau^{BE}}{[1/q^S]^{BE} - 1} = \left| d\left(\frac{k_+^R}{k_+^S}\right)^{BE} / d\nu' \right|. \quad (58)$$

³⁸I use $[1/q^S]^{BE} \equiv 1 + r^S$ for the baseline economy.

The absolute value of the derivative of k_+^R/k_+^S w.r.t. ν' is continuous in δ both for this economy and for the baseline economy. \square

Results of Proposition 3 imply that the magnitude in the response of TFP to an increase in ν will be larger in this economy relative to the baseline economy because aggregate output, given total inputs, is strictly increasing in $[k^R/k^S] \in (0, 1)$ for both of the two economies holding the number of risky firms, N_t , to constant. Here key is that the marginal cost of capital for a risky firm conditional on being hit by the jump-default shock is higher in this economy than in the baseline economy. Both in this economy and in the baseline economy, a risky firm hit by the jump-default shock defaults. In this economy, the risky firm self-finances its marginal investment and therefore loses, upon default, 100% of its marginal (undepreciated) capital; therefore, roughly speaking, the marginal cost of capital for the risky firm increases by one unit in response to one unit increase in ν . Meanwhile, in the baseline economy, the bondholder provides a risky firm with capital and loses $\tau^{BE} < 1$ fraction of (undepreciated) capital in the event of default, which is less than 100% loss of equity holders in this economy as discussed above; thus, roughly speaking, the marginal cost of capital for the risky firm increases by $\tau^{BE} < 1$ units in response to one unit increase in ν in the baseline economy.

In short, an increase in the marginal cost of capital for a risky firm in response to an increase in its default probability ν is larger in the case of internal financing relative to the debt financing³⁹. As a result, the magnitude of capital reallocation in response to an increase in ν is larger in this economy relative to the baseline economy, and so are changes in TFP and output, which will be discussed in more detail in the later section of simulation results.

5.3 Calibration

This section discusses how to calibrate parameter values for the internal-financing model, labeled as either the *Internal Financing* model or simply *IF* model, so that the model should match long-run averages of key aggregates for the U.S. economy over the period 1964-2009. One period in the model corresponds to one quarter in the data. The benchmark parameter values are listed in table 12.

One set of parameters $(\nu, \beta, \alpha, \delta, \theta, \omega, \psi)$ in Panel A of table 12 are held the same as for the benchmark values used for the baseline model. The other set of parameters $(\lambda, M, \tau, B, \kappa, \chi)$ in Panel B of table 12 are calibrated based on implications of the internal financing model.

First, the mass of safe firms λ is set to 0.477 to match the size of newly issued safe bonds relative to the total bonds as done for the baseline-model calibration. As the bond size is the

³⁹Note that the above result in this paper is obtained even in the absence of tax advantage of debts.

Table 12: Benchmark Parameter Values: Internal Financing Model

PARAMETER	VALUE	TARGET
Panel A: Same as for the Baseline Model		
ν : Prob. of $z(i) = 0$ for Risky Firm i	.0048	Default Rate of Risky Bonds
β : Discount Factor	.9909	Real Return to Aaa Bonds
α : Returns to Scale	.87	Khan and Thomas (2011)
δ : Depreciation Rate	.015	Benchmark
θ : Capital Share	.33	Investment/GDP
ω : Curvature of Labor-Disutility	.30	Benchmark
ψ : Level of Labor-Disutility	3.32	$h_{ss} = .3333$
Panel B: Unique to the Internal Financing Model		
λ : Measure of Safe Firms	.477	Volume-Share of Safe Bonds
M : Constant Measure of Risky-Entrant Firms	.00251	$N = 1 - \lambda$
$\tilde{\tau}$: Liquidation Costs	.30	Ramey and Shapiro (2001)
B : Size of Debt	14.79	Recovery Rate of Risky Bonds
κ : A Firm's Expected Endowment	.354	Leverage Ratio
χ : Creation & Entry Costs of a Risky Firm	35.20	Value of an Entrant Risky-Firm

same between risky and safe firms in this economy, λ is simply equal to the relative size of safe bonds to the total, which is 0.477, when the steady state mass of risky firms N is normalized to $(1 - \lambda)$. Then $M = .00251$ is chosen because $M = \nu \cdot N$ where $N = 1 - \lambda$.

Second, the per-capital liquidation cost $\tilde{\tau}$ is set to .30 as benchmark, which basically captures the high discount rates applied to the market price of a liquidated asset. $\tilde{\tau}$ is calibrated based on estimates of the discount rate for a liquidated asset reported in the literature. There is a wide range of estimates of such discount rates. For instance, Ramey and Shapiro (2001) estimate that the “ratio of sales price to replacement cost” is about 10-40%, which amounts to 60-90% for $\tilde{\tau}$ in the IF model. Taking into consideration that Ramey and Shapiro (2001)'s sample is limited to a number of, but not the entire, industries and that liquidation does not necessarily involve reemployment of capital, e.g., selling off the equity of the defaulting firm to other investors, I choose $\tilde{\tau} = .30$ as benchmark.

Third, the constant size of rolled-over debt B is chosen by targeting the recovery rate in the event of default. Recall that the bondholders receives $[1 - \tilde{\tau}][1 - \delta]k^R / B$, which corresponds to the recovery rate, per unit of defaulted risky-bond via liquidation process. The recovery rate for risky corporate bonds is again about 40% on average during the period 1982-2009. Given (τ, δ) and the model-implied capital held by a risky firm k^R , $B = 14.79$ is chosen to match the average recovery rate for risky corporate bonds of 40%.

Fourth, the expected firm-level endowment κ is set as twice large as the interest expenses for a survived risky firm, which results in that the leverage ratio, i.e., the ratio of debt to the market

value of equity plus debt, is 33% for a survived risky firm, 20% for a safe firm and 26-27% in either aggregate or cross-sectional average, which is broadly in line with the data⁴⁰.

5.4 Results

This section discusses simulated results of the IF model, see table 13.

Table 13: Statistics of Key Aggregates: Internal Financing Model

x	$\sigma(y)$	$\sigma(x)/\sigma(y)$					$corr(x, y)$				
	y	h	k	TFP	c	i	h	k	TFP	c	i
Data	1.56	1.27	0.34	0.53	0.54	3.35	0.88	0.15	0.45	0.84	0.91
Baseline	0.93	0.74	0.74	0.66	0.59	5.32	0.81	0.18	0.86	0.37	0.88
IF	1.34	0.98	0.38	1.07	1.02	3.53	0.06	0.44	0.84	0.75	0.54

Note: The line headed ‘IF’ refers to the internal financing model and the line headed ‘Baseline’ does the same for the baseline model of 100% debt financing. $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable x to that of output $\sigma(y)$ and $corr(x, y)$ refers to the correlation coefficient between x and output. All variables refer to their log deviations.

The main message of table 13 is that shocks to ν account for quite a large part of fluctuations in output and TFP. Thus, the availability of the internal-financing does not overturn the main message of the earlier simulated results of the baseline model: fluctuations in the corporate credit spread, mainly driven by shocks to the default risk for risky firms, are an important source of fluctuations in the U.S. economy.

One caveat in interpreting the above results of the IF model is that shocks to current ν_t affects directly the next-period aggregate output y_{t+1} via changes in the measure of risky firms in the next-period N_{t+1} holding all else constant:

$$y_{t+1} = \lambda \left[[k_{t+1}^S]^\theta [h_{t+1}^S]^{1-\theta} \right]^\alpha + N_{t+1} \left[[k_{t+1}^R]^\theta [h_{t+1}^R]^{1-\theta} \right]^\alpha + \lambda \kappa + N_{t+1} \kappa$$

where the last term $N_{t+1}\kappa$ refers to endowments earned by risky firms, which are under the direct influence of changes in ν_t through the changes in N_{t+1} because of $N_{t+1} = (1 - \nu_t)N_t + M$.

Below I discuss how to decompose changes in output and TFP so that in the IF model, effects of shocks to ν on output and TFP via *endogenous* changes in resource-allocation are disentangled

⁴⁰In the model, as a safe firm has no default risk, in the absence of taxes on corporate incomes, internal cash flows and debts make no difference for a safe firm’s cost of capital, i.e., the Modigliani-Miller theorem holds. As a result, in the model, the leverage ratio is meaningful for a risky firm but not for a safe firm. In the COMPUSTAT data for publicly traded firms, the leverage ratio measured as the book value of total liability relative to the market value of total assets is about 37% for both risky and safe firms, comparable the above 33% leverage ratio for a risky firm in the model. For a firm’s riskiness, volatility of the firm’s own stock returns is used, and the cross-sectional median value of this riskiness is used as the criterion for the classification of firms into either risky or safe.

from effects of *exogenous* changes in endowment $N_{t+1}\kappa$ on output and TFP. To control for effects owing to changes in $N_{t+1}\kappa$, alternative measures of output and TFP are proposed as follows:

$$\tilde{y}_t \equiv y_t - [N_t - N_{ss}]\kappa, \quad \widetilde{TFP}_t \equiv TFP_t \cdot \frac{\tilde{y}_t}{y_t}.$$

where \tilde{y}_t measures changes in output induced by the endogenous changes in resource-allocation and \widetilde{TFP}_t does the same for TFP. Then percentage standard deviations of \tilde{y}_t and \widetilde{TFP}_t are presented in table 14.

Table 14: Percentage Standard Deviations of Output and TFP

Output			TFP		
Data	Baseline	IF: \tilde{y}	Data	Baseline	IF: \widetilde{TFP}
1.56	0.93	1.02	0.83	0.60	0.75

Note: \tilde{y}_t measures changes in output induced by the endogenous changes in resource-allocation in the IF model and \widetilde{TFP}_t does the same for TFP. All variables refer to their log deviations.

As shown by table 14, effects of shocks to ν on output and TFP in the IF model are still sizable compared to the data even after changes in the endowment term, $N_t \cdot \kappa$ are controlled for; moreover, those effects are slightly larger compared to results of the baseline economy of 100% debt financing. The key in generating the difference in quantitative results for output and TFP between the IF model and the baseline model is that the marginal cost of capital for a risky firm responds to an increase in ν more sensitively in the IF model than in the baseline model (recall results of Proposition 3). As a result, capital allocation k^R/k^S is more sensitive to an increase in ν in the IF model than in the baseline model, see table 15.

Table 15: Percentage Standard Deviation of Capital Allocation k^R/k^S

Baseline Model	IF Model
39.1	53.5

Table 15 highlights that as predicted in Proposition 4, capital allocation k^R/k^S indeed responds to ν -shocks more sensitively in the internal financing model than in the baseline model.

Lastly, volatilities of the (equity) value for risky and safe firms are discussed, see table 16.

As shown in table 16, the equity value for a risky firm is as about twice volatile as that for a safe firm. Risky firms are exposed to time-varying default-risk, resulting in the large volatility of the risky-firm's equity value. By contrast, a safe firm has no such risk. Therefore, the equity value should be more volatile for a risky firm relative to a safe firm.

Table 16: Percentage Standard Deviation of the Market Value of Equity

Risky Firm	Safe Firm
10.1	5.2

Note: The market value of equity is measured at the final subperiod for a survived risky and safe firms.

All variables refer to their log deviations.

6 Conclusion

This paper studies corporate credit spreads and business cycles in a general equilibrium model. Motivated by the fact that corporate credit spreads are significantly countercyclical, this paper focuses on addressing the question of what are causes and consequences of an increase in the corporate credit spread. A simple model of corporate credit spreads is developed; in the model, firms finance their capital by issuing non-contingent one-period bonds. The key feature of the model is that firms are exposed to the different level of default risk, which with the default-cost distortion results in the credit spread too high and resource misallocation between the high-risk and low-risk firms. In the model, an increase in the default risk for high-risk firms causes, even though every firm's expected productivity held constant, both output and TFP to drop because the extent of the resource misallocation becomes larger due to the increased credit spread between high-risk and low-risk firms.

This paper is the first quantitative study to incorporate the default rate dynamics of corporate bonds by credit ratings into the standard growth model. More specifically, the earlier key mechanism is embedded into an otherwise standard growth model, which is then calibrated and simulated to data on the U.S. economy and corporate bond market during the period 1964-2009. Simulated results show that fluctuations in corporate credit spreads, mainly driven by shocks to the default risk for high-risk firms, account for a large part of fluctuations in output and TFP. Note that in this paper, the sizable endogenous fluctuations in output and TFP are obtained even in the absence of exogenous shocks to the aggregate productivity; therefore, the model in this paper also sheds some lights on the question of what shocks drive the output fluctuations, one of classical issues discussed in the business cycle literature.

As in the literature, I imposed the restriction that one-period bond is the only available financial instrument for an external debt financing. Simulated results for the benchmark case of 100% short-term debt financing are quite similar even for the case in which firms use internal financing for their investments and roll over their constant size of one-period bond. It would be interesting, even though beyond the scope of this paper, to extend my analysis to the case in which firms can adjust the maturity structure of their long-term bonds.

Lastly, the household's portfolio in this paper is restricted to securities issued by domestic-

private-production firms. More generally, the household could also invest to public and foreign sectors as well as the domestic-private-production sector. In this case, fluctuations in corporate credit spreads would still imply significant fluctuations in output via more the channel of aggregate investment and less the TFP channel. In addition, with the availability of the third security, the government bond in particular, factors other than default risk would be also important in determining the allocation of capital to and within domestic-private-production sector: for instance, liquidity premium, taxes, foreign-exchange-risk premium and so on. It is important, although left for future work, to study how such non-default-risk factors would interact with default-risk in determining the corporate credit spread and allocation of capital across firms.

7 Appendix

7.1 Mathematical Appendix

First, proof of Lemma 1 is as follows.

Proof. Combining the first order conditions of risky and safe firm w.r.t. hiring labor with the equilibrium wage rate condition $[1 - \nu]w^R = w^S$, I obtain: $\frac{h^R}{h^S} = \left[\frac{k^R}{k^S}\right]^{\frac{\alpha\theta}{1-\alpha(1-\theta)}}$. The above labor-ratio of a risky firm relative to a safe firm is substituted to the capital-ratio equation of a risky firm relative to a safe firm: $\frac{k^R}{k^S} = \left[\frac{[1-\nu]r^R}{r^S}\right]^{-\frac{1-\alpha(1-\theta)}{1-\alpha}}$. Lastly, $\frac{[1-\nu]r^R}{r^S} = 1 + \frac{\tau\nu[1-\delta]}{r^S}$ is derived from the equalization of the expected returns between risky and safe bonds. \square

Second, proof of Proposition 1 is as follows.

Proof. It immediately follows from results of Lemma 1. Given r^S , it is obvious that r^R is increasing in ν from the equilibrium condition for equalization of the expected returns between safe and risky bonds. Next, differentiating k^R/k^S w.r.t. ν results in:

$\frac{dk^R/k^S}{d\nu} = -\frac{1-\alpha(1-\theta)}{1-\alpha} \frac{\tau[1-\delta]}{r^S} \left[1 + \frac{\tau\nu[1-\delta]}{r^S}\right]^{-\frac{1-\alpha(1-\theta)}{1-\alpha}-1} < 0$ given r^S . As h^R/h^S is strictly increasing in k^R/k^S , it is obvious that $d[h^R/h^S]/d\nu < 0$. Lastly, $dy/d\nu < 0$ follows from that given aggregate capital and labor, aggregate output y is an increasing in $\forall[k^R/k^S] < 1$ because y is maximized at $[k^R/k^S] = 1$. \square

Last, proof of Proposition 2 is as follows.

Proof. It is essentially the same as for the proof of Proposition 1. Differentiating k^R/k^S w.r.t. τ results in: $\frac{dk^R/k^S}{d\tau} = -\frac{1-\alpha(1-\theta)}{1-\alpha} \frac{\nu[1-\delta]}{r^S} \left[1 + \frac{\tau\nu[1-\delta]}{r^S}\right]^{-\frac{1-\alpha(1-\theta)}{1-\alpha}-1} < 0$ given r^S . Other parts immediately follow. \square

7.2 Data Sources

COMPUSTAT Variables Investment is measured as expenditures on ‘Property, Plant and Equipment Capital’ (COMPUSTAT data item #30) plus ‘Acquisition’ (item #129) so that my measure of investment captures both intensive margin and extensive margin of capital increase well. For robustness check, I also use the item #30 only as in the literature and obtain the same message (available in the online appendix). The book value of assets is the item #6.

Next, for additional firm characteristics, marginal product of capital mpk is measured as the ratio of net sales (item #12) to capital stock (item #8), and leverage ratio lvg as the ratio of the book value of total liabilities (item #181) to assets. Cash flows cf is measured as the income

before extraordinary items (item #18) plus depreciation and amortization (item #14), and then expressed as the ratio to the lagged assets. Lastly, Tobin’s Q is measured as the ratio of the market value of assets to the book value of assets where the market value of assets = (item #181) + (item #25) × (item #199) + (item #10)+(item #19). See Covas and Haan (2011) for further discussion.

NIPA Variables I basically use the ‘Table 1.1.3. Real Gross Domestic Product, Quantity Indexes’ of NIPA, which provides quantity indexes for output, consumption and investment where the index for each item is seasonally adjusted and relative to the quarterly average over the year 2005. Then I convert the quantity index to the variables in terms of billions of chained 2005 dollars by using the quarterly average over the year 2005 based on ‘Table 1.1.6. Real Gross Domestic Product, Chained Dollars’.

Moody’s Dataset on Corporate Bond Market The default rate provided by Moody’s is measured by the ratio of the number of issuers defaulted for a year relative to the total number of issuers of the bond-cohorts, which is not a volume-weighted default rate as used by Giesecke et al. (2011) and Altman and Karlin (2010). I measure the default rate of risky bonds as the weighted average of default rates for the Baa, Ba and B grade bonds where the number-of-issuers shares by letter grades are used as the weights. Similarly, the default rate of safe bonds is measured as the weighted average of default rates for the Aaa, Aa and A grade bonds.

I next present the number-of-issuers shares of corporate bonds by their credit grades.

Table 17: Percentage Shares of Corporate Bonds During the Period 1997-2000

Aaa	Aa	A	Baa	Ba	B	Caa-C
3.3	13.2	25.4	21.9	13.7	18.5	4.0

Note: shares of corporate bonds are measured by number of issuers rather than by the bond-volumes.
Source: Hamilton (2001), the exhibit 11 on page 13.

Lastly, I discuss how recovery rates are measured and present summary statistics of them. Recovery rate of a corporate bond is measured by the post-default price of that corporate bond relative to the price of the corporate bond in some years prior to default. The recovery rates reported in the above table are averages of them measured in 1, 2, 3, 4, and 5 years prior to default (see Moody’s (2010) for more detailed discussion).

The BofA Merrill Lynch Corporate-Bond Yields Downloadable from:
<http://research.stlouisfed.org/fred2/categories/32347>.

7.3 Calibration

Recovery Rate The recovery rate for risky corporate bonds in the data is calculated as a weighted average of recovery rates for Baa, Ba and B grade bonds, which results in about 41.3% on average over the period 1982-2009. Taking into consideration that Caa-C grade bonds are omitted, of which recovery rates are about 34% on average for the same period, I adjust the above 41.3% recovery rate slightly downward to 40% and take it as my estimate of the long-run recovery rates of the risky corporate bonds.

Below I discuss how to measure the recovery rate in the model economy as in the data. In the model, one unit of defaulted risky bond returns $[1 - \delta][1 - \tau]$ units of undepreciated capital: the price of defaulted risky bond is, in terms of the final good, $[1 - \delta][1 - \tau]$. Meanwhile, the price of a risky bond prior to default equals one over the promised gross return $1/[1 - \delta + r^R]$. As in the data, recovery rate for a risky bond is measured as the ratio of defaulted risky-bond price relative to the prior-to-default price: $\text{recovery rate} = [1 - \delta][1 - \tau] / [1/[1 - \delta + r^R]]$.

Bond Size of Safe vs. Risky Bonds Next, I describe the procedure for calculating the volume share of safe bonds which consist of Aaa, Aa and A grade bonds, or the investment-grade excluding Baa grade bonds. Note that the volume share of the speculative-grade bonds in the U.S. newly-issued, per year, corporate bond market is about 27.5% on average during the period 1993-2009 according to Altman and Karlin (2010). Therefore, the volume share of the investment-grade bonds is about 72.5% on average in this period because corporate bonds are categorized as either investment-grade or speculative-grade. The Baa grade corporate bonds account for, on average, about 34.25% of the number of outstanding investment-grade bond-issuers during the period 1997-2000 according to Hamilton (2001). Assuming that volume shares are proportional to the number-of-issuers shares for the investment-grade bonds, I compute the volume-share of safe bonds as: $0.725 \times [1 - 0.3425] = 0.4767$.

Shock Process for ν I discretize the space of ν by partitioning the interval $(0, 0.04)$ by an equal distance of 0.001 and then set the lower bound ν_1 to 0.00025. Note that the maximum quarterly default rate for a risky bond is 4% in the calibrated model, larger than the counterpart in the data of 2%.

For discretization of the continuous process for ν , the uniform distribution $f^U(\nu'|\nu)$ is represented by the probability assigned to a single point, the minimum level $\nu_1 = 0.025\%$, which is enough for my purpose to capture the default rate clustered around zero as in the data. The truncated normal distribution $f^{TN}(\nu'|\nu)$ is discretized to the transition matrix $\pi^{TN}(\nu', \nu)$. Lastly, the state-dependent weight on the uniform distribution $\phi(\nu)$ is simplified such that $\phi(\nu) = \phi_1$

for $\nu = \nu_1$ and $\phi(\nu) = \phi_2$ for $\nu > \nu_1$ so that there are essentially two regimes, which eventually leads to that in the model, the risky-bond default rate is clustered around zero and also mean-reverting.

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