Proximity versus Comparative Advantage: A Quantitative Theory of Trade and Multinational Production

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March 2011

Notation

- N countries. i : origin, l: location, n : destination
- X_{iln} : sales of firms from i, producing in l, selling to n
- Y_l : total production by all firms in location l
  - $\sum_{i,n} X_{iln} = Y_l$
- X_n : total income/spending (CA balance) of country n
  - $\sum_{i,l} X_{iln} = X_n$

Setup

- Measure of L_i consumers/workers, and measure M_i of firms (exogenous if no free entry)

- **Consumers**
  - Dixit-Stiglitz preferences over varieties, elasticity of substitution σ
  - Income from labor and profits from national firms (profits zero if free entry)

Setup

- **Firms**
  - Monopolists over their variety.
    - Enter in i with fixed cost $w_i f_i^e$.
    - This is the cost of "product innovation."
  - Firms can serve n from any location l at "marketing" cost $w_n F_n$.
  - Linear production technology. Productivity in location l is z_l, use $\mathbf{z} \equiv (z_1, ..., z_N)$
  - τ_{ln} iceberg trade costs, γ_{il} iceberg MP costs
  - Marginal production and shipping cost from l to n

$$C_{iln} = \frac{\gamma_{il} w_l \tau_{ln}}{z_l} \equiv \frac{\xi_{iln}}{z_l}$$

Firm Productivities

- Productivity vector $\mathbf{z}$ is drawn from a multivariate Pareto distribution

$$\Pr(Z_1 \leq z_1, ..., Z_N \leq z_N) = G_i(z_1, ..., z_N) = 1 - \left(\sum_{l=1}^{N} \left[T_{il} z_l^{-\theta} \right]^{\frac{1}{1-\rho}} \right)^{1-\rho},$$

with $z_l \geq \widetilde{T}_i^{1/\theta}$, where $\widetilde{T}_i \equiv \left[\sum_l T_{il}^{1/(1-\rho)} \right]^{1-\rho}$, $\rho \in [0, 1[$, and $\theta > \sigma - 1$.

- Properties of the distributions
  - Marginals are not Pareto, but do have Pareto tails – for $z_l \geq a > 1$,

$$\Pr(Z_l \geq z_l \mid Z_l \geq a) = (z_l / a)^{-\theta}$$

  - As $\rho \longrightarrow 1$ then perfect correlation among z_l, $G_i(z_1, ..., z_N) = 1 - \max_l T_{il} z_l^{-\theta}$
  - If $\rho = 0$, then all the mass is at the boundary - like having l determined with probabilities $T_{il} / \widetilde{T}_i$ and Z_i drawn from $1 - \widetilde{T}_i z_l^{-\theta}$ with $z_l \geq \widetilde{T}_i^{1/\theta}$.
- With no free entry and $\gamma_{il} \longrightarrow \infty$ $\forall i \neq l$ then this is the Chaney-EKK version of Melitz.

Firm Productivities

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with $z_l \geq \widetilde{T}_i^{1/\theta}$, where $\widetilde{T}_i \equiv \left[\sum_l T_{il}^{1/(1-\rho)} \right]^{1-\rho}$, $\rho \in [0, 1[$, and $\theta > \sigma - 1$.

- Later we make the following assumptions:
    - *Assumption 1:* $T_{il} = T_i^I T_l^P$ *and* $\sum_v \left(T_v^P \right)^{1/(1-\rho)} = 1$. (This implies $\widetilde{T}_i = T_i^I$)
    - *Assumption 2:* $f_i^e = f^e$.
- Looking ahead: country i has CA in innovation if it has a high ratio of $\left(T_i^I \right)^{1+\theta/(1-\rho)}$ to $A_i \equiv \left(T_i^P \right)^{1/(1-\rho)} / L_i$

Firm Problem: Entry

- Firm i chooses to sell to n from l if

$$\arg\min_{v} C_{ivn} = l \ \cap \min_{v} C_{ivn} \leq c_n^*$$

where

$$c_n^* = \left(\frac{\sigma w_n F_n}{\widetilde{\sigma} X_n} \right)^{1/(1-\sigma)} P_n$$

and where P_n is the Dixit-Stiglitz price index.

Firms

Lemma

The probability that a firm from i will serve market n from l is

$$\Pr\left(\arg\min_{v} C_{ivn} = l \ \cap \min_{v} C_{ivn} = c\right) = \theta \Psi_{in}^{-\frac{\rho}{1-\rho}} \left(T_{il}\xi_{iln}^{-\theta}\right)^{\frac{1}{1-\rho}} c^{\theta-1}$$

where $\Psi_{in} \equiv \left[\sum_{v} \left(T_{iv}\xi_{ivn}^{-\theta}\right)^{\frac{1}{1-\rho}}\right]^{1-\rho}$.

- Conditional on selling to n from l, distribution of $c \leq c_n^*$ is $\left(c/c_n^*\right)^{\theta}$

Trade Flows

Since the sales of a firm with cost c in a market n are $\widetilde{\sigma} X_n P_n^{\sigma-1} c^{1-\sigma}$, the previous results imply that

$$X_{iln} = \psi_{iln} \lambda_{in}^{E} X_n,$$

where

$$\lambda_{in}^{E} \equiv \frac{\sum_l X_{iln}}{\sum_{j,l} X_{jln}} = \frac{M_i \Psi_{in}}{\sum_j M_j \Psi_{jn}}$$

and

$$\psi_{iln} \equiv \frac{\left(T_{il} \xi_{iln}^{-\theta} \right)^{1/(1-\rho)}}{\sum_v \left(T_{il} \xi_{ivn}^{-\theta} \right)^{1/(1-\rho)}}$$

Trade and MP shares

- Expenditure shares of consumers in n on goods produced in l (trade shares)
$$\lambda_{ln}^{T} = \frac{\sum_i X_{iln}}{\sum_{j,l} X_{jln}} = \sum_i \psi_{iln} \lambda_{in}^{E}$$
- Production shares of firms from i in l (MP shares)
$$\lambda_{il}^{M} = \frac{\sum_n X_{iln}}{\sum_{j,n} X_{jln}} = \frac{\sum_n \psi_{iln} \lambda_{in}^{E} X_n}{Y_l}$$

Equilibrium

- Let $\widetilde{\Pi}_{iln}$ be the profits (net of marketing costs) realized by firms from i through their production in country l for market n, and let $\Pi_{il} \equiv \sum_n \widetilde{\Pi}_{iln}$
- Current Account balance

$$X_i = Y_i + \sum_{l \neq i} \Pi_{il} - \sum_{j \neq i} \Pi_{ji}$$

- Labor market clearing

$$Y_i - \sum_j \Pi_{ji} = w_i (L_i - M_i f_i^e)$$

- Free entry

$$\sum_l \Pi_{il} = M_i w_i f_i^e$$

Equilibrium

- **Lemma:** Profits are a constant fraction $\eta \equiv (\sigma - 1) / (\sigma\theta)$ of aggregate sales, $\Pi_{iln} = \eta X_{iln}$
- Under free entry, equilibrium entails $w_i L_i = X_i$, so we have a system of $2N$ equations in $2N$ unknowns (**M** and **w**)

$$w_i L_i = (1 - \eta) \sum_n \lambda_{in}^T w_n L_n + \eta \sum_n \lambda_{in}^E w_n L_n$$

$$\eta \sum_n \lambda_{in}^E w_n L_n = M_i w_i f_i^e$$

Special Cases: No MP

- In this case we have

$$\lambda_{in}^{T} = \frac{M_i T_i^I T_i^P \left(w_i \tau_{in}\right)^{-\theta}}{\sum_j M_j T_j^I T_j^P \left(w_j \tau_{jn}\right)^{-\theta}}$$

  - Like EK but with their T_i replaced by the $M_i T_i^I T_i^P$.
- Under free entry we get

$$M_i = \eta L_i / f_i^e$$

  - With no MP, trade has no effect on M_i (standard feature of Melitz/Chaney, also EK 2001)
  - $r_i \equiv L_i^e / L_i = \eta$, independent of entry costs – model without MP cannot account for differences in r_i.
- With MP then countries with a "CA in innovation relative to production" will have

$$r_i > \eta$$

Special Cases: Frictionless World

- Assume that $f_i^e = f^e$ all i, let $A_i \equiv \left(T_i^P\right)^{1/(1-\rho)} / L_i$ and assume that for all i we have

$$\frac{A_i / \left(T_i^I\right)^{1+\theta/(1-\rho)}}{\sum_j \delta_j A_j / \left(T_j^I\right)^{1+\theta/(1-\rho)}} < \frac{1}{1-\eta}$$

where

$$\delta_j \equiv \frac{l_j T_j^I}{\sum_v l_v T_v^I}.$$

Then

$$r_i = 1 - (1-\eta) \frac{A_i / \left(T_i^I\right)^{1+\theta/(1-\rho)}}{\sum_j \delta_j A_j / \left(T_j^I\right)^{1+\theta/(1-\rho)}}$$

Home Market Effects

- Two activities: production and innovation.
- Production HME is enhanced by trade costs, innovation HME is enhanced by MP costs.
- Consider two countries that are identical except for size, with $L_1 > L_2$ and $\left(T_i^P\right)^{1/(1-\rho)} = l_i$. Assume also that $\tau_{12} = \tau_{21} = \tau$ and $\gamma_{12} = \gamma_{21} = \gamma$.
    - If $\tau = \gamma = 1$ or either $\tau = \infty$ or $\gamma = \infty$ then $r_1 = r_2 = \eta$.
    - Assume $\gamma = 1$. As τ increases from 1, r_1 decreases from η and then increases to reach η again as $\tau \to \infty$.
    - Assume $\tau = 1$. As γ increases from 1, r_1 increases from η and then decreases to reach η again as $\gamma \to \infty$.

Welfare: Exogenous Entry

- The gains from openess with exogenous entry are

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-(1-\rho)/\theta} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\rho/\theta} \left(\frac{X_n}{Y_n}\right)^{1+\frac{\theta-(\sigma-1)}{\theta(\sigma-1)}}$$

  - With no MP we have $X_n = Y_n$ and $\sum_i X_{inn} = \sum_l X_{nln} = X_{nnn}$,

$$GO_n = \lambda_{nn}^{-1/\theta}$$

- In the special case with $\rho = 0$ we can show that

$$\frac{X_{iln}}{X_{ill}} = \frac{X_{kln}}{X_{kll}}$$

for all k, i. This implies that $\frac{X_{nnn}}{X_n} = \lambda_{nn}^M \lambda_{nn}^T$ and hence

$$GO_n = \left(\lambda_{nn}^T\right)^{-\frac{1}{\theta}} \left(\lambda_{nn}^M\right)^{-\frac{1}{\theta}} \left(\frac{X_n}{Y_n}\right)^{1+\frac{\theta-(\sigma-1)}{\theta(\sigma-1)}}$$

Welfare: Free Entry

- The gains from openess with exogenous entry are

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} \left(\frac{M_n}{\widetilde{M}_n}\right)^{\frac{1}{\theta}}$$

- But

$$\frac{M_n}{\widetilde{M}_n} = \frac{X_n / Y_n}{1 - \eta + \eta X_n / Y_n}$$

so

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} \left(\frac{X_n / Y_n}{1 - \eta + \eta X_n / Y_n}\right)^{\frac{1}{\theta}}$$

Commanding Heights?

- Consider a two country world with exogenous entry, with $m_1 \equiv M_1/L_1 > M_2/L_2 \equiv m_2$
- What happens to country 1 workers as MP costs decline? Can w_1/P_1 fall?
- Consider the move from frictionless trade but no MP to a fully frictionless world.
- The worse scenario has $\rho \to 1$. We can show that if

$$1 + \theta > \sigma > \frac{(1+\theta)^2}{1+\theta+\theta^2}$$

  then for $m_1 = m_2 + \varepsilon$ then w_1/P_1 falls.
- X_1/P_1 can also fall, but less likely because profits increase.
- As long as there is an interior solution, w_1/P_1 does not decline under free entry.