

Fixed Costs, Retirement and the Elasticity of Labor Supply*

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Abstract

We analyze the forces that can generate retirement in different versions of standard life cycle models of labor supply. While nonconvexities in production can generate retirement, we show that the size of nonconvexities needed increases sharply as the intertemporal elasticity of substitution for labor decreases.

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1. Introduction

A large literature uses life cycle profiles of hours and wages to estimate the intertemporal elasticity of substitution for labor supply, which we henceforth refer to as the *IES*. Virtually all of this literature restricts attention to the prime-age portion of the life cycle, and most of it has concluded that the *IES* is quite small.¹ In this paper we argue that retirement, specifically the direct movement of a worker from full time work to little or no work, also contains important information about the size of the *IES*. The connection between retirement and the *IES* is intuitive. The *IES* determines the extent to which individuals value a smooth profile for leisure over the life cycle. But since retirement represents a very dramatic change in leisure, the fact that individuals willingly incur such a dramatic change in leisure should provide information about their willingness to intertemporally substitute.

In this paper we explore the connection between retirement and the *IES* in a class of models that feature retirement as an optimal property of life cycle labor supply. As in French (2005) the key element that generates retirement as an optimal property of life cycle labor supply is the presence of nonconvexities. We consider three different sources of nonconvexities that the literature has emphasized: fixed time and consumption costs associated with work, and nonlinear wage-hours schedules. We derive simple closed form expressions that link the extent of nonconvexities, the level of hours worked just prior to retirement, and

¹See for example the early papers by MaCurdy (1981) and Altonji (1986), both of which produce small estimates. A notable exception is Imai and Keane (2004), who allow for human capital accumulation. We discuss this feature more later on in the paper.

the value of the *IES* that are consistent with retirement representing optimal behavior on the part of the individual. Our key finding is that given empirically reasonable values for the extent of nonconvexities, it is very difficult to rationalize retirement from full-time work with values of the *IES* that are below .75.

Our results are consistent with recent work that has argued for various reasons that the *IES* is significantly larger than suggested by earlier estimates. For example, Domeij and Floden (2006) argue that abstracting from credit constraints leads to a bias that is on the order of a factor two, while Low (2005) argues that precautionary savings motives also creates a negative bias. Imai and Keane (2004) argue that the failure to account for endogenous human capital accumulation can result in a bias that is almost a factor of ten. Chetty (2010) argues that optimization frictions such as adjustment costs could also account for a bias as large as a factor of two. A noteworthy feature of our estimates is that they are robust to the presence of credit constraints for young workers and endogenous human capital accumulation.

Consistent with the previous literature on life cycle labor supply, a key feature of our analysis is that hours are assumed to be a continuous choice variable for individual workers. However, the non-convexities in our analysis can generate outcomes in which certain intervals of work hours are not observed, even though the underlying choice set is continuous. Our goal in this paper is to extend the standard analysis that does assume a continuous hours choice to include the retirement decision and assess how it affects our inference regarding the *IES*. An important issue for future work is to assess the possible role of other elements that

might lead to restricted choice sets, such as firms offering a restricted set of hours choices due to the need to coordinate schedules across workers.

A novel contribution of our analysis is to point out that adjustment along the extensive margin has important information about individual preference parameters. In particular, it is well known that in the extreme case in which individual workers can only choose between full time work (say at 40 hours) and no work, then the individual preference parameter that determines the individual *IES* cannot be identified. Put somewhat differently, models in which all adjustment is along the extensive margin do not allow one to uncover the curvature over leisure in the individual utility function. This has lead many researchers to think that one needs to observe adjustment along the intensive margin in order to estimate parameters of the individual utility function for leisure. We show that adjustment along the extensive margin also provides important information about the individual preference parameter that determines the *IES*.

An outline of the paper follows. In Section 2 we present evidence on the nature of retirement. The key result of this section is to show that for a large set of workers, retirement is characterized as a direct movement from full time work to effectively no work. Section 3 describes the standard life cycle model without any nonconvexities and discusses why it is very difficult to account for retirement in the context of this model, and in particular how the difficulty is inversely related to the *IES*. Section 4 extends the standard model to include various nonconvexities in production and characterizes the level of the *IES* that is consistent with retirement given the degree of nonconvexities. Section 5 presents

sensitivity analysis and Section 6 concludes.

2. Retirement in the Data

At age 60, roughly 60% of men work in excess of 1750 hours over the course of the year, while about 25% do not work at all. By age 70, only about 10% work in excess of 1750 hours, while almost 75% report working zero hours.² In this section we examine the nature of this transition from full time work to not working. Our main finding is that the dominant transition from full time work to not working is an abrupt one, with individuals moving from full time work to little or no work with at most one intermediate value in between. To the extent that individuals may initiate the transition from full time work to little or no work at various points over the year, having one intermediate value is consistent with individuals moving directly from full time work to little or no work.

We begin by presenting pooled cross-section data for annual hours worked in the previous year from the CPS for the years 2002-2004. As we discuss below, a key limitation of this data is that it is cross-sectional rather than panel. Nonetheless, we think it is worth looking at this data before we consider panel data because the latter suffers from a relatively small sample size. By presenting the data from the CPS we can verify that the PSID data remains representative in terms of aggregates. Table 1 presents the distribution of males across hours intervals.³

²As detailed below, these statistics are based on the 2002-2004 CPS.

³While our data analysis will focus on males, we note that the same features that we find for males also hold for the overall population.

Table 1
Distribution of Male Annual Hours by Age, CPS

Age	Annual Hours					
	0	(0, 250)	[250, 750)	[750, 1250)	[1250, 1750)	≥ 1750
60	.26	.02	.03	.05	.05	.59
61	.29	.02	.03	.05	.06	.55
62	.36	.02	.04	.06	.05	.47
63	.43	.02	.04	.06	.05	.39
64	.48	.03	.06	.06	.04	.34
65	.54	.03	.05	.07	.04	.27
66	.59	.03	.05	.06	.04	.22
67	.63	.04	.04	.06	.05	.19
68	.65	.03	.05	.06	.04	.16
69	.69	.03	.04	.05	.04	.15
70	.75	.03	.03	.04	.03	.12

Several features of this table are worth noting. First, as noted earlier, virtually all of the change in the distribution between the ages of 60 and 70 occurs in the $h = 0$ and $h \geq 1750$ bins. The former increases by roughly fifty percentage points, while the latter shrinks by roughly fifty percentage points. The fraction of the population that is accounted for by these two bins exceeds .80 at all ages. While this evidence certainly suggests that movement from full time work to no work is the typical transition as individuals age over this period of life, the fact that it is based on cross-sectional data limits the conclusions that one can draw. There are

two distinct issues. First, the above numbers do not preclude the possibility that individuals are transiting sequentially between the bins over a period of several years. Second, it could be that individuals oscillate between full time work and no work rather than simply transiting from full time to no work. With regard to the first issue, the numbers in Table 1 suggest that this possibility is limited in scope. To see why, note that between the ages of 60 and 65, the fraction of people who move out of full time work (i.e., $h \geq 1750$) is roughly .32. However, looking only at individuals of age 65, the fraction of individuals who have hours between 250 and 1750 is only .16. So even if these individuals are transiting from full time to little or no work, the transition must be happening fairly rapidly. The second point cannot be addressed without panel data.

To address these issues we next consider evidence from the PSID. We use the sample created by Heathcote et al (2009), and focus on male heads of households. We include only those individuals who have data for hours worked for all years from age 60 to age 70. This limits the sample size to 307. We begin by presenting the same statistics as in Table 1.

Table 2
Distribution of Male Annual Hours by Age, PSID

Age	Annual Hours						$\frac{\text{hours}}{\text{worker}}$
	0	(0, 250)	[250, 750)	[750, 1250)	[1250, 1750)	≥ 1750	
60	.09	.01	.03	.04	.11	.72	2036
61	.11	.03	.03	.03	.13	.67	1974
62	.18	.03	.02	.08	.09	.61	1949
63	.25	.04	.05	.09	.06	.52	1860
64	.32	.04	.09	.06	.07	.42	1731
65	.41	.06	.05	.08	.08	.33	1672
66	.50	.09	.10	.09	.06	.17	1249
67	.59	.07	.09	.07	.06	.13	1241
68	.65	.07	.07	.07	.05	.10	1172
69	.69	.06	.06	.07	.03	.10	1129
70	.71	.06	.06	.05	.04	.09	1161

One difference between the two samples is that the PSID has far fewer individuals with zero hours at age 60, and more with hours of 1750 or greater. This probably reflects that fact that the PSID sample begins in 1968, whereas our CPS sample was from 2002-2004. As is well known, labor force participation of older males has declined over this period. Nonetheless, Table 2 exhibits the same key patterns that we found in Table 1. Specifically, the single most important dynamic as individuals age is the movement from hours of 1750 or more to zero hours. While at some ages the PSID data has a greater fraction of individuals

with hours of intermediate levels, for all ages except one the fraction of individuals in the two extreme hours categories exceeds .70.

Having established that the key aggregate movement is a decrease in the fraction working full time and an increase in the fraction with little or no work, we next examine the transitions at the individual level. To do this we focus on those individuals in our sample who have hours of at least 1750 at age 60. This yields a sample of 222. We then ask how many in this group make a direct transition from full time work ($h \geq 1750$) to a state of little or no work ($h < 250$) at ages 69 and 70. To allow for the possibility that this transition may occur during the middle of a calendar year, we allow individuals to have one year of intermediate hours as part of the transition. We find that 151, or 72.2% of the subsample of individuals who worked at least 1750 hours at age 60 fit this criterion.⁴ Recalling that almost 10% of the overall sample is still working at least 1750 hours at age 70, we conclude that the dominant transition for these workers is to move directly from working at least 1750 hours to working less than 250 hours. Table 3 reports some additional statistics of interest for this group of 151 individuals.

⁴We have also considered alternative sample selection rules and criterion. For example, in order to identify persistent full time workers at age 60 we considered individuals who worked at least 1750 hours for at least four of the five years between ages 56 and 60. And then to single out individuals who had transitioned to a persistent state of little or now work we considered individuals who had less than 250 hours at each of ages 68 through 70. In this case the overall sample size was 108 workers, and of these 75% exhibited the direct transition from 1750 or more hours to no more than 250 with at most one intermediate value.

Table 3

	Age 60	Age 69	Age 70
Mean Hours	2152	5	7
Median Hours	2048	0	0

We conclude from Table 3 that to first approximation one can think of the typical retirement transition for those individuals still working full time at age 60 as consisting of an individual moving from working around 2000 hours per year to effectively zero hours. These values will be relevant for the calculations that we consider in the rest of this paper. The last column of Table 2 reports mean hours per worker for all workers with positive hours. It is relevant to note that especially for ages beyond 65, mean hours for all workers is much less than hours for full time workers. Although the amount of part-time work does not change that much over this age range, part-time work is a greater fraction of total work for later ages precisely because many full time workers have transited to no work. Given that our goal is to understand the abrupt transition from full time work to no work that dominates the transitions during this age range, it is important to note that average hours for all workers is not the key statistic of interest.

Blau and Shvydko (2010) find similar behavior using data from the Health and Retirement Survey (HRS). The HRS data also allows them to present some additional correlations of interest. First, while it is true that some of the transitions from full time work to no work are associated with deteriorating health, the majority of these transitions occur for individuals who report being in good health both before and after the transition. Second, they show that abrupt transitions

from full time work to no work are the dominant pattern even among workers who do not have defined benefit pension plans and have retiree health benefits. So while it is the case that health status and pension benefits may play a role in the labor supply decisions for some older workers, abrupt transitions from full time work to no work seem to be a more pervasive phenomenon.

3. Retirement in the Standard Life Cycle Model

In this section we consider a standard life cycle model of labor supply, by which we mean an individual who maximizes a time separable and strictly concave utility function subject to a convex budget set. We show why it is very difficult for this model to generate retirement when standard functional forms are used. While this result is probably not surprising to anyone familiar with these models, it is useful to consider the issue of retirement in this model since it allows us to focus on a key tension that will also be present in more complex settings considered later. Based on the data presented in the previous section, we use the term retirement to describe the case of an individual who chooses a large, abrupt and persistent decrease in their hours of work following a lengthy period of full time work. The most extreme form of this phenomenon is the case in which a worker who has worked full time for thirty or more years chooses to move from full time market work in one period to no market work in all subsequent periods. For ease of exposition, in what follows we will focus almost exclusively on the extreme form of retirement, where a worker moves from full time work to no work, though we will report one illustrative calculation to show that generating a phased transition

into retirement faces similar issues.

The standard life cycle model necessarily generates a motive to smooth consumption and leisure over time. Movements in relative prices can induce individuals to choose profiles in which consumption and leisure (and hence work hours) change over time, but in the face of constant prices, and assuming that the interest rate exactly offsets the agent's discounting, the individual will choose constant sequences for consumption, leisure and work hours. Viewed from the perspective of this standard model, retirement is a puzzling phenomenon, since it represents anything but a smooth profile for leisure and work.

To facilitate discussion it is useful to consider a specific model, purposefully simplified so as to make the main points more transparent. We consider the perfect foresight utility maximization problem of a finitely lived individual who faces markets for labor and consumption, and is allowed to borrow and lend freely. For now we assume that there are no policies in place that involve taxes or transfers, either explicitly or implicitly. That is, there is no social security and there is no private pension plan. Let c_t and h_t denote consumption and hours of work at age t , and normalize the total time endowment to equal one each period, so that leisure at age t is given by $1 - h_t$. We assume that the individual has preferences described by:

$$\sum_{t=0}^T [u(c_t) + \alpha_t v(1 - h_t)] \tag{3.1}$$

where T is the length of life, assumed to be known with certainty. For now we assume preferences are separable between consumption and leisure; later on in the paper we show that this is not important for our results. Both u and v

are assumed to be twice continuously differentiable, strictly increasing, strictly concave and have infinite derivative at 0. The parameter α_t is included to allow for the possibility that the marginal rate of substitution between consumption and leisure changes with age. To simplify exposition we have assumed that the individual does not discount the future, but will also assume that the interest rate is zero.⁵ The present value budget equation faced by this individual is given by:

$$\sum_{t=0}^T c_t = \sum_{t=0}^T w_t h_t + Y \quad (3.2)$$

where Y is the present value of non-labor income for the individual.

Letting μ be the Lagrange multiplier on the budget equation and assuming an interior solution, the first order condition for h_t is:

$$\alpha_t v'(1 - h_t) = \mu w_t \quad (3.3)$$

If the solution for h_t is interior, the optimal solution for h_{t+1} is interior if and only if the following inequality holds:

$$v'(1) < v'(1 - h_t) \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t} \quad (3.4)$$

⁵Alternatively, we could assume that the individual discounts at a positive rate but that the interest rate is positive and perfectly offsets this discounting. All of our analysis would carry over to this case, but the algebra is somewhat simpler in the zero discounting case. More generally, we could assume that the interest rate and discount factor are not perfectly offsetting. This induces slopes to the life cycle profiles for hours of work and consumption. While there is some empirical support for the presence of these effects they are not central to the issues we focus on here, and so in the interest of simplicity we abstract from them.

Our focus is to understand how to account for retirement, as defined above, in this framework. That is, assuming that h_t is a number corresponding to full time work, we are interested in examining how this framework would rationalize that h_{t+1} equals zero. As just noted, the solution for h_{t+1} is zero if:

$$v'(1) \geq v'(1 - h_t) \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t} \quad (3.5)$$

If $v'(1) = 0$ then $h_{t+1} = 0$ if and only if $w_{t+1} = 0$. But if $v'(1) > 0$ it is possible that $h_t > 0$, and $h_{t+1} = 0$ even with $w_{t+1} > 0$.

A simple calculation is informative to provide a quantitative perspective on this issue. Consider a standard choice for the function v :

$$v(1 - h) = \frac{1}{1 - \frac{1}{\gamma}} (1 - h)^{1 - \frac{1}{\gamma}} \quad (3.6)$$

The parameter γ denotes the intertemporal elasticity of substitution for leisure. There is of course an extensive literature that has estimated this value, largely focusing on the labor supply behavior of prime aged males. In connecting with the empirical literature it is preferable to consider the intertemporal substitution elasticity for labor rather than for leisure. The implied intertemporal elasticity of substitution for labor is different by a factor of $(1 - h)/h$. In what follows we will use the abbreviation *IES* to *always* refer to the elasticity for *labor*, which as just noted is in general not equal to γ . There is a voluminous literature that has estimated the *IES* using variation in hours and wages over the life cycle in panel data. Early examples include Heckman and MaCurdy (1980), MaCurdy

(1981) and Altonji (1986). The early literature found relatively small estimates for males, on the order of .3 or less, but much larger values for women. (See Pencavel (1986) for a survey of early work.) Subsequent work, including recent papers by Kimball and Shapiro (2003), Pistaferri (2003) and Domeij and Floden (2006) have refined these estimates in various ways, and found larger estimates, in the range of .7 – 1.0 for males. (See Hall (2007) for a critical survey of the recent literature.) In a model that assumed human capital accumulation, Imai and Keane (2004) found an *IES* that exceeds 3, though Wallenius (2007) argues that this estimate is likely to be biased upward, and that adding human capital accumulation does not lead to estimates of the *IES* that are much greater than 1.0.⁶

The first property that we highlight is the connection between the value of the *IES* and the difficulty in accounting for retirement. This connection is intuitive. As noted above, the concavity of $v(1 - h)$ generates a motive to smooth leisure over time, and the lower the value of the *IES*, the greater is this motive. Retirement constitutes a dramatic departure from smoothness in the leisure profile, and the greater the desire for smoothness, the more difficult it is to account for this departure from smoothness. To quantify this we proceed as follows. Denote the ratio $\frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t}$ in equation (3.5) by R_{t+1} , which can be interpreted as the return to work in period $t + 1$ relative to period t . Equation (3.5) tells us the highest value of R_{t+1} consistent with $h_{t+1} = 0$ given a value of γ and a value of h_t . Assuming a weekly endowment of discretionary time equal to 100 hours and annual hours of

⁶Although we do not consider human capital accumulation in our analysis, Wallenius (2009) contains some results about the degree of nonconvexities required in that setting.

work equal to 2000, we have $h_t = .385$. Table 4 gives the maximum value of R_{t+1} that is consistent with inducing retirement at age $t + 1$.

Table 4
Value of R_{t+1} to Induce Retirement

$IES=2.0$	$IES=1.0$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.68	.46	.36	.22	.04	.0004	.0000

The values in the table are revealing. Even with a very large value of the IES , say equal to 2, one would still require a drop of more than 30% in the return to work between consecutive years to generate retirement. If one focuses on values of the IES that are commonly used in the literature, such as those that are .25 and below, the message is that one needs almost a 100% drop in the return to work in order to generate retirement. Even for values that are at the upper end of estimates for males, such as .75 and 1.00, one still needs the return to work to drop by more than 50% between consecutive years. The basic message is clear—in this framework, the only way that one can generate retirement is to assume dramatic decreases in wages or dramatic increases in the disutility of working precisely at the time of retirement. It is important to note that the above analysis has abstracted from uncertainty. While allowing for shocks to w , α , or Y does change the analysis somewhat, the basic message is that only large shocks to these values can induce retirement. While it is undoubtedly true that some individuals experience shocks to market opportunities, wealth, and/or health that might rationalize retirement in this context, the available evidence does not support this as the prime cause

of retirement (see, e.g., Blau and Shvydko (2010)). Put somewhat differently, although shocks may alter an individual's plans for retirement, they do not seem to be the underlying explanation for why individuals plan to retire in the first place.

Although we abstracted from social security and pension programs, the previous analysis can also be used to gauge how large the change in effective tax rates associated with these features would need to be in order to induce retirement. In particular, assuming no change in the return to work, i.e., constant wages and disutility of working, Table 4 tells us the required magnitude of the increase in either the implicit or explicit effective tax rate on earnings to induce retirement. Once again, the message is that these values are extremely large.⁷ One issue to note regarding implicit tax rates associated with private pension plans is that these rates are specific to the job and so are typically not relevant if the individual considers working for a different employer. In this case the relevant calculation would be the value of R_{t+1} based on the other job opportunities for this individual.

The above calculations were based on the assumption that the individual moves from full-time work to no work at all. How are these values affected by someone who moves from full-time work to part time work? Or by assuming that someone moves from part-time work to pure retirement? Tables 5 and 6 show the results, using $h = .20$ to reflect part time work.⁸

⁷While typically not the case in the US, in other countries individuals can find themselves in a situation where they face dramatic increases in effective tax rates from one year to the next. In particular, systems in which one must retire in order to collect social security benefits can induce large changes in effective tax rates at the normal retirement age.

⁸In Table 6 we use the same value for γ as in Table 5, i.e., we continue to assume that the mapping from γ to the IES was evaluated at full time work.

Table 5

Value of R_{t+1} to Induce Transition from Full-Time to Part-Time

$IES=2.0$	$IES=1.0$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.81	.66	.57	.43	.19	.02	.0002

Table 6

Value of R_{t+1} to Induce Retirement from Part-Time

$IES=2.0$	$IES=1.0$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.84	.70	.62	.49	.24	.03	.001

Comparing Table 5 with Table 4 we see that although the required values of R_{t+1} are not as small, it remains true that these represent dramatic changes in either preferences or opportunities between consecutive years, even for values of the IES on the high end. A similar message applies to the results in Table 6. That is, inducing pure retirement even for an individual who is currently working part-time still requires a dramatic change in the economic returns to work.

The simple conclusion that we want to emphasize from the above analysis is that it is very difficult to reconcile retirement with the standard model of life cycle labor supply. While this statement seems to apply to all empirically reasonable values of the IES , we also want to note that the lower the IES the more difficult it becomes to account for retirement in this framework.

4. Nonconvexities as a Source of Retirement

The analysis in the previous section illustrated the difficulty in generating retirement in a model with a strictly concave time separable utility function and a convex budget set. An obvious alternative is to relax one of these two assumptions so as to generate a nonconvexity in the consumer's optimization problem. Because nonconvexities can lead to discontinuities in the decision rule for hours, this alternative would seem to overcome the key problem encountered in the previous section.⁹ As noted in the introduction, much recent work on retirement assumes that budget sets are nonconvex, implicitly because of some underlying nonconvexities in production. In this section we describe two variations of a model that features nonconvexities in production. While it is true that this model diminishes the tension that we found in the standard life cycle model, we show that the same tension is very much present. Specifically, it remains true that the smaller the *IES*, the more difficult it is to generate retirement, in the sense that the required degree of underlying nonconvexity is larger.

4.1. Fixed Time Costs

We begin with a version of the model recently put forth by Prescott et al (2009). Because we want to use standard methods to characterize the optimal retirement decision it is convenient to formulate the model in continuous time. While one loses the notion of a period in the continuous time formulation, when it comes time

⁹Cogan (1981) is a classic reference for empirical work on the implications of fixed costs, though he did not focus on retirement.

to interpret some of the model features our preferred interpretation is to think in terms of a period being a year, so that we want to interpret labor supply during a period as annual hours of work. Normalizing the length of life to 1, preferences are now given by:

$$\int_0^1 [u(c(t)) + \alpha(t)v(1 - h(t))]dt \quad (4.1)$$

We consider a nonconvexity that takes the form of a fixed time cost associated with work, which we denote by \bar{h} . If an individual gives up $h(t)$ units of leisure at time t this will lead to $\max\{0, h(t) - \bar{h}\}$ units of labor that can be sold in the labor market. Letting $w(t)$ denote the wage at time t , the present value budget equation for this individual now reads:

$$\int_0^1 c(t)dt = \int_0^1 w(t) \max[0, h(t) - \bar{h}]dt + Y \quad (4.2)$$

Whereas in the previous section one required changes in at least one of $\alpha(t)$ or $w(t)$ to generate retirement, with fixed costs one can generate retirement without any variation in these factors. In order to focus on the forces associated with the nonconvexity it is convenient to initially assume that $\alpha(t) = \alpha$ and $w(t) = w$ for all t . We will return to consider the more general case in a later section. With w and α constant over time, and the interest rate and discount factor perfectly offsetting each other, the optimal timing of work for the individual is indeterminate. That is, the individual could choose to do all of the work at the beginning of life, all at the end of life, or all in the middle, etc.... As such, the model may not appear

to be a good model of retirement per se.¹⁰ However, this is an artifact of the extreme but useful assumption that there is no change in the return to work over time. If, for example, there is even an arbitrarily small positive slope to $\alpha(t)$, or negative slope to $w(t)$ (even if only at later ages), then the indeterminacy would vanish.¹¹ So while we will work with a specification in which the timing of work is indeterminate, we will focus on the solution in which work occurs at the beginning of life, followed by retirement.

Independently of the optimal labor supply decision, the optimal consumption decision for this individual is to smooth consumption perfectly.¹² The optimal solution for the hours profile can take one of two forms. The first corresponds to a solution in which it is optimal for the individual to have positive hours in all periods. By symmetry, the solution in this case will entail a constant amount of work at each time t . This case applies if the nonconvexity is not sufficiently large to overcome the forces that favor smooth leisure. The second and more interesting case is the one in which the individual chooses to work at some but not all dates.¹³ Once again by symmetry, the individual will work the same amount of time in all periods with positive hours. If an individual works for measure e periods and

¹⁰See also Ljungqvist and Sargent (2010) for further discussion of this issue in the context of a model that features learning-by-doing human capital accumulation and depreciation of skills when not working.

¹¹Alternatively, the indeterminacy could be resolved if the interest rate does not perfectly offset discounting.

¹²Later in the paper we discuss how the model can address the documented drop in consumption at retirement. The model could also be extended in different ways to generate a hump-shape in consumption during working life, but because this does not appear to be central to the issue of generating retirement, we do our analysis in the simpler specification.

¹³To be more precise we are interested in the case where the individual chooses positive hours for a positive measure of time but strictly less than measure one. In our discussion we will ignore the issues associated with deviations on sets of measure zero.

gives up $h > \bar{h}$ units of leisure at each date, he or she will have a present value of income equal to:

$$e(h - \bar{h}) + Y \quad (4.3)$$

We can thus write the utility maximization problem as:

$$\max_{e,h} u[e(h - \bar{h})w + Y] + e\alpha v(1 - h) + (1 - e)\alpha v(1) \quad (4.4)$$

Our main interest is to determine the conditions necessary for an interior solution for e , since this corresponds to there being retirement. Assuming interior solutions for both e and h we obtain the following two first order conditions:

$$u'(e(h - \bar{h})w + Y)(h - \bar{h})w = \alpha[v(1) - v(1 - h)] \quad (4.5)$$

$$u'(e(h - \bar{h})w + Y)w = \alpha v'(1 - h) \quad (4.6)$$

Dividing these two equations by each other we obtain:

$$h - \bar{h} = \frac{v(1) - v(1 - h)}{v'(1 - h)} \quad (4.7)$$

This equation is similar in spirit to equation (3.5) that we derived in the previous section to characterize the conditions necessary to generate retirement at a given age. Assuming the same functional form as in the last section:

$$v(1 - h) = \frac{1}{1 - \frac{1}{\gamma}}(1 - h)^{1 - \frac{1}{\gamma}} \quad (4.8)$$

equation (4.7) becomes:

$$h - \bar{h} = \frac{1}{1 - \frac{1}{\gamma}} [1 - (1 - h)^{1 - \frac{1}{\gamma}}] (1 - h)^{\frac{1}{\gamma}} \quad (4.9)$$

If we choose a value of γ and specify the level of work during working years, h , this expression gives the value of \bar{h} that is required for the optimal solution to display both the level of work h while working and retirement. Note that the solution for \bar{h} is independent of the age at which retirement occurs. That is, subject to requiring that the optimal solution entails working hours of h when working, the size of the fixed cost that is required to generate retirement is independent of whether one wants the worker to retire at age 40 or age 65. Conditional upon an interior solution for e and the level of h , the length of working life is determined by the values of α, w and Y .

Proceeding as before, we consider the same range of values for the *IES* as in the previous section, and once again consider the case where h is equal to .385 while working. Table 7 provides the results.

Table 7

Value of \bar{h} Required for Retirement

<i>IES</i> =2.0	<i>IES</i> =1.0	<i>IES</i> =.75	<i>IES</i> =.50	<i>IES</i> =.25	<i>IES</i> =.10	<i>IES</i> =.05
.07	.13	.16	.20	.28	.34	.37

One interpretation of the fixed cost is that it represents commuting costs. Estimates of commuting times would suggest that one hour per day is an upper

bound for the average value, suggesting a value of $\bar{h} = .05$. From this perspective all of the above values would seem high, with the possible exception of the $IES = 2.0$ case. However, there are several issues that should be noted in the context of interpreting \bar{h} as commuting costs. First, it is not clear that average commuting costs are the appropriate measure to use in this calculation. We are not aware of any evidence to suggest that retirement does not occur for individuals with relatively low commuting costs. For example, commuting times tend to be much less in smaller urban areas, but it seems that retirement remains a prominent feature of life cycle labor supply in these settings as well as larger urban areas. This being the case, we need to understand why retirement occurs not only for an individual with a high value of commuting costs but also for those individuals with low values. From this perspective we may want to evaluate equation (4.7) using a value of \bar{h} much lower than the average value of time spent commuting.

Second, although commuting costs are often mentioned as the classic example of a fixed cost, how they should be interpreted here depends very much on how one interprets a period. If one interprets a period to be a day, then commuting costs are clearly a fixed cost. But if one thinks of the period as a year, and an individual adjusts annual hours by adjusting the number of days worked, then commuting costs become a proportional time cost rather than a fixed cost, and therefore would not give rise to a nonconvexity and would not generate retirement.

Prescott et al (2009) argued that the fixed costs in this specification were intended to capture set-up costs that a worker experiences at work. Even without taking a strong stand on the appropriate level of setup costs, the results in Table

7 clearly suggest a tension. If one considers values of the IES that are .25 and below, it seems very hard to rationalize the level of fixed time costs required to generate retirement since the implication is that over two thirds of the individual's time is devoted to set-up costs. This tension has implicitly appeared in empirical work on labor supply. French (2005) estimates a life cycle model of labor supply that includes accounting for retirement, and assumes fixed time costs. When he estimates this specification he finds a relatively small intertemporal elasticity of substitution for labor, but a very large value of the fixed time costs. In fact, his estimate of the fixed time cost using annual data is more than 1200 hours per year. We note that our calculations suggest a stronger tension than the results in French. The reason for this is that our calculations focus on generating retirement from full time work, whereas French only required that his model match the average hours of all workers. As previously noted, average hours for all workers is substantially below average hours for full time workers, especially for workers above age 65. In any case, while a fixed hours cost of 1200 is the value that is required to make the model consistent with the data, this value seems hard to justify. In the next subsection we consider an alternative formulation which provides a somewhat sharper comparison with the data.

4.2. An Alternative Formulation

The previous subsection assumed that the nonconvexity took the form of a fixed time cost. Whether this time cost reflects time getting to and from work or time getting set up at work, it necessarily induces a nonconvex relationship between

the hours of leisure that the individual gives up and the earnings per hour of leisure sacrificed. In this section we consider another form for the relationship between time taken away from leisure and earnings. Specifically, we assume that the individual faces a nonlinear wage schedule for the wage per unit of time as a function of time spent working in a particular period. In particular we assume that the wage schedule $w(h)$ is given by:

$$w(h) = w_0 h^\theta \tag{4.10}$$

where $\theta \geq 0$. If $\theta = 0$ this reduces to the standard case in which the wage per unit of time worked is independent of the number of hours worked, and implies a convex budget set. The advantage of this specification is that there has been some empirical work to guide us in thinking about reasonable values of the parameter θ . (See, for example, Moffitt (1984), Keane and Wolpin (2001) and Aaronson and French (2004).) While there are some important issues involved in estimating this parameter and there is by no means a definitive estimate, the value suggested by this work is $\theta = .4$.¹⁴ This is also the value that French (2005) assumed when considering this specification. A simple way to gauge the magnitude of the implied nonconvexity is to compute the hourly wage for different annual hours of work. In our benchmark calculations we will use 2000 annual hours as our measure of full time work. If someone were to work 1500 annual hours, $\theta = .4$ would imply a wage penalty of slightly more than 10%, while if someone were to work 1000 annual

¹⁴The value of θ is likely to vary across occupations. This would influence the length of the full time work week across occupations as well as the incidence of part time work.

hours then the wage penalty would be almost 25%. If we were to increase θ to .5, then these penalties would be approximately 13% and 29%. In this subsection we consider this alternative formulation and solve for the value of θ that is required to generate retirement as a function of the preference parameter γ and the time devoted to work when working.

The relevant maximization problem now becomes:

$$\max_{e,h} u(e w_0 h^{1+\theta} + Y) + e \alpha v(1-h) + (1-e) \alpha v(1) \quad (4.11)$$

Repeating the same steps as before, we arrive at the expression:

$$\frac{h}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}} [1 - (1-h)^{1-\frac{1}{\gamma}}] (1-h)^{\frac{1}{\gamma}} \quad (4.12)$$

which now gives us a required value of θ given values for γ and h . The results are contained in Table 8.

Table 8

Value of θ Required for Retirement

<i>IES</i> =2.0	<i>IES</i> =1.0	<i>IES</i> =.75	<i>IES</i> =.50	<i>IES</i> =.25	<i>IES</i> =.10	<i>IES</i> =.05
.23	.48	.67	1.09	2.64	8.38	18.37

The qualitative pattern is the same as that found in Table 7: the smaller the *IES* the larger the nonconvexity needs to be in order to generate retirement as part of an optimal choice for the individual. If we take the value of $\theta = .4$ as a

guideline for a reasonable magnitude, we see that any values of the *IES* that are 1.00 or below are not consistent with retirement given this degree of nonconvexity.

We previously argued that commuting costs are probably not a source of nonconvexity from the perspective of annual labor supply. Nonetheless, we can consider the case in which we allow for a fixed time cost associated with getting to and from work in addition to the nonlinear wage schedule that applies to hours at work. It is easy to assess how this affects the numbers in Table 8. If one assumes a fixed time cost of \bar{h} in addition to the nonlinear wage schedule, so that only $h - \bar{h}$ hours are productive, one obtains the following expression:

$$\frac{h - \bar{h}}{1 + \theta} = \frac{1}{1 - \frac{1}{\gamma}} [1 - (1 - h)^{1 - \frac{1}{\gamma}}] (1 - h)^{\frac{1}{\gamma}} \quad (4.13)$$

Assuming that commuting costs represent 10% of total working time, the new values of θ are given in Table 9.

Table 9

Value of θ Required for Retirement When $\bar{h} = .1h$

<i>IES</i> =2.0	<i>IES</i> =1.0	<i>IES</i> =.75	<i>IES</i> =.50	<i>IES</i> =.25	<i>IES</i> =.10	<i>IES</i> =.05
.10	.34	.51	.89	2.28	7.44	16.44

While this adjustment does influence the magnitude of the required values of θ , the overall picture does not change much. There are two very different potential interpretations of the tradeoff that these tables describe. If one wants to take as given that the *IES* is .50 or smaller, and that θ is around .40, then Tables 8

and 9 suggest that this model is not a good model of retirement, since it cannot generate retirement for empirically reasonable parameters. Alternatively, if one accepts this model as correct and assumes that $\theta = .4$ is reasonable, we would take this as evidence in support of a value for the *IES* that is either near to 1.0 or even slightly above 1.0. This value is much larger than many of the traditional estimates derived from looking at changes in hours and wages for prime age males, and at the upper end of more recent estimates that challenge the earlier estimates. We argue below that the above calculations are immune to the issues of credit constraints and human capital accumulation that are problematic for traditional estimates.

4.3. Fixed Consumption Costs of Working

The literature has also argued that there are fixed consumption costs associated with working. See for example, the work of Banks et al (1998) and Aguilera et al (2008). Intuitively, the presence of fixed consumption costs associated with working should reduce the magnitude of the other nonconvexities that are required to induce retirement. In this section we consider this extension. In particular, we assume that there is a fixed consumption cost \bar{c} that is incurred in any period in which hours are positive, and which yields no utility to the individual. While we proceed under this assumption, we think it is important to note again that from the perspective of annual labor supply a substantial part of these costs may be better modeled as a variable cost that is related to the number of days or weeks worked. However, since our basic message will be that even the extreme case in

which all of these costs are interpreted to be fixed costs does not significantly change our conclusions, we think it is best to focus on this case.

The objective of the individual is now to choose e and h so as to maximize the value of:

$$\max_{e,h} u(ew_0(h - \bar{h})^{1+\theta} - e\bar{c} + Y) + e\alpha v(1 - h) + (1 - e)\alpha v(1) \quad (4.14)$$

Proceeding as before, we conclude that a necessary condition for an interior solution for e is:

$$\frac{(1 + \theta)w_0(h - \bar{h})^\theta}{w_0(h - \bar{h})^{1+\theta} - \bar{c}} = \frac{v'(1 - h)}{v(1) - v(1 - h)} \quad (4.15)$$

To facilitate calculations, it is useful to parameterize the level of the fixed cost as a fraction \hat{c} of labor income, i.e., $\bar{c} = \hat{c}w_0h^{1+\theta}$. With this parameterization the above expression simplifies to:

$$\frac{(1 + \theta)}{(1 - \hat{c})(h - \bar{h})} = \frac{v'(1 - h)}{v(1) - v(1 - h)} \quad (4.16)$$

This expression shows that fixed consumption costs associated with working act very much like fixed time costs associated with working. Table 10 presents the quantitative implications of considering various levels of the fixed consumption costs, assuming a fixed time cost equal to 10% of working time.

Table 10

 θ Required for Retirement With Fixed Consumption Costs and $\bar{h} = .1h$

	<i>IES</i> =2.0	<i>IES</i> =1.0	<i>IES</i> =.75	<i>IES</i> =.50	<i>IES</i> =.25	<i>IES</i> =.10	<i>IES</i> =.05
$\hat{c} = .025$.08	.30	.47	.84	2.19	7.23	16.00
$\hat{c} = .05$.05	.27	.43	.79	2.11	7.02	15.56
$\hat{c} = .10$.00	.20	.36	.70	1.95	6.60	14.69

When interpreting the value of \hat{c} it is important to note that we have expressed it relative to labor earnings. Assuming that individuals spend roughly 2/3 of their lives in employment and begin with zero wealth, consumption smoothing would imply that average consumption is equal to 2/3 of labor earnings when working. It follows that to convert from consumption costs expressed relative to labor earnings to consumption costs expressed relative to consumption, to first order one should adjust \hat{c} by a factor of 1.5. So, for example, a value of $\hat{c} = .10$ represents fixed consumption costs that are roughly 15% of total consumption expenditures. Comparing Tables 9 and 10 one can see that the inclusion of fixed consumption costs does serve to lower the extent of nonconvexities that are required to generate retirement. However, even with fixed consumption costs equal to 10% of labor earnings and a 10% fixed time cost we see that it is still difficult to reconcile values of the *IES* below .75 with current estimates of the nonconvexities.

5. Sensitivity Analysis

In this section we assess the extent to which some of the special features assumed above are influencing the quantitative results that we derived. We consider six

separate issues: age varying wages or disutility of work, preferences that are not separable between leisure and consumption, the possibility of credit constraints, the possibility of human capital accumulation, age varying *IES* and the presence of social security. We deal with each in turn.

5.1. Age-Varying Wages and Utility from Leisure

Adding age-varying wages or utility does not matter at all for the results derived above if we assume that these profiles are continuous. We demonstrate this in the context of an age-varying utility from leisure, given by $\alpha(t)$. Consistent with our desire to focus on retirement, i.e., that the period of not working in the market occurs at the end of life, we assume that the $\alpha(t)$ profile is increasing.¹⁵ It is no longer the case that hours of work when working are constant, so we will now have an hours of work profile $h(t)$. The maximization problem is now:

$$\max_{e, h(t)} u\left(\int_0^e w(h(t) - \bar{h})dt + Y\right) + \int_0^e \alpha(t)v(1 - h(t))dt + \int_e^1 \alpha(t)v(1)dt \quad (5.1)$$

Assuming an interior solution the first order condition for e is:

$$u'\left(\int_0^e w(h(t) - \bar{h})dt + Y\right)w(h(e) - \bar{h}) = \alpha(e)v(1) - \alpha(e)v(1 - h(e)) \quad (5.2)$$

¹⁵In fact, our analysis would go through unchanged if we instead assumed that this profile were u-shaped, thereby potentially generating a period of nonwork at the beginning of life as well.

Of particular interest is the first order condition for the optimal level of hours at the time of retirement, $h(e)$. The first order condition for this value is:

$$u'(\int_0^e w(h(t) - \bar{h})dt + Y)w = \alpha(e)v'(1 - h(e)) \quad (5.3)$$

Dividing these two expressions gives:

$$h(e) - \bar{h} = \frac{v(1) - v(1 - h(e))}{v'(1 - h(e))} \quad (5.4)$$

It follows that our previous calculations all go through exactly, as long as we understand that the level of hours that we use in the calculation refers to the level of hours worked at the time of retirement. But with this one proviso, the calculation is entirely unchanged.¹⁶

We can also extend this analysis to handle the case of a single discontinuity in the α profile. If retirement occurs at the point of the discontinuity then what matters is not what the hours were just prior to retirement, but rather what the hours worked would have been at e had the individual not retired. The first order condition for optimal hours tells us that this value must satisfy

$$\alpha(e)v'(1 - h(e)) = \lim_{t \rightarrow e} \alpha(t)v'(1 - h(t)) \quad (5.5)$$

The discontinuity in the α profile can reduce the needed nonconvexity through lowering the appropriate h to feed into the calculations. Basically, the implied

¹⁶Rogerson and Wallenius (2009) assume that productivity varies with age, but do not target working time at retirement in their calibration, so this result does not apply to their calculations.

level of hours is the value that equates marginal disutility of work at the margin with those periods in which the individual chose to work. However, as we know from our earlier calculations, for relatively small elasticities the effect of even moderate discontinuities on h is small, and moreover the effect of a small change in h on the required nonconvexity is also small. We conclude that the previous calculations are not much affected by allowing for time changing α or w , unless we allow for very large discontinuities.

5.2. Non-separable Preferences

Next we consider whether separability between consumption and leisure matters. Following Trabandt and Uhlig (2009) and Shimer (2010), we adopt the following specification of preferences that are consistent with balanced growth at the same time that they exhibit a constant Frisch labor supply elasticity:

$$\frac{1}{1-\eta}c(t)^{1-\eta}[1-\kappa(1-\eta)h(t)]^{1+\frac{1}{\phi}\eta} \quad (5.6)$$

where η , κ , and ϕ are all positive. The Frisch labor supply elasticity is equal to ϕ . In the interests of space we only consider the case of a fixed time cost. In the appendix we show that in order for the optimal solution to exhibit retirement, the value of \bar{h} must satisfy:

$$\bar{h} = \frac{1}{\phi}h$$

where h is the time devoted to production at the time of retirement. For comparisons with our earlier analysis it is convenient to express this relative to total

time devoted to work:

$$\frac{\bar{h}}{h + \bar{h}} = \frac{1}{1 + \phi}$$

Similar to our previous analysis based on the separable specification, neither the exponent on consumption nor the length of working life influences the extent of nonconvexity that is required to generate retirement. Using our previous benchmark value of $h = .385$, Table 11 shows the required value of \bar{h} for various values of the *IES*.

Table 11

Value of $\bar{h}/(h + \bar{h})$ Required for Retirement With Nonseparable Preferences						
<i>IES</i> =2.0	<i>IES</i> =1.0	<i>IES</i> =.75	<i>IES</i> =.50	<i>IES</i> =.25	<i>IES</i> =.10	<i>IES</i> =.05
.33	.50	.57	.67	.8	.91	.95

Relative to our earlier results, we see that this specification requires even larger nonconvexities in order to generate retirement. The reason for this is that the preference specification in equation (5.6) implies that marginal disutility of work is zero when work is zero. Relative to our earlier specification in which we had a non-zero marginal utility of leisure at zero work, this makes it even more difficult to generate retirement.

There is a large literature that seeks to both measure and account for the drop in consumption that is associated with retirement. (see Laitner and Silverman (2005)). If the value of the parameter η is greater than one then one can show that the above specification does entail a drop in consumption at retirement. However, what the above calculation shows is that the value of η has no impact

on the extent of the nonconvexity required to generate retirement. In the context of this specification, generating retirement is a completely separate issue from generating a drop in consumption at the time of retirement.

5.3. Borrowing Constraints When Young

Recent work by Low (2005) and Domeij and Floden (2006) argues that standard attempts to infer the value of the *IES* from life cycle variation in hours and wages are sensitive to assumptions about credit constraints facing younger workers. The intuition is clear: in the data younger workers have significantly lower wages than older workers but still work quite a bit relative to older workers. If there are no credit constraints, then this implies that the *IES* must be relatively small. However, if younger workers are credit constrained, i.e., cannot borrow against future earnings, or want to accumulate assets for precautionary motives due to uninsurable income risk, then the fact that their hours of work are relatively high need not convey any information about the *IES*. Domeij and Floden argue that the true *IES* might reasonably be twice the value that is estimated when this feature is ignored.

The results that we have derived are independent of whether young workers are credit constrained. All that matters for our results is that individuals are not credit constrained at the time of retirement. To see why, note that because our problem is recursive, we could have the individual resolve their optimal life cycle labor supply problem at any date during their life without affecting the solution for the remaining part of the life cycle. It follows that as long as there is some

age prior to retirement at which the individual is not credit constrained, then our key equations remain unaffected.

5.4. Human Capital Accumulation

Work by Heckman (1976), Imai and Keane (2004), and Wallenius (2007) has argued that allowing for human capital accumulation can also have a large impact on values of the *IES* estimated from life cycle data. Similar to the comment regarding borrowing constraints, our results are not affected by the presence of human capital accumulation if human capital accumulation ceases to be important at the time of retirement. That is, as long as there is some point prior to retirement at which the evolution of wages from that point forward can be taken as exogenous, then our equations would remain completely valid. This is because the key data for our calculations is simply the hours worked, preferences and market opportunities at the time of retirement. That is, our equations that link the *IES* and other parameters are valid at the time of retirement even if there have been additional factors that played a role in shaping life cycle labor supply earlier in life. A standard assumption in the literature on human capital is that the incentive for human capital accumulation among older workers is very small, so that wage changes for these workers are effectively exogenous. See for example, Heckman et al (1998) and Huggett et al (2010).

5.5. Age-Varying *IES*

Our benchmark specification assumed that the preference parameter γ was constant over the life cycle, with age affecting preferences solely through α . Basically, our formulation allows for level effects of age on the marginal utility of leisure or equivalently, the marginal disutility of work. An alternative possibility is that age affects the slope of the marginal utility of leisure. In particular, one might argue that relative to younger workers, older workers get tired at a faster rate as hours of work increase. One way to capture this is to assume that the curvature in the utility of leisure function increases with age. Consistent with previous discussion, this has no effect on the expression that we derived, as it characterizes the relationship between preference parameters and hours of work at the time of retirement. Hence, our calculations would then be relevant for the value of the *IES* at retirement. The *IES* is decreasing in the amount of curvature in the utility from leisure function. To the extent that the utility function for leisure might exhibit more curvature at older ages than at younger ages, our estimates represent a lower bound for the value of the *IES* at younger ages.

5.6. Social Security and Pensions

Thus far our analysis has abstracted from Social Security and private pensions. Here we argue that our results are robust to the inclusion of these features in the US context. For individuals in the US who retire at full retirement age (or beyond), Social Security is effectively a lump-sum transfer of income, since for most individuals there is effectively no restriction on the amount of work that

he or she can do. The expressions that we have derived above allowed for the possibility of an income transfer in addition to labor income, and so are consistent with this case. For individuals who retire prior to full retirement age, the situation is somewhat more complicated. In this case individuals are subject to an earnings test and their Social Security benefits are reduced if earnings exceed a threshold. However, if benefits are reduced because of the earnings test, then future benefits are actually increased in a fashion that is approximately actuarially fair. Hence, even at earlier ages, Social Security is still approximately a lump-sum transfer independently of the individuals labor supply decision. However, the one potential issue concerns the case in which an individual is credit constrained at age 62 and chooses to retire because they are not allowed to borrow against future social security benefits. In this case our expressions would not be valid. While we believe this case does occur in reality, it does not pose a problem for our calculations unless one thinks that effectively all of the transitions from full time work to no work are characterized by this situation. But this is clearly not the case, both as documented by Blau and Shvydko (2010) as well as evidenced by the fact that a substantial part of retirement occurs outside of the early retirement window. We conclude that given the structure of the Social Security system in the US, the presence of Social Security is not of first order importance for our calculations.¹⁷ We note however, that this would not be the case in a country in which Social

¹⁷Rust and Phelan (1997) argue that some workers postpone retirement until age 65 due to the fact that Medicare is available at age 65. We interpret this as evidence that the high price of private health insurance for non-employed individuals and the relatively low price of health insurance for those over 65 distorts the optimal retirement decision, but is not the fundamental source of retirement.

Security benefits are contingent on not working, since in this case the discontinuity in the benefits system can induce large changes in the effective return to work and hence serve as the driving force in generating retirement.

In contrast to the provisions of Social Security in the US, there are many companies that offer private pensions in which the return to continuing to work at the company can drop virtually to zero at a particular point. Specifically, at a certain point an individual may have accumulated sufficient experience that he or she could retire from the company and receive a pension that is the same as his or her current salary. For the purposes of our calculation there are several issues to note. First, as documented in Blau and Shvydko (2010), this is not the dominant source of abrupt retirements in the data. Second, as emphasized earlier in this section, what is important for our calculations is the change in the return to work at the time of retirement. So in the context of the previous example, what matters is how the return to work is changing at the time of retirement. Although pension plans do involve discontinuities, the issue is the size of the discontinuity. Second, the above calculations focused only on the return to work at the same firm, and not the return to working elsewhere. Even if an individual faces little or no return to working at their current firm, what matters for the labor supply decision is the return to working elsewhere. To the extent that the pension is anticipated, its value is already incorporated in the life cycle labor supply decision. What becomes relevant for our calculation is to incorporate the extent of the drop in wages that the individual must accept when moving to alternative employment. However, as emphasized earlier in the paper, even wage drops as large as 25%

have virtually no power to induce retirement for low values of the IES .

6. Conclusion

The typical pattern at retirement consists of a worker making an abrupt transition from full time work to no work. We have studied a class of life cycle labor supply models that generate this type of transition despite the fact that individuals face a continuous choice for hours of work. The key element that gives rise to this abrupt change in hours worked are nonconvexities in the problem that an individual worker faces. We show that generating retirement in this class of models implies sharp restrictions on possible combinations of values for hours of work prior to retirement, the intertemporal elasticity of labor supply and the extent of nonconvexities. We argue that based on existing estimates of the size of nonconvexities and measures of full time work prior to retirement, it is hard to rationalize values of the IES that are less than .75. Our estimates are robust to allowing for human capital accumulation and credit constraints, factors which the previous literature have argued are important sources of negative bias in earlier estimates of the IES . A novel aspect of our analysis is the observation that in a model that features both intensive and extensive margins, adjustment along the extensive margin is also an important source of information about curvature over leisure in the individual utility function.

We have explored the connection between retirement and the IES in a simple model in order to facilitate transparency. It is also of interest to explore this issue in richer settings. One extension of interest for future work is to allow for

other factors that might be relevant in generating abrupt transitions, such as firms offering limited possibilities for hours of work.¹⁸ For example, workers may face a restricted choice, say between full time and half-time work (possibility with a wage penalty). While our existing results do not directly apply to this case, we think that the key insight from our analysis will still remain, since we need to know why it is that workers transit from full time work to no work instead of transiting through part time work. Another extension of interest to allow for home production in the analysis in order to provide a richer description of how time allocations change at retirement.

¹⁸See Hurd (1996) for a discussion of this issue.

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Appendix

In this appendix we derive the expression for \bar{h} in the model that assumes nonseparable preferences. The optimization problem for an individual can be written as:

$$\max_{e, h, c_w, c_r} eu(c_w, h + \bar{h}) + (1 - e)u(c_r, 0)$$

subject to the lifetime budget constraint:

$$ec_w + (1 - e)c_r = eh$$

where c_w is consumption while working, c_r is consumption while retired, h is time devoted to production, e is the fraction of life spent in employment and \bar{h} is the fixed time cost associated with working. As noted in the text, the utility function is given by:

$$u(c, h) = \frac{1}{1 - \eta} c^{1 - \eta} [1 - \kappa(1 - \eta)h^{1 + \frac{1}{\phi}}]^\eta.$$

Letting λ denote the Lagrange multiplier on the budget equation and assuming interior solutions for all four choice variables we have the following first order conditions:

$$c_w : c_w^{-\eta} [1 - \kappa(1 - \eta)(h + \bar{h})^{1 + \frac{1}{\phi}}]^\eta = \lambda \quad (\text{A1})$$

$$c_r : c_r^{-\eta} = \lambda \quad (\text{A2})$$

$$e : \frac{1}{1 - \eta} \{c_w^{1 - \eta} [1 - \kappa(1 - \eta)h^{1 + \frac{1}{\phi}}]^\eta - c_r^{1 - \eta}\} = \lambda [c_w - c_r - h] \quad (\text{A3})$$

$$h : c_w^{1 - \eta} [1 - \kappa(1 - \eta)h^{1 + \frac{1}{\phi}}]^\eta \kappa (1 + \frac{1}{\phi})(h + \bar{h})^{\frac{1}{\phi}} = \lambda \quad (\text{A4})$$

In characterizing the solution to this set of equations it is useful to define two terms:

$$A = [1 - \kappa(1 - \eta)h^{1+\frac{1}{\phi}}] \quad (\text{A5})$$

and

$$B = (1 - \eta)\kappa \left(1 + \frac{1}{\phi}\right)(h + \bar{h})^{\frac{1}{\phi}} \quad (\text{A6})$$

Dividing equation (A1) by equation (A2) one obtains:

$$c_w = Ac_r \quad (\text{A7})$$

Using this expression in the lifetime budget equation gives:

$$c_r = \frac{eh}{Ae + (1 - e)} \quad (\text{A8})$$

Using equation (A8) in equation (A2) yields:

$$\lambda = \left[\frac{eh}{Ae + (1 - e)}\right]^{-\eta} \quad (\text{A9})$$

Substituting equations (A7), (A8) and (A9) into (A3) and simplifying gives:

$$A = \frac{\eta + e - 1}{e} \quad (\text{A10})$$

Substituting equations (A7), (A8) and (A9) into equation (A4) one obtains:

$$B = \frac{1 - \eta}{\eta} \frac{Ae + (1 - e)}{eh} \quad (\text{A11})$$

Note from equations (A5) and (A6) that A and B are related by:

$$A = 1 - \frac{B(h + \bar{h})}{1 + \frac{1}{\phi}} \quad (\text{A12})$$

Substituting equations (A10) and (A11) into equation (A12) and simplifying yields:

$$\bar{h} = \frac{1}{\phi}h.$$