Housing Dynamics over the Business Cycle[*]

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Abstract

Over the U.S. business cycle, fluctuations in residential investment are well known to systematically lead GDP. These dynamics are documented here to be specific to the U.S. and Canada. In other developed economies residential investment is broadly coincident with GDP. Nonresidential investment has the opposite dynamics, being coincident with or lagging GDP. These observations are in sharp contrast with the properties of nearly all business cycle models with disaggregated investment. Including mortgages and interest rate dynamics aligns the theory more closely with U.S. observations. Longer time to build in housing construction makes residential investment coincident with output.

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1 Introduction

Over the U.S. business cycle, fluctuations in residential investment are well known to systematically lead real GDP (e.g., Leamer, 2007). These dynamics are found to be specific to the U.S. and Canada. In other developed economies, residential investment is, more or less, coincident with GDP. Nonresidential investment, on the other hand, has exactly the opposite dynamics in our sample of countries, being either coincident with or lagging GDP.

Such international evidence is in sharp contrast with the properties of nearly all business cycle models that disaggregate investment into residential and nonresidential. The home production models of Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), and McGrattan, Rogerson, and Wright (1997) predict exactly the opposite pattern: that home investment *lags* the cycle and business investment *leads* the cycle. A business cycle model of Gomme and Rupert (2007), featuring a more detailed disaggregation of investment and investment-specific shocks, and a multi-industry model of housing construction of Davis and Heathcote (2005) also exhibit this anomaly.[1] Gomme, Kydland, and Rupert (2001) demonstrate that while longer time to build in nonresidential—than residential—construction can reduce the discrepancy between models and data, it is not strong enough to overturn the lead-lag pattern. Fisher (2007) explores the potential role of complementarities between home and business capital. He shows that a traditional home production model can be consistent with the data if home capital positively affects labor productivity in the market sector.[2]

The first objective of this paper is to provide further empirical evidence on the dynamics of residential and nonresidential investment. We establish that even though the strong

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[1] The reason why the models predict the opposite pattern to that in the data is that output produced by business capital has more uses than output produced by home capital: the former can be either consumed or invested in both business and home capital, whereas the latter can only be consumed (e.g, as housing services). Investment in business capital thus allows greater future consumption of both types of goods, market and home. This provides a strong incentive to invest in business capital first, in response to a positive total factor productivity shock.

[2] Edge (2000), Li and Chang (2004), and Dressler and Li (2009) construct monetary models with a focus on the different responses of the two types of investment to monetary policy shocks, pointed out by Bernanke and Gertler (1995). Namely, that residential investment responds faster to such shocks than nonresidential investment.

lead of residential investment observed in the U.S. is shared only by Canada, international evidence generally does not support the lead-lag pattern inherent in business cycle models; other countries in our sample have the two types of investment, more or less, coincident with GDP. These patterns in international data are confirmed by robustness checks based on bootstrapping.

We then scrutinize the data in more detail in order to narrow down the potential sources of the cyclical dynamics of residential investment. Further analysis of U.S. data reveals that: (i) the cyclical lead of residential investment cannot be entirely attributed to Regulation Q; (ii) the lead in residential investment is driven by those structures that rely on mortgage finance; and (iii) it is primarily fixed-rate mortgages that are used to finance growth in residential investment ahead of GDP growth. In addition, the observed dynamics of the 30-year mortgage interest rate suggest that mortgages are relatively cheap ahead of an economic upturn—a feature of mortgage rate data observed also in other countries. At the same time, international data on housing starts provide insight into the cross-country differences in residential investment dynamics. In particular, they show that there is much more uniformity across countries in the dynamics of housing starts than in the dynamics of residential investment. Nearly all countries in our sample exhibit housing starts leading real GDP, which suggests that there are significant cross-country differences in residential time to build—a period over which expenditures on investment projects are incurred and recorded in national accounts. Such a possibility is confirmed by available data for the U.S. and the U.K.: U.K. time to build in residential construction appears to be twice as long as in the U.S.

After describing the data, we calibrate a business cycle model with disaggregated investment (based on Gomme et al., 2001) and show that the presence of mortgage finance in residential investment, together with the observed interest rate dynamics, aligns the theory more closely with U.S. data.[3] In particular, the model exhibits lead-lag patterns of residential and nonresidential investment similar to those in the data, while also being in

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[3]Debt finance in our model is used only for residential investment. This assumption is justified by the observation that in major developed economies, on average, nonfinancial corporations finance only 16-28% of their fixed assets through debt (Rajan and Zingales, 1995).

line with standard business cycle moments as much as other models in the literature. The quantitative effects of mortgage finance on investment dynamics are then analyzed in more detail. We summarize the effects of mortgages on the equilibrium in the form of a wedge in the Euler equation for residential capital, which resembles an ad-valorem tax on residential investment. Changes in the dynamic behavior of this wedge help us understand complicated interactions between various aspects of mortgage finance and the dynamic behavior of investment variables. Mortgage finance has not only direct effects on residential investment, but through general equilibrium it also affects nonresidential investment as households try to keep consumption relatively smooth. While mortgage finance is crucial for producing residential investment leading output, increasing time to build in residential construction pushes residential investment towards being more coincident with output, even as housing starts lead output.

Following Iacoviello (2005), a number of authors have studied housing and housing finance in business cycle models. The models of this tradition, however, usually consider only residential capital.[4] In addition, housing finance in this literature involves a sequence of one-period loans. Although it makes the models tractable, this form of finance misses a number of important features of mortgage contracts. In particular, their very long repayment periods (up to 30 years) during which the principal is gradually amortized; constant period payments (certainly in the case of a traditional fixed-rate mortgage, and in the absence of interest rate shocks also in the case of an adjustable-rate mortgage); and heavy front-loading of interest payments. We propose a fairly accurate approximation of mortgage contracts, which captures all of these three features. The approximation has only three state variables and two, easy to calibrate, parameters. Its parsimonious nature thus provides a simple way of introducing mortgages into business cycle/DSGE models that other researchers may find

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[4]The absence of nonresidential capital in these models (one of the few exceptions is, for instance, Iacoviello and Pavan, 2011) is perhaps motivated by a different focus of that literature, being predominantly concerned with the interaction between borrowing constraints, home equity loans, consumption, and monetary policy. However, as is clear from the home production literature, the presence of nonresidential capital has important implications for the equilibrium dynamics of residential capital.

useful.[5]

The paper proceeds as follows. The next section presents the empirical findings. Section 3 describes the model. Section 4 defines the equilibrium and characterizes the wedge due to mortgage finance. Section 5 calibrates the model to U.S. data and presents quantitative findings for the U.S. economy. Section 6 then investigates the quantitative effects of the various features of mortgage finance on investment dynamics and extends the model to include residential time to build. Section 7 concludes with a summary of our results and a discussion of some avenues for future research. The paper has three appendixes. Appendix A provides a description of the international data used in Section 2. Appendix B contains some additional derivations related to Section 4 and describes the computation of the equilibrium. Finally, Appendix C contains estimates of exogenous stochastic processes used for computational experiments in Sections 5 and 6.

2 Leads and lags in investment data

Our empirical analysis is based on quarterly data for the following countries and periods: Australia (1959.Q3-2006.Q4), Belgium (1980.Q1-2006.Q4), Canada (1961.Q1-2006.Q4), France (1971.Q1-2006.Q4), the U.K. (1965.Q1-2006.Q4), and the U.S. (1958.Q1-2006.Q4). Although the sample is somewhat limited, these are the only countries for which the breakdown of total investment into residential and nonresidential components is available from at least 1980 (we regard a period of about 25 years as the shortest that allows us to talk sensibly about business cycles).[6]

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[5]In the literature on housing tenure choice, Chambers, Garriga, and Schlagenhauf (2009) model mortgages in a lot more detail than we do. Their focus, however, is on steady-state analysis.

[6]Concerning other developed economies, the data are available as follows: Austria from 1988.Q1, Denmark from 1990.Q1, Finland from 1990.Q1, Germany from 1991.Q1 (annually from 1970), Ireland from 1997.Q1 (annually from 1970), Italy from 1990.Q1, the Netherlands from 1987.Q1, New Zealand from 1987.Q2, (annually from 1972), Portugal from 1995.Q1, and Spain from 1995.Q1. (The data sources are the OECD Main Economic Indicators database, the OECD National Accounts database, and national statistical agencies.) The data are also available for Japan from 1980.Q1, Norway from 1978.Q1, and Sweden from 1980.Q1. However, for the available time periods residential investment in these countries does not exhibit 'cyclical' fluctuations. Instead, in each of these countries the data are dominated by one episode: the financial and housing market crises in Norway (1987-1992) and Sweden (1990s) and the late 1980s housing boom and early

All investment data are measured as chained-type quantity indexes. The reported statistics are for logged data filtered with the Hodrick-Prescott filter; i.e., the statistics are for percentage deviations from 'trend'.[7] The cyclical behavior of a variable x is then conveniently summarized by its correlations with real GDP at various leads and lags; i.e., by $\text{corr}(x_{t+j}, GDP_t)$ for $j = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, where x_{t+j} and GDP_t are deviations from trend. We adopt the following terminology, common in modern business cycle literature: we say that a variable is *leading* the cycle (meaning leading real GDP) if the highest correlation is at $j < 0$, as *lagging* the cycle if the highest correlation is at $j > 0$, and as *coincident* with the cycle if the highest correlation is at $j = 0$.

2.1   Total, residential, and nonresidential investment

To set the stage, we start with correlations for total investment, usually referred to in national accounts as gross fixed capital formation (GFCF), one of the five main expenditure components of GDP. The correlations are presented graphically in Figure 1 (the figure caption contains the volatilities of the data). As the figure shows, in all six countries total investment is coincident with GDP. In addition, the volatility of total investment is between 2.5 times to 4 times the volatility of GDP; that is, in the ballpark of the much-cited volatility of U.S. investment, which is about 3 times as volatile as GDP. Such volatilities are also broadly in line with the prediction of a prototypical business cycle model with typical calibration.

Figure 2 displays the cross-correlations for residential and nonresidential structures (volatilities are reported in the figure caption). Residential structures include houses, apartment buildings, and other dwellings, whereas nonresidential structures include office buildings, retails complexes, production plants, etc. Together with equipment and software, residential and nonresidential structures make up GFCF. We will often refer to residential structures as 'residential investment' and to nonresidential structures as 'nonresidential investment'.[8] The

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1990s bust in Japan.

[7]Similar results are obtained also for the Christiano and Fitzgerald (2003) band-pass filter.

[8]In the case of Belgium and France the cross-correlations are for the sum of nonresidential structures and equipment and software as the two series are not available individually.

well-known empirical regularity that over the U.S. business cycle residential structures lead GDP clearly jumps out of the chart for the U.S. This chart also shows that nonresidential structures have the opposite dynamics, lagging GDP over the business cycle. Such a stark difference in the dynamic properties of residential and nonresidential investment is to a lesser extent observed also in Canada, but in the remaining countries the two types of investment tend to be, more or less, coincident with GDP.

Even though the cross-correlations in Figure 2 are useful descriptive statistics summarizing the dynamic properties of the historical data, it would be useful to have a handle on how robust these empirical regularities are. For example, in the case of Belgium, although not clearly leading (based on our definition), residential structures tend to be more strongly correlated with GDP at leads than at lags and nonresidential structures are in fact lagging GDP a little. In order to assess the significance of the leads and lags in the data, we carry out the following robustness check. Using a block bootstrap method (e.g., Hardle, Horowitz, and Kreiss, 2001), 10,000 artificial data series of the same length as the historical data are drawn for each country. Like the historical data, each artificial series is logged and filtered with the Hodrick-Prescott filter, the cross-correlations are computed, and the lead or lag (i.e., $j \in \{-4, ..., 0, ..., 4\}$) at which the highest correlation occurs is recorded. Figure 3 plots the histograms of these occurrences at the different j's.[9] For residential structures, the U.S. and Canada are the only countries for which the highest correlation is at a lead (i.e., at $j < 0$) in at least 95% of the draws, while for nonresidential structures only the U.S. has the highest correlation at a lag (i.e., at $j > 0$) in at least 95% of the draws. Nevertheless, with the exception of Belgium, all countries exhibit residential investment either leading or coincident with GDP; i.e., the highest correlation occurring at $j \leq 0$ in more than 95% of the draws. And, with the exception of the U.K., they exhibit nonresidential investment either lagging or coincident with GDP; i.e., the highest correlation occurring at $j \geq 0$ in more than 95% of the draws. The predictions of business cycle models with dis-

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[9]The length of each block in the bootstrap is set equal to 20 quarters, which is sufficient to address the serial correlation in the historical data.

aggregated investment, as reviewed in the Introduction, are thus not supported by available international data. (Note that even in Belgium residential investment is not lagging, based on the 95% confidence level, and in the U.K. nonresidential is not leading, based on the same confidence level.)

2.2 Housing starts

While the U.S. and Canada look clearly different from the other countries in terms of the cyclical lead of residential structures, there is much more uniformity across the six countries in terms of the dynamics of housing starts.[10] The start of construction is defined across countries consistently as the beginning of excavation for the foundation of a residential building (single family or multifamily) and every month detailed surveys of home builders record the number of such activities.

The top half of Figure 4 plots the cross-correlations with GDP for the historical data (volatilities are in the figure caption). As is immediately apparent, housing starts lead GDP in all countries, possibly with the exception of Belgium. Using a similar robustness check as in the case of structures, the lead occurs in at least 95% of the draws in the cases of Canada, the U.K., and the U.S. And if the significance level is lowered to 90%, then also in the case of Australia and France, as the bottom half of Figure 4 shows.[11] Together with the data on residential investment, the data on housing starts suggest cross-country differences in completion times (time to build) in residential construction. Longer time to build means that investment expenditures on a housing project are recorded in national accounts over a

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[10]The time periods used for housing starts differ slightly from the time periods used for residential structures due to different data availability. Housing starts are for the following periods: Australia (1965.Q3-2006.Q4), Belgium (1968.Q1-2006.Q4), Canada (1960.Q1-2006.Q4), France (1974.Q1-2006.Q4), and the U.S. (1959.Q1-2006.Q4). For the U.K., residential building permits are used instead of starts as the data on starts are available only from 1990.Q1. Based on a strong comovement between the two data series during the period 1990.Q1-2006.Q4, we take permits as a proxy for starts. For all countries the data come from the OECD MEI database.

[11]In the case of Belgium, even though starts do not lead, residential building permits lead by three quarters, based on the cross-correlogram for historical data (based on the bootstrap test, however, a lead is significant only at a 70% confidence level). In the other countries, building permits and starts exhibit essentially the same lead.

longer period of time. Residential investment thus may not exhibit a cyclical lead in countries with longer time to build even when housing starts do. Empirical evidence on cross-country differences in residential time to build is discussed below.

2.3 Further details on the dynamics of residential structures

Available details on the different types of residential construction in the U.S., and a comparison of the data across time periods, provide an insight into the potential sources of the cyclical lead of U.S. residential investment. A comparison of some of the details with available evidence from other countries also provides information about the sources of the cross-country differences documented above. We first discuss the relevant characteristics of the different types of residential structures and time periods and then present the findings.

2.3.1 Single family vs multifamily structures

Most of residential construction in the U.S. is accounted for by single family structures (houses). Their share in residential investment is five times as large as the share of multi-family structures (mainly apartment buildings). Whereas new houses are primarily built for owner occupancy, most apartment buildings are built to rent (historical data from Census Bureau's Survey of Construction).[12] For our purposes, the main differences between the two types of structures are two-fold. First, time to build is longer for multifamily than for single family structures. Based on historical data from the Survey of Construction, the average period from start to completion for a typical single family structure is 6.2 months (5.6 months if only built-for-sale houses, as opposed to custom-built houses, are counted). For multifamily structures the average construction time is 10 months for all structure types and 13 months for 20+ unit structures, which make up the majority of multifamily construction.

Second, ownership of a house is financed differently from ownership of a multifamily

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[12]Most of the historical data from the Survey of Construction used in this section are from either early 1960s or early 1970s to 2006.

structure.[13] House purchase finance is relatively simple and standardized. Based on historical data from the Survey of Construction, on average 76% of new houses are financed through a 30-year conventional mortgage (this includes also subprime and Alt-A mortgages not reported separately), 18% through FHA/VA insured mortgages, and 6% are paid for with cash. And the average loan-to-value ratio of conventional mortgages for newly-built homes has been relatively stable at 76% (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).[14] Debt thus plays a major role in financing newly-built house purchases and its importance has been relatively stable over time. In contrast, financing acquisitions of new multifamily structures is more involved, heterogenous, and, as discussed below, has changed dramatically over time.

2.3.2 Structural changes in housing finance in the 1980s

There are two reasons for splitting the U.S. sample period 1959.Q1-2006.Q4 into two sub-periods in 1984. First, it is often argued that Regulation Q was responsible for residential construction booms and busts in the U.S. before the 1980s, causing boom and bust cycles in the wider economy (e.g., Bernanke, 2007). This regulation set ceilings on interest rates that savings banks and savings and loans—the main mortgage lenders at the time—were allowed to pay on deposits. Regulation Q was eventually abolished in 1980 and largely phased out during the following four years. Second, the method of financing multifamily housing changed dramatically. As discussed by Bradley, Nothaft, and Freund (1998) and Colton and Collignon (2001), up until mid- to late 1980s limited partnerships, financing apartment housing through mortgages, have been the dominant form of apartment ownership in the U.S. Since then, however, they have been replaced by equity real estate investment trusts (REITs). As a result there has been substantial substitution of equity for debt as a

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[13]Construction, as opposed to the ultimate ownership, is in both cases typically financed by a short-term construction loan obtained by a home builder or a developer from a bank.

[14]The data on loan-to-value ratios exclude subprime and Alt-A mortgages. Their importance in the aggregate has been, however, isolated only to the last three years of our sample.

means of financing apartment housing.[15]

2.3.3 Findings

The first two panels of Table 1 report the cross-correlations with GDP, as well as volatilities, for key data related to single family and multifamily housing investment in the U.S. The first panel is for the period 1958.Q1-1983.Q4, while the second panel is for the period 1984.Q1-2006.Q4. The first two rows in each panel are for the single family and multifamily components of residential investment in national accounts, followed by starts and completions. These 'construction data' are then complemented with 'financing data'. Namely, the net change in real mortgage debt outstanding obtained from the Flow of Funds Accounts, Table F.217.[16]

From the first panel of Table 1 we see that single family structures clearly lead GDP in the first period (1958.Q1-1983.Q4), with the highest correlation coefficient of 0.73 at $j = -2$. Multifamily structures are, in contrast, coincident with GDP, with the highest correlation coefficient of 0.51 at $j = 0$. In terms of starts, however, both types of structures lead GDP, with both having the highest correlation coefficient at $j = -2$ (0.70 and 0.61, respectively). The reason why multifamily structure investment from the national accounts is coincident with the cycle is a longer time to build. As noted above, it takes about four quarters to complete most multifamily housing construction, compared with just two quarters (at the most) for single family houses. This is reflected in the dynamics of completions: while completions of single family structures peak at $j = -1$, one quarter after the peak of starts, completions of multifamily structures peak at $j = 2$, four quarters after the peak of starts. Notice also that both single family and multifamily mortgages lead GDP, with the highest

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[15]Significant changes occurred also in the market for single family housing finance. These changes, however, occurred on the side of mortgage lenders—deregulation of the primary mortgage market and development of a liquid secondary mortgage market through securitization (see, e.g., Green and Wachter, 2005). Mortgage debt, nevertheless, remained the main source of finance.

[16]Flow of Funds tables report *home* mortgages, defined as mortgages for 1-4 family properties, and *multifamily* mortgages, defined as mortgages for 5+ family properties. The fraction of new construction accounted for by 2-4 family properties is, however, negligible (completions data from the Survey of Construction). Home mortgages are thus a good proxy for single family property mortgages.

correlation coefficients of 0.69 and 0.46, respectively, at $j = -2$, the same as that for starts.

There are three key observations concerning the second period (1984.Q1-2006.Q4). First, investment in single family structures still leads GDP, even though the cross-correlations at all leads and lags are weaker than in the first period. Starts, completions, and single family mortgages have also similar dynamics to those in the first period, even though again the correlations are weaker.[17] Thus, although Regulation Q likely played a role in the cyclical dynamics of residential investment in the first period, perhaps accounting for the stronger correlations with GDP, it cannot be the only reason for why movements in residential investment precede movements in GDP. An additional argument against Regulation Q being the main source of such dynamics is that a clear lead in residential investment is observed also in Canadian data, especially for single family structures (the third panel of Table 1). Unlike U.S. mortgage lenders, Canadian banks did not face constraints such as those imposed by Regulation Q (Lessard, 1975).

Second, multifamily residential investment in the second period behaves like nonresidential investment in the sense that it lags GDP; starts are coincident with GDP and completions lag GDP by three quarters.[18] Interestingly, this is despite the fact that mortgages for multifamily housing still lead GDP, even though, like in the case of single family housing, they are much more volatile and the correlations are weaker than in the first period. Such decoupling between mortgage finance and construction in the multifamily sector is consistent with the increased role of equity finance in multifamily housing noted above.

Third, the lead in single-family mortgages is due to fixed-rate mortgages (FRMs) rather than adjustable-rate mortgages (ARMs). Up until early 1980s, the only mortgage type available in the U.S. was essentially a 30-year FRM. But with the start of the 1980s, ARMs

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[17]The mortgage data are especially substantially less correlated with GDP at all leads and lags than in the first period. In addition, they are much more volatile. This is even after home equity loans (broadly available from 1991) have been stripped out of the data. A likely explanation for the low correlations and the high volatility is refinancing, which became much more accessible during the 1980s.

[18]The generally weaker cross-correlations of multifamily structures with GDP in the second period are likely due to shocks specific to that market segment that occurred in the early and mid- 1980s. As discussed by Colton and Collignon (2001), changes in the U.S. tax code in 1981 (Economic Recovery Tax Act) provided strong incentives for apartment construction. Most of these incentives were, however, eliminated by the 1986 Tax Reform Act.

became an integral part of the U.S. mortgage market. Accounting on average for about 30% of all mortgages for newly-built single-family homes (FHFA, Monthly Interest Rate Survey, Table 18), their popularity has fluctuated over time. As Table 1 reveals, over the business cycle their fraction in total mortgages for single family newly-built homes moves in tandem with GDP without any lead or lag. This leaves the bulk of the lead in single family mortgages to be accounted for by FRMs.

We close this subsection by following up on our previous discussion regarding cross-country differences in completion times. The bottom panel of Table 1 reports the dynamics of starts and completions in the U.K (the only other country for which completions data are available; unlike in the U.S., direct measurement of completion times is not available). As we can see, U.K. completions tend to peak three to four quarters after starts, an indication of possibly twice as long time to build in the U.K. than in the U.S. (single family homes).[19]

To sum up, we draw the following lessons from this subsection: (i) the cyclical lead of U.S. residential investment cannot be entirely attributed to Regulation Q; (ii) the lead is driven by those structures that rely on mortgage finance; and (iii) it is primarily FRMs that are used to finance growth in residential investment ahead of GDP growth. In addition: (iv) there may be significant differences in residential time to build across countries, perhaps due to technological, supply chain, or regulatory constraints, or a different composition of residential investment in terms of single- and multifamily structures; Ball (2003) provides an overview of the structure and practices of housebuilding industries in different countries, which points to a large variation across countries along these dimensions.

2.4 Dynamics of mortgage rates

The last piece of empirical observation we report concerns the cyclical dynamics of the mortgage rate—the nominal interest rate on mortgage loans. Even though by itself it does not reflect the true costs of mortgage finance to consumers—which, as we show in the next

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[19]Completions data for the U.K. come from the Department of Communities and Local Government, Housing Statistics, Table 222.

section, depend on the present value of real mortgage payments (interest and amortization) over the lifetime of the mortgage—the mortgage rate may indicate how the costs behave over the business cycle. According to the literature (e.g., Scanlon and Whitehead, 2004; Calza, Monacelli, and Stracca, forthcoming) the countries in our sample can be described as either FRM or ARM countries. For each country we use the interest rate for the country's most common mortgage product, as reported in the above studies.

The cross-correlations of mortgage rates with GDP (and their volatilities) are reported in the first panel of Table 2, which reveals a common pattern across countries: mortgage rates are generally negatively correlated with future GDP and positively correlated with past GDP. Thus, on average, mortgage rates are relatively low before a GDP peak, tend to increase as GDP increases, and reach their peak a few quarters after a peak in GDP. The second panel, which reports the same statistics for government bond yields, shows that the cyclical dynamics of mortgage rates reflect the general behavior of nominal interest rates over the business cycle, rather than factors specific to the mortgage market (for FRM countries we take par yields on coupon government bonds of maturities close to the periods for which FRM mortgage rates are fixed; for ARM countries we take 3-month Treasury bill yields, as mortgage rates on ARMs are set, after some initial period, as a constant margin over a short-term government bond yield).[20] Because it is real, not nominal, mortgage payments that matter in the model of the next section, the last panel of Table 2 reports the dynamics of inflation rates. We see that, with the exception of Belgium, the lead-lag pattern of inflation rates is similar to that of nominal interest rates.

3   A business cycle model with mortgages

The findings of the previous section suggest that mortgage finance may be a key factor behind the observed lead of residential investment in the U.S. business cycle. Time-to-build in residential construction may then affect the extent of such lead. To evaluate these

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[20]For future reference we also include for the U.S. the yield on 3-month Treasury bills.

conjectures within a theoretical framework, we introduce mortgages into a business cycle model with disaggregated investment. We build on Gomme et al. (2001), henceforth referred to as GKR, which shares with other models in the literature the property that in equilibrium residential investment lags and nonresidential investment leads output (refer to footnote 1 for the intuition behind this common result).

Before getting into details, it is worth pointing out two aspects of the model. First, mortgage and inflation rates are exogenous—they follow a joint VAR(n) process with TFP (a government ensures that the economy's resource constraint is satisfied when these prices are exogenous). This is motivated by practical considerations: given our question, it is important to capture the lead-lag relationship between output on one hand and interest and inflation rates on the other, as summarized by Table 2. Unfortunately, existing literature does not provide a mechanism generating such dynamics endogenously.[21] Second, we do not model the underlying frictions giving rise to mortgages—mortgage finance is simply imposed on residential investment. Modeling demand for housing finance from first principles would make the model too large (in terms of the state space) for business cycle analysis. For similar reasons we also abstract from refinancing and default.[22]

3.1 Preferences and technology

A representative household has preferences over consumption of a market-produced good c_{Mt}, a home-produced good c_{Ht}, and leisure, which is given by $1 - h_{Mt} - h_{Ht}$, where h_{Mt} is time spent in market work and h_{Ht} is time spent in home work. The preferences are

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[21]For a discussion of this issue see Canzoneri, Cumby, and Diba (2007), Atkeson and Kehoe (2008), and Sustek (2011).

[22]Gervais (2002), Rios-Rull and Sanchez-Marcos (2008), and Chambers et al. (2009) develop models with many of the micro-level features we abstract from. Their focus, however, is on steady-state analysis. Campbell and Cocco (2003) model a single household's mortgage choice that includes refinancing. Corbae and Quintin (2011) and Chatterjee and Eyigungor (2011) construct models with foreclosures, focusing on steady-state equilibria. Iacoviello and Pavan (2011) construct a model with similar features as in Gervais (2002) and with aggregate shocks. Housing finance in their model, however, takes the form of trading a one-period bond.

summarized by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - h_{Mt} - h_{Ht}\right), \quad \beta \in (0,1), \tag{1}$$

where $u(.,.)$ has all the standard properties and c_t is a composite good, given by a constant-returns-to-scale aggregator $c(c_{Mt}, c_{Ht})$. Time spent in home work is combined with home capital k_{Ht} to produce the home good according to a production function

$$c_{Ht} = A_H G(k_{Ht}, h_{Ht}), \tag{2}$$

where $G(.,.)$ has all the standard properties. In contrast to the home production literature, we abstract from durable goods and equate home capital with residential structures when mapping the model to data. We will therefore refer to home capital as 'residential capital'.[23]

Output of the market-produced good y_t is determined by an aggregate production function

$$y_t = A_{Mt} F(k_{Mt}, h_{Mt}), \tag{3}$$

operated by identical perfectly competitive firms. Here, A_{Mt} is total factor productivity (TFP) and k_{Mt} is market capital, which we will refer to as 'nonresidential capital'.[24] Firms rent labor and capital services from households at a wage rate w_t and a capital rental rate r_t, respectively. The market-produced good can be used for consumption, investment in residential capital, x_{Ht}, and investment in nonresidential capital, x_{Mt}. Nonresidential capital has a J-period time to build, where J is an integer greater than one. Specifically,

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[23]c_{Ht} is thus consumption of housing services and h_{Ht} is interpreted as time devoted to home maintenance and leisure enjoyed at home, rather than in a bar. Under enough separability in utility and production functions, which will be imposed under calibration, the period utility function can be rewritten such that it is a function of c_{Mt}, h_{Mt}, and k_{Ht} (Greenwood, Rogerson, and Wright, 1995). This makes it comparable with models that put housing directly in the utility function.

[24]Notice that in contrast to A_{Mt}, which is time varying (due to shocks), A_H is constant. GKR show that under enough separability in utility and production functions, which will be imposed under calibration, shocks to A_H do not affect market variables (i.e., time spent in market work, consumption of the market-produced good, and accumulation of the two types of capital). This is convenient as it allows us to abstract from home-production TFP shocks, which cannot be measured outside of the model.

an investment project started in period t becomes a part of the capital stock only in period $t + J$. However, the project requires resources throughout the construction process from period t to $t + J - 1$. In particular, a fraction $\phi_j \in [0, 1]$ of the project must be invested in period $t + J - j$, $j \in \{1, ..., J\}$, where j denotes the number of periods from completion and $\sum_{j=1}^{J} \phi_j = 1$. Let s_{jt} be the size of projects that in period t are j periods from completion. Total nonresidential investment (i.e., investment across all on-going projects) in period t is thus

$$x_{Mt} = \sum_{j=1}^{J} \phi_j s_{jt} \tag{4}$$

and the projects evolve as

$$s_{j-1,t+1} = s_{jt}, \quad j = 2, \ldots, J, \tag{5}$$

$$k_{M,t+1} = (1 - \delta_M)k_{Mt} + s_{1t}, \tag{6}$$

where $\delta_M \in (0, 1)$. For now, residential capital is assumed to have only one-period time to build and therefore

$$k_{H,t+1} = (1 - \delta_H)k_{Ht} + x_{Ht}, \tag{7}$$

where $\delta_H \in (0, 1)$. The assumptions regarding time to build of the two types of capital are the same as in GKR. They are motivated by the observation that in the U.S. nonresidential structures take much longer to complete than residential structures, especially single-family houses.

3.2   Mortgages

So far the setup is exactly the same as in GKR. What makes the current model different is that residential investment is subject to a financing constraint

$$l_t = \theta p_t x_{Ht}, \tag{8}$$

where l_t is the nominal value of mortgage loans, $\theta \in [0, 1)$ is a loan-to-value ratio, and p_t is the aggregate price level (the price of the market-produced good in dollars). Mortgage debt requires that the household makes regular payments throughout the life of the mortgage. The household's budget constraint is thus

$$c_{Mt} + x_{Mt} + x_{Ht} = (1 - \tau_r)r_t k_{Mt} + (1 - \tau_w)w_t h_{Mt} + \delta_M \tau_r k_{Mt} + \frac{l_t}{p_t} - \frac{m_t}{p_t} + \tau_t, \qquad (9)$$

where τ_r is a tax rate on income from nonresidential capital, τ_w is a tax rate on labor income, m_t are mortgage payments on outstanding mortgage debt, and τ_t is a lump-sum transfer.[25] Mortgage payments are given as

$$m_t = (R_t + \delta_{Dt})d_t, \qquad (10)$$

where d_t is nominal mortgage debt outstanding, R_t is an *effective* net interest rate on the outstanding mortgage debt, and $\delta_{Dt} \in (0, 1)$ is an *effective* amortization rate of the out-standing mortgage debt. Notice that $\delta_{Dt} \in (0, 1)$ implies that $m_t > R_t d_t$; i.e., a part of the outstanding debt is amortized each period. The variables d_t, R_t, and δ_{Dt} are state variables evolving recursively according to these laws of motion

$$d_{t+1} = (1 - \delta_{Dt})d_t + l_t, \qquad (11)$$

$$\delta_{D,t+1} = (1 - \nu_t)\delta_{Dt}^\alpha + \nu_t \kappa, \qquad \alpha, \kappa \in (0, 1), \qquad (12)$$

$$R_{t+1} = \begin{cases} (1 - \nu_t)R_t + \nu_t i_t & \text{if FRM,} \\ i_t & \text{if ARM.} \end{cases} \qquad (13)$$

Here, $\nu_t \equiv l_t/d_{t+1}$ is the share of current loans in the new stock of debt and $(1 - \nu_t) \equiv (1 - \delta_{Dt})d_t/d_{t+1}$ is the share of outstanding unamortized debt in the new stock of debt. In

[25] τ_r and τ_w are constant and, as in the rest of the home production literature, are introduced into the model purely for calibration purposes; τ_t is time-varying and its role is to ensure that the economy's resource constraint holds.

addition, i_t is the net interest rate (either fixed or adjustable) on current loans and α and κ are parameters controlling the evolution of the amortization rate, which is described in further detail below. Notice that the assumption $\alpha, \kappa \in (0, 1)$ implies that $\delta_{Dt} \in (0, 1)$ for all t, as assumed above. Notice also that combining equations (10) and (11) gives the evolution of mortgage debt in a more familiar form: $d_{t+1} = (1 + R_t)d_t - m_t + l_t$. Given that most countries can be characterized as either FRM countries or ARM countries, the household in the model operates only under either FRM or ARM environment.

3.2.1 An example and assessment of the mortgage

It is worth pausing here to explain in a little more detail the laws of motion (11)-(13) and their implications for the time path of mortgage payments, given by equation (10). For this purpose, let us suppose that the representative household has no outstanding mortgage debt and takes a fixed-rate mortgage in period $t = 0$ in the amount $l_0 > 0$. Let us further assume that the household does not take any new mortgage loans in subsequent periods (i.e., $l_1 = l_2 = ... = 0$). Equations (10)-(13) then yield the following path of mortgage payments: In period $t = 1$, the household's outstanding debt is $d_1 = l_0$, the initial amortization rate at which this debt will be reduced going into the next period is $\delta_{D1} = \kappa$, and the effective interest rate is $R_1 = i_0$. Mortgage payments in $t = 1$ are thus $m_1 = (R_1 + \delta_{D1})d_1 = (i_0 + \kappa)l_0$. In period $t = 2$ the outstanding debt is $d_2 = (1 - \kappa)l_0$ and is reduced at a rate $\delta_{D2} = \kappa^\alpha > \kappa$ going into the next period. The interest rate R_2 is again equal to i_0. Mortgage payments in $t = 2$ are thus $m_2 = (R_2 + \delta_{D2})d_2 = (i_0 + \kappa^\alpha)(1 - \kappa)l_0$ and so on. Notice that whereas the interest part of mortgage payments, $R_t d_t$, declines as debt gets amortized, the amortization part, $\delta_{Dt}d_t$, may increase if, for a given κ, α is sufficiently small. The parameter α thus allows us to calibrate the model such that m_t is approximately constant for a 'sufficiently long' period, thus approximating the constant mortgage payments during the lifetime of a typical mortgage contract.

Figure 5 provides a numerical example to illustrate these points further and to assess

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how well the mortgage in the model approximates a real-world contract. Here, one period corresponds to one quarter, $l_0 = \$250,000$, $i_0 = 9.28\%/4$ (the long-run average mortgage rate for a U.S. 30-year conventional FRM), $\alpha = 0.9946$, and $\kappa = 0.00162$. Panels A and B plot mortgage payments, m_t, and outstanding debt, d_t, respectively, for 120 quarters. Panel C then plots the shares of interest payments, $R_t d_t$, and amortization payments, $\delta_{Dt} d_t$, in mortgage payments, m_t. For comparison, the panels also plot the same variables obtained from a Yahoo mortgage calculator for a U.S. 30-year conventional FRM in the same amount and with the same interest rate. We see that the model captures two key features of the conventional mortgage. First, mortgage payments based on the calculator are constant; in the model they are approximately constant for the first 70 or so periods (17.5 years). Second, interest payments are front-loaded: they make up most of mortgage payments at the beginning of the life of the mortgage and their share gradually declines; the opposite is true for amortization payments.[26] How good is this approximation? By comparing the time paths in panel A, one may conclude that the approximation is poor, as after the 70th period the payments in the model significantly deviate from the payments in the real-world contract. Such conclusion would, however, be misguided. This is because mortgage payments far out are heavily discounted and thus matter little for decisions in period 0. A more suitable metric is therefore to measure the deviations in present value terms (we use $1/i$ as the annual discount factor), normalized by the size of the loan (i.e., \$250,000). This metric is plotted in panel D of the figure, which shows that throughout the 120 periods the approximation error is of the order of magnitude of $1e^{-4}$. The sum of these present-value errors is equal to about 1% of the size of the loan. For comparison, when all monetary transaction costs of obtaining a real-world mortgage are counted (costs that we abstract from), they usually add up to at least 3% of the amount borrowed.[27]

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[26]If α was equal to one, the share of interest payments in m_t would be constant and m_t would decline linearly throughout the lifetime of the mortgage.

[27]If we were to plot the time paths of m_t and d_t in the model beyond period 120, the picture would show that both indeed converge to zero, making also the approximation error in panel D to converge to zero.

3.2.2 The general case

So far we have only considered once-and-for-all investment. Of course, in response to shocks, the representative household adjusts x_{Ht}, and thus l_t, every period. In this case, δ_{Dt} and R_t are the effective amortization and interest rates, respectively, on the economy-wide stock of mortgage debt. Over time the effective amortization rate evolves as the weighted average of the amortization rates on the stock and the flow, with the weights being the relative sizes of the stock and the flow. Similarly, the effective interest rate evolves as the weighted average of the interest rates on the stock and the flow.

The advantage of our approximation lies in its parsimonious nature. It effectively replaces 120 vintages of mortgage debt, each with a different amortization and interest rate, with just three state variables and two parameters. This should make it easy to introduce mortgages into a variety of models, including those with a host of different frictions and shocks.[28]

3.3 Exogenous process and closing the model

As mentioned above, the inflation rate $\pi_t \equiv \log p_t - \log p_{t-1}$ and the current mortgage rate i_t follow a joint VAR(n) process with market TFP: $z_{t+1}b(L) = \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim N(0, \Sigma)$, $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$, $b(L) = I - b_1 L - ... - b_n L^n$ (L being the lag operator), and $\Sigma = BB'$. The model is closed by a government budget constraint. The government collects revenues from capital and labor taxes and operates the mortgage market by providing mortgage loans and collecting mortgage payments. Each period the government balances out its budget by lump-sum transfers to the household: $\tau_t = \tau_r r_t k_{Mt} + \tau_w w_t h_{Mt} - \tau_r \delta_M k_{Mt} + m_t/p_t - l_t/p_t$.

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[28]Even though, following Iacoviello (2005), many DSGE models include housing and housing finance, they do not have debt contracts resembling mortgages. Instead, housing is financed by a sequence of one-period loans. The interest rate applied to a loan is either the current short-term interest rate (e.g., Iacoviello, 2005, and many others), a weighted average of the current and past interest rates (Rubio, 2011), or evolving in a sticky Calvo-style fashion (Graham and Wright, 2007). Calza et al. (forthcoming) model FRMs as two-period contracts in which half of the principal and half of the total interest is paid each period; an ARM contract in their model is a one-period loan.

4 Equilibrium effects of mortgages

This section defines the equilibrium and shows how the equilibrium effects of mortgages can be conveniently summarized by a wedge in an Euler equation for housing investment. In the following sections this wedge will help us understand the interactions between the parameters of mortgage finance and the dynamics of residential and nonresidential investment in the model. Due to space constraints, equilibrium conditions that are not essential for the current discussion are relegated to Appendix B. This appendix also describes the computation of the equilibrium.

The equilibrium is defined as follows: (i) the representative household solves its utility maximization problem, described below, taking all prices and transfers as given; (ii) r_t and w_t are equal to their marginal products; (iii) the government budget constraint is satisfied; and (iv) the exogenous variables follow the VAR(n) process. The aggregate resource constraint, $c_{Mt} + x_{Mt} + x_{Ht} = y_t$, then holds by Walras' Law. To characterize the equilibrium, it is convenient to work with a recursive formulation of the household's problem. The Bellman equation is

$$V\left(s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right) = \max\{u\left(c_t, 1 - h_{Mt} - h_{Ht}\right)$$

$$+\beta E_t V\left(s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\},$$

subject to (2) and (4)-(13). After substituting the constraints into the Bellman equation, the maximization is only with respect to $(h_{Mt}, h_{Ht}, s_{Jt}, x_{Ht})$. Here, x_{Ht} affects the period utility function, through its effect on l_t in the budget constraint, and the value function, through its effect on the laws of motion for $k_{H,t+1}$, d_{t+1}, $\delta_{D,t+1}$, and R_{t+1}. There is enough separability in this problem that the variables related to mortgage finance (l_t, d_t, δ_{Dt}, R_t, i_t, π_t) show up only in the first-order condition for x_{Ht}, which is

$$u_{1t}c_{1t}(1 - \theta) - \theta\beta E_t\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}. \quad (14)$$

Here, $\zeta_{Dt} \equiv \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right) / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$, $\widetilde{d}_t \equiv d_t/p_{t-1}$ and $V_{kH,t}$, V_{dt}, $V_{\delta_D,t}$, and V_{Rt} are the derivatives of the value function with respect to the state variables specified in the subscript.[29] The variables $V_{d,t+1}$ and d_t are transformed in order to ensure their stationarity. It is convenient to rearrange the first-order condition as

$$u_{1t}c_{1t}(1+\tau_{Ht}) = \beta E_t V_{kH,t+1}, \tag{15}$$

where

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t \widetilde{V}_{d,t+1}}{u_{1t}c_{1t}} + \frac{\beta\left[\zeta_{Dt}(\kappa - \delta_{Dt}^\alpha)E_t V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)E_t V_{R,t+1}\right]}{u_{1t}c_{1t}} \right\} \tag{16}$$

is an *endogenous* 'wedge'. For $\tau_{Ht} = 0$, equation (15) has a simple interpretation: it equates marginal utility of market consumption today with discounted expected marginal lifetime utility of housing. The wedge acts like an ad-valorem tax, making an additional unit of housing more or less expensive in terms of current market consumption (the wedge can be positive or negative, depending on parameter values and exogenous shocks). In GKR, there is no mortgage finance. Indeed, if $\theta = 0$, the wedge is equal to zero and the equilibrium is the same as in their model. Thus, under $\theta = 0$ the model exhibits the same dynamics as in GKR: x_{Ht} lagging and x_{Mt} leading. The question is if for $\theta \in (0,1)$, calibrated to the data, the wedge moves in such a way as to overturn this results and reproduce the lead-lag pattern in the data.

The derivatives of the value function are given by Benveniste-Scheinkman conditions. Here we focus only on $V_{kH,t}$ and $\widetilde{V}_{dt}$ ($V_{\delta_D,t}$ and V_{Rt} are contained in Appendix B). For $V_{kH,t}$ the condition is $V_{kH,t} = u_{1t}c_{2t}A_H G_{1t} + \beta(1-\delta_H)E_t V_{kH,t+1}$. It states that marginal lifetime utility of housing is given as the expected discounted sum of per-period marginal utilities of

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[29]We also adopt the convention of denoting, for example, by u_{2t} the first derivative of the u function with respect to its second argument.

housing over its lifetime. For $\widetilde{V}_{dt}$, the Benveniste-Scheinkman condition is

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\frac{R_t + \delta_{Dt}}{1 + \pi_t} + \beta\frac{1 - \delta_{Dt}}{1 + \pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\delta_{Dt}^\alpha - \kappa)V_{\delta_D,t+1} + \zeta_{xt}(R_t - i_t)V_{R,t+1}\right],$$
(17)

where $\zeta_{xt} \equiv \theta x_{Ht}/\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$. Notice that this condition simplifies when either i) new loans are the same as old loans (i.e., $\delta_t^\alpha = \kappa$ and $R_t = i_t$) or ii) we consider again a once-and-for-all house purchase, implying that $\zeta_{Dt} = 0$ and $\zeta_{x,t+j} = 0$ for all $j = 1, 2, \dots$ In these special cases equation (17) becomes

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\left(\frac{R_t + \delta_{Dt}}{1 + \pi_t}\right) + \beta\left(\frac{1 - \delta_{Dt}}{1 + \pi_t}\right)E_t\widetilde{V}_{d,t+1},$$
(18)

which has a simple interpretation: the marginal value of mortgage debt is given as the expected discounted sum of marginal per-period real mortgage payments, weighted by the marginal utility of market consumption, over the lifetime of the mortgage debt. Notice that if mortgages were modeled as a one-period loan, this condition would simplify further to a familiar $\widetilde{V}_{dt} = -u_{1t}c_{1t}(1 + R_t)/(1 + \pi_t)$, where $R_t = i_{t-1}$.

Similarly, in the special cases (i) or (ii), the expression for the wedge (16) simplifies to

$$\tau_{Ht} = -\theta\left[1 + \beta E_t\widetilde{V}_{d,t+1}/(u_{1t}c_{1t})\right].$$
(19)

Combining this equation with equation (18) provides a clear interpretation of the wedge: the wedge is equal to $-\theta$ times the difference (as $\widetilde{V}_{d,t+1}$ is negative) between the equity cost of financing an additional unit of housing, which is a foregone unit of market consumption today, and the debt cost of doing so, which is the present value of foregone market consumption in the future. Other things being equal, when the debt cost declines (i.e., $\widetilde{V}_{d,t+1}$ declines in absolute value), the wedge declines, leading to more residential investment.

Of course, the household in the model chooses x_{Ht} every period in response to shocks and new (i.e., marginal) debt has a different amortization rate and a different interest rate

(under FRM) than outstanding debt. The terms $\zeta_{Dt}(\kappa - \delta^{\alpha}_{Dt})V_{\delta_D,t+1}$ and $\zeta_{Dt}(i_t - R_t)V_{R,t+1}$ in the general expression for the wedge (16), and the terms $\zeta_{xt}(\delta^{\alpha}_{Dt} - \kappa)V_{\delta_D,t+1}$ and $\zeta_{xt}(R_t - i_t)V_{R,t+1}$ in the general Benveniste-Scheinkman condition (17), account for this fact. Without these terms the first-order condition for x_{Ht} would state that the marginal effect on period mortgage payments of *new* mortgage debt, l_t, is $R_t + \delta_{Dt}$, i.e., the sum of the effective interest and amortization rates on the *outstanding* stock. The term $\zeta_{Dt}(\kappa - \delta^{\alpha}_{Dt})V_{\delta_D,t+1}$ in equation (16), for instance, 'corrects' for the fact that new debt has a lower amortization rate than outstanding debt ($\kappa < \delta^{\alpha}_{Dt}$). (Put differently, the extra terms appear in the optimality condition because, even though the optimization is with respect to the state variables d_{t+1}, $\delta_{D,t+1}$, and R_{t+1}, it is only the effect on lifetime utility of the new loan that matters for the optimal decision.)

For future reference we note that the wedge depends on the following features of mortgage finance: (i) the loan-to-value ratio θ; (ii) how fast the loan is amortized (governed by α and κ); (iii) whether the loan is a FRM or ARM (which matters for the future paths of R_t); and (iv) the dynamics of $[\log A_{Mt}, i_t, \pi_t]$, imbedded in the conditional expectation operator E_t, which is based on the exogenous VAR(n) process.

5   Quantitative results for the U.S. economy

This section presents quantitative findings for the U.S. economy, which we take as a benchmark case. After describing the calibration we report the results, leaving much of the explanation of how the various model features affect the results for the next section.

5.1   Calibration

The parameter values are summarized in Table 3. One period in the model corresponds to one quarter and the functional forms are as in GKR: $u(.,.) = \omega \log c + (1-\omega) \log(1 - h_M - h_H)$; $c(.,.) = c^{\psi}_M c^{1-\psi}_H$; $G(.,.) = k^{\eta}_H h^{1-\eta}_H$; and $F(.,.) = k^{\varrho}_M h^{1-\varrho}_M$. The parameter A_H is normalized to be equal to one and the value of A_{Mt} in a nonstochastic steady state is chosen so that y_t

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in the nonstochastic steady state is equal to one.

As mentioned above, we abstract from consumer durable goods. The data equivalent to y_t is thus GDP less expenditures on consumer durable goods. Nonresidential capital in the model is mapped into the sum of nonresidential structures and equipment & software (equipment & software is, more or less, coincident with GDP, although it is more strongly positively correlated with GDP at lags than at leads). If only nonresidential structures were used as the data equivalent to k_{Mt}, the share of capital income in GDP, ϱ, would be too low, making the model's dynamic properties difficult to compare with the literature. Because k_{Mt} includes equipment & software, we set J equal to 4 and ϕ_j equal to 0.25 for all j. These are the same choices as those of GKR. The parameter ϱ is set equal to 0.283, based on measurement from the National Income and Product Accounts (NIPA) obtained by Gomme, Ravikumar, and Rupert (2011). We also use their NIPA-based estimate of $\tau_w = 0.243$. The depreciation rates are given as the average ratios of investment to the corresponding capital stocks. This yields $\delta_H = 0.0115$ and $\delta_M = 0.0248$. These are a little higher than the average depreciation rates from BEA Fixed Assets Accounts because the model abstracts from long-run population and TFP growth.

The parameter θ is set equal to 0.76, the average loan-to-value ratio for conventional single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10, 1963-2006). As noted in Section 2.3.1, this ratio has been fairly stable over time. The values of the steady-state mortgage interest rate i and of the parameters α and κ are the same as those in Section 3.2.1: $i = 9.28\%$ per annum, $\alpha = 0.9946$, and $\kappa = 0.00162$.[30] Given these values, the law of motion (12) implies a steady-state amortization rate of 0.0144, which, as in the U.S. economy, is higher than the depreciation rate for residential structures. The law of motion for debt (11) then implies a steady-state debt-to-GDP ratio of 1.64, which is close to the average ratio (1958-2006) of home mortgages to GDP, which is 1.71 (for GDP

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[30]As in the previous section, the model is transformed so that it is expressed in terms of an inflation rate. The steady-state inflation rate is set equal to 4.54% per annum, the average inflation rate for 1971-2006, which is the period for which the mortgage rate data are available. This implies a steady-state real mortgage rate of 4.74%.

less consumer durable goods).

The discount factor β, the share of consumption in utility ω, the share of market good in consumption ψ, the share of capital in home production η, and the tax rate on income from nonresidential capital τ_r are calibrated jointly. Namely, by matching the average values of h_M, h_H, k_M/y, k_H/y, and the after-tax real rate of return on nonresidential capital, using the steady-state versions of the first-order conditions for h_M, h_H, s_J, and x_H (see Appendix B), and the model's after-tax real rate of return on nonresidential capital, $(1 - \tau_r)(F_1 - \delta_M)$, evaluated at the steady state. According to the American Time-Use Survey (2003), individuals aged 16 and over spent on average 25.5% of their available time working in the market and 24% in home production. We assume that half of home hours correspond to our notion of h_H. The average capital-to-GDP ratios are 4.88 for nonresidential capital and 4.79 for residential capital (in both cases consumer durable goods are subtracted from GDP). The average (annual) after-tax real rate of return on nonresidential capital is measured by (Gomme et al., 2011) to be 5.16%. These five targets yield $\beta = 0.988$, $\omega = 0.47$, $\psi = 0.69$, $\eta = 0.30$, and $\tau_r = 0.61$. As is common in models with disaggregated capital, the tax rate is higher than the statutory tax rate or a tax rate obtained from NIPA.

The parameterization of the exogenous stochastic process is based on point estimates of the parameters of a VAR(3) process, obtained for the relatively stable, post-reform, period 1984.Q1-2006.Q4 (see Appendix C for details). The economy's resource constraint $c_t + x_{Mt} + x_{Ht} = y_t$ implies constant unitary rates of transformation between the three uses of output. This makes the two types of investment extremely sensitive to the VAR shocks. To address this issue, we adopt the *intratemporal* adjustment costs of Huffman and Wynne (1999), which make the production possibilities frontier concave. Namely, we assume $c_t + x_{Mt} + q_t x_{Ht} = y_t$, where $q_t = \exp(\sigma(x_{tH} - x_H))$, with $\sigma > 0$ and x_H being the steady-state residential investment.[31] Increasing x_{Ht} above x_H is thus increasingly costly in terms of foregone c_t or x_{Mt}. Such costs reflect the costs of changing the composition of the economy's production

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[31] Of course, x_{Ht} is then multiplied by q_t through out the model. The household takes q_t as given; i.e., q_t depends on the aggregate x_{Ht}.

and construction (Huffman and Wynne, 1999), as well as constraints on available residential land in a given period, on which an increasing stock of housing can be placed (for instance, Davis and Heathcote, 2007, document that residential land grows at a fairly constant rate). The curvature parameter σ is then chosen by matching the ratio of the standard deviations (for HP-filtered data) of residential investment (single family structures) and GDP, which, for the period 1984.Q1-2006.Q4, is 8.4. This yields $\sigma = 6.4$.

5.2 Cyclical behavior of the model economy

Table 4 reports the cyclical behavior of the model economy for the above calibration. It reports the standard deviations (relative to that of y_t) of the key endogenous variables and their cross-correlations with y_t at various leads and lags. The first thing to notice is that the introduction of mortgage finance into the model does not significantly affect the behavior of the basic variables, y_t, c_{Mt}, x_t, and h_{Mt}. These variables behave pretty much like in other business cycle models: market consumption is roughly 50% as volatile as output, total investment is about four and a half times as volatile as output, and market hours are roughly 60% as volatile as output; in addition, all three variables are strongly positively correlated with output contemporaneously, without any lead or lag.

Second, unlike in other models, residential and nonresidential investment exhibit dynamics similar to those in the data. As in the data, x_{Ht} is twice as volatile as x_{Mt}, it is less contemporaneously correlated with output than x_{Mt}, and leads output. The lead in the model is one quarter, compared with two quarters in the data. x_{Mt}, in contrast, although not lagging, is more strongly correlated with output at lags than at leads (in the data x_{Mt} lags by one quarter). Thus, even though the lead-lag patterns in the baseline experiment are not as pronounced as in the data, the results present a major improvement upon the literature.[32] The reason why residential investment leads output in the model can be understood

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[32]As the model, like other models in the literature, lacks an internal propagation mechanism—a hump-shaped response of output to TFP shocks, present in the data—it will always produce a less pronounced lead-lag pattern than that in the data.

from the behavior of the wedge. As discussed in the previous section, the wedge captures the relative cost of mortgage finance. In Section 2 (Table 2) we saw that the 30-year mortgage rate leads output negatively and lags positively, a dynamics that we match through the exogenous VAR process estimated on the data. This dynamics transmits into the dynamics of the wedge, which exhibits a similar lead-lag pattern as the interest rate, but is an order of magnitude more volatile. This induces more residential investment ahead of an increase in GDP. While the wedge generates a lot of action outside of the steady state, our calibration implies that in steady state its value is close to zero ($\tau_H = -0.0117$), producing essentially the same steady state as that in GKR.

6 The role of mortgages and time to build

In order to gain further understanding of the results, this section disentangles the quantitative effects of the various model features on the lead-lag patterns of the investment variables. The results of these experiments are reported in Table 5, where, for the ease of comparison, the first panel repeats the results for the benchmark economy of the previous section.

6.1 Mortgages

We start by removing mortgage finance from the model ($\theta = 0$). The exogenous VAR process, however, stays the same. This guarantees that the underlying probability space of the current economy is the same as that of the benchmark economy and, thus, that the two economies differ only in terms of the value of θ (even though there is no mortgage finance under $\theta = 0$, households care about the mortgage and inflation rates to the extent that these variables help forecast future TFP). We see that for $\theta = 0$ the lead-lag patterns disappear: both x_{Ht} and x_{Mt} become coincident with output, with very strong contemporaneous correlations; in addition, x_{Ht} becomes much less volatile. Even though the behavior of its components changes, the behavior of total investment, x_t, stays, more or less, the same. In fact, the dynamics of x_t stay broadly unchanged across all our experiments. This is because consumption smoothing

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constrains the response of total investment to shocks. For this reason, x_{Ht} and x_{Mt} can both be coincident with output only if at least one of the two becomes substantially less volatile than in the benchmark (for the same reason, x_{Mt} has to lag output in the benchmark, when x_{Ht} leads output with high volatility).

Next, we consider again the case of no mortgage finance, but, in addition, assume a linear production possibilities frontier ($\sigma = 0$). This makes changes in the output mix less costly than in the previous case and the benchmark. This economy is essentially the GKR model (subject to small differences in calibration and the VAR process). We see that in this case the 'inverted' lead-lag pattern present in most existing models re-appears. As GKR show, this inverted lead-lag pattern would be even stronger if there was no time to build in nonresidential capital.

When $\alpha = \kappa = 1$, the mortgage reduces to a one-period loan. In this case the model behaves as if $\theta = 0$. This is because the wedge is too smooth and too little correlated with output to significantly affect the dynamics of the two investment components (the movements in the wedge in this case are essentially driven by $E_t(1 + i_t)/(1 + \pi_{t+1})$; i.e., the one-period ex-ante real interest rate implied by the VAR process).[33]

Finally, we consider FRM vs ARM. Under ARM, the mortgage interest rate is reset every period (equation (13)). Whereas FRM interest rates are closely tied to the yields on long-term government bonds—in the case of the U.S., for instance, at a roughly constant spread of 2%, except the early 1980s—ARM interest rates are set, after some initial period, as a constant margin over a short-term government bond yield. We therefore equalize i_t with the yield on a 3-month U.S. Treasury bill. It would, however, be incorrect to simply replace the estimated VAR for the 30-year mortgage rate with an estimated VAR that contains the 3-month T-bill yield instead. Such a strategy would change the underlying probability space.

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[33]We have also experimented with an approximation to a 15-year FRM, which is common in France (Calza et al., forthcoming). Similarly, we have experimented with values of θ that are suitable for countries like France and Belgium, which have much lower mortgage debt to GDP ratio than the U.S. and the other countries in our sample (Calza et al., forthcoming). In both cases, the volatility of x_{Ht} is significantly reduced. In the former case the lead-lag pattern of x_{Ht} stays essentially the same as in the benchmark, whereas in the latter case the pattern becomes less pronounced.

In order to keep the probability space constant when comparing the model's behavior under FRM and ARM, we estimate a four-variable VAR for $z_t = [\log A_{Mt}, i_t^{FRM}, \pi_t, i_t^{ARM}]$ (point estimates are reported in Appendix C). Under FRM, households care about i^{ARM} to the extent that it helps forecast the other three exogenous variables; i^{FRM} plays a similar role under ARM.

Table 5 contains the results for the four-variable VAR for both FRM and ARM. Under FRM the lead-lag pattern of x_{Ht} is even more pronounced, and closer to that in the data, than in the benchmark. This improvement comes from the fact that the four-variable VAR captures the joint dynamics of $\log A_{Mt}$, i_t^{FRM}, and π_t better than the three-variable VAR. Under ARM x_{Ht} is less volatile than under FRM and leads output by way too much (in the table this shows up as positive correlations at $j = -4$ and $j = -3$, but the correlations peak at $j = -6$, not shown in the table). As a result of the long lead, the contemporaneous correlation is negative. This behavior of x_{Ht} can be again understood from the dynamics of the wedge, in conjunction with the dynamics of the 3-month T-bill rate. As Table 2 shows the T-bill rate has similar dynamics as the 30-year mortgage rate in the sense that it is negatively correlated with future output and positively correlated with past output. Because the wedge in period t depends on expected future mortgage rates, expectations of higher interest rates in the future make the wedge start to increase even when the current T-bill rate is still relatively low. This is why the wedge in Table 5 starts to be much less negatively correlated with output (or even positively correlated) at $j's$ at which the T-bill rate is still negatively correlated with output. This in turn starts to reduce residential investment even before output begins to increase. Because the dynamics of the T-bill rate is governed by a stationary VAR process, the T-bill rate is expected to mean revert, following a shock. Thus, although the T-bill rate is somewhat more volatile than the 30-year mortgage rate (see Table 2), its mean reversion over the lifetime of the mortgage makes the wedge less volatile than under FRM (which has the mortgage rate of period t applied for its entire lifetime). This translates then into lower volatility of x_{Ht}.

Although the workings of the mechanism under ARM seem sensible, they generate predictions that are at odds with the data: the share of ARMs in U.S. mortgage lending for newly-built homes, reported in Table 1, is coincident with GDP; and ARM countries, like Australia and the U.K. do not exhibit negative contemporaneous correlations of residential investment (or housing starts) with GDP, despite the fact that their mortgage rate dynamics are similar to those of the U.S. T-bill rate. In concluding remarks we suggest an avenue for how to potentially make residential investment under ARM more coincident with output.[34]

6.2 Residential time to build

So far we assumed no time to build in residential construction (or, more precisely, we assumed the standard one-period time to build). When residential construction takes more than one period, we need to distinguish between finished and unfinished houses, and between the prices of finished houses and residential construction. We treat unfinished houses in a similar way as unfinished nonresidential investment projects: the household invests in residential projects and, upon completion, sells finished houses at a market price q_t^*. The household also buys finished houses for its own use (think of the household as a homebuilder who likes houses of a different color than those it builds). Let n_t^* be the number of houses the household wants to purchase for its own use. With these changes, the budget constraint becomes:

$$c_{Mt}+x_{Mt}+q_tx_{Ht}+q_t^*n_t^* = (1-\tau_r)r_tk_{Mt}+\tau_r\delta_Mk_{Mt}+(1-\tau_w)w_th_{Mt}+q_t^*n_{1t}+l_t/p_t-m_t/p_t+\tau_t,$$

where $l_t = \theta p_t q_t^* n_t^*$ and $x_{Ht} = \sum_{\iota=1}^{N} \mu_\iota n_{\iota t}$, with $n_{\iota t}$ denoting residential projects ι periods from completion and the μ's sum up to one. The stock of houses for the household's own use evolves as $k_{H,t+1} = (1 - \delta_H)k_{Ht} + n_t^*$ and the on-going residential projects evolve as $n_{\iota,t+1} = n_{\iota+1,t}$. In equilibrium, $n_t^* = n_{1t}$. Notice that the economy's resource constraint is the same as before: $c_{Mt} + x_{Mt} + q_tx_{Ht} = y_t$, except that x_{Ht} now consists of investment expenses on houses at different stages of completion.

The bottom panel of Table 5 reports the results for the model with residential time to

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[34]Koijen, Van Hemert, and Van Nieuwerburgh (2007) argue that the dynamics of ARM vs FRM origination over the business cycle is driven by bond risk premia.

build. We use $N = 4$, the same number of periods as for nonresidential capital (which, as noted earlier, corresponds in the data to the sum of nonresidential structures and equipment & software). We treat the μ's and ϕ's symmetrically, setting $\mu_\iota = \phi_\iota = 0.25 \ \forall \iota$. In addition to the usual variables, x_t, x_{Ht}, x_{Mt}, and τ_{Ht}, the table also reports results for housing starts n_{4t} and completions $n_{1,t-1}$ (we denote completions in the table by n_{0t}; that is, the number of houses that in period t become a part of the usable housing stock). As the table shows, x_{Ht} now reaches the highest correlation at $j = 0$, while starts lead by two quarters and completions lag by two quarters, patterns similar to those in the data for the U.S. multifamily structures (first subperiod) or the U.K.

7 Conclusion

A well known feature of the U.S. business cycle is that residential investment leads and nonresidential investment lags GDP. We document that in most other developed economies both types of investment are, more or less, coincident with GDP. There is much more uniformity across countries, however, when residential construction activity is measured by housing starts: almost all countries in our sample exhibit housing starts leading GDP. In contrast, a strong internal mechanism present in most business cycle models produces residential investment occurring only after an increase in GDP, once enough business capital has been built up. Our empirical analysis points to mortgage finance as a potential reason why actual economies exhibit the opposite dynamics. In order to evaluate this channel within a quantitative-theoretical framework, we introduce mortgages into an otherwise standard business cycle model with home production. The complexity of such an extension is greatly reduced by devising a (fairly accurate) approximation of mortgages. Feeding into the model the observed dynamics of mortgage interest rates over the business cycle produces dynamics of residential and nonresidential investment similar to those in U.S. data. Cheap mortgage finance ahead of future GDP growth, summarized by a decline in a wedge in an Euler equation for housing, induces households to invest in residential capital before GDP

peaks. Consumption smoothing than dictates that investment in nonresidential capital has to be delayed. Whereas in the U.S. residential construction is fairly rapid, in other countries the process appears to be much slower. Introducing time to build in residential capital into the model confirms than longer completion times in residential construction make residential investment more coincident with GDP.

A broader lesson from the analysis is that interest rate dynamics, in conjunction with long-term mortgage contracts, have a quantitatively significant effect on the economy. In our framework this shows up only in the composition of total investment, not in other aggregate variables. It would, therefore, be worth exploring if such effects can transmit also into aggregate output. This, of course, would require a richer framework than the one used here. Our way of modeling mortgages, however, should make it relatively easy to introduce mortgage finance into a variety of DSGE models more suitable for studying this issue. It is also beyond the scope of this paper to answer the question what drives the observed movements of mortgage rates. We have shown that their cyclical behavior is very similar to that of government bond yields. The dynamics of government bond yields can be partly due to monetary policy, but other sources, such as time-varying risk or risk aversion, are also a possibility. Building a more structural model would allow the analysis of, for instance, the effects of monetary policy on the economy through the mortgage channel.

Even though our model is consistent with the data when fixed-rate mortgages are used, it produces a way too long lead in residential investment under adjustable-rate mortgages. One aspect of mortgage finance that may be relevant for this deviation from the data, and which we have abstracted from, is risk. Our model is solved under certainty equivalence. But even if it was solved 'exactly', risk would play little role as the preferences used here are the standard time-additive CRRA preferences. An interesting extension would therefore be to study the dynamics of residential investment under preferences for which risk quantitatively matters. It is possible that under such preferences adjustable-rate mortgages may be riskier for households than fixed-rate mortgages. If that is the case, under adjustable-rate mort-

gages, households may respond relatively less to changes in the mortgage rate and relatively more to changes in income, making residential investment move more closely with GDP. We leave this, as well as the questions of the role of monetary policy, for future research.

Appendix A: Data used in Section 2

Only those data for which details were not already provided in Section 2 are listed here. These are data on GDP, total investment, residential and nonresidential structures, and mortgage and interest rates. **Australia**. REAL QUANTITIES: GDP, private GFCF, private GFCF nondwelling construction total, private GFCF dwellings total (all in chained dollars, SA, 1959.Q3-2006.Q4, *Australian Bureau of Statistics, National Accounts*); MORTGAGE RATE: standard variable housing loans lending rate, banks (1959.Q3-2006.Q4, *Reserve Bank of Australia*); INTEREST RATE: 3-month T-bill yield (1960.Q1-2006.Q4, *Global Financial Data*). **Belgium.** REAL QUANTITIES: GDP at market prices, GFCF total, GFCF in dwellings, GFCF by enterprises, self-employed workers and non-profit institutions (all in chained 2006 euros, SA, 1980.Q1-2006.Q4, *BelgoStat Online, National Accounts*); MORTGAGE RATE: fixed rate on loans for house purchasing (1980.Q1-2006.Q4, *Global Financial Data*); INTEREST RATE: 3-month T-bill yield (1980.Q1-2006.Q4, *Global Financial Data*). **Canada.** REAL QUANTITIES: GDP, residential structures, nonresidential structures, single dwellings, multiple dwellings (all in chained 2002 dollars, SA, *Statistics Canada, National Accounts*, 1961.Q1-2006.Q4, except for single and multiple dwellings, which are for 1981.Q1-2006.Q4); MORTGAGE RATE: conventional mortgage lending rate, 5-year term (1961.Q1-2006.Q4, *Statistics Canada*); INTEREST RATE: 3-month T-bill yield (1961.Q1-2006.Q4, *Global Financial Data*). **France.** REAL QUANTITIES: GDP, total GFCF, GFCF of non financial enterprises—including uninc. entrep., GFCF of households—excluding uninc. entrep. (all in chained euros, SA, 1971.Q1-2006.Q4, *INSEE, National Accounts*); MORTGAGE RATE: mortgage lending rate (1978.Q1-2006.Q4, *Global Financial Data*); INTEREST RATE: money market rate (1971.Q1-2006.Q4, *International Financial Statistics* and *Datastream*). **United Kingdom.** REAL QUANTITIES: GDP at market prices, GFCF total, GFCF dwellings, GFCF other new buildings and structures (all in chained 2002 pounds, SA, 1965.Q1-2006.Q4, *Office for National Statistics, United Kingdom Economic Accounts*); MORTGAGE RATE: sterling standard variable mortgage rate to households (1995.Q1-2006.Q4, *Bank of England*); INTEREST RATE: 3-month T-bill yield (1965.Q1-2006.Q4, *Office for National Statistics*). **United States.** REAL QUANTITIES: GDP, private fixed investment, private residential fixed investment, private fixed investment single family, private fixed investment multifamily, private fixed investment structures (all in chained 2000 dollars, SA, 1958.Q1-2006.Q4, *FRED* and *Bureau of Economic Analysis, National Income and Product Accounts*); MORTGAGE RATE: 30-year conventional mortgage rate (1971.Q1-2006.Q4, *FRED*); INTEREST RATE: 3-month T-bill yield (1958.Q1-2006.Q4, *FRED*).

Appendix B: Equilibrium—details and computation

This appendix provides the full set of optimality conditions for the household's problem of Section 4 and describes the method used to compute the equilibrium of the model.

The household's optimal decisions are characterized by four first-order conditions for h_{Mt}, h_{Ht}, s_{Jt}, and x_{Ht}. These are, respectively,

$$u_{1t}c_{1t}(1 - \tau_w)w_t = u_{2t},$$

$$u_{1t}c_{2t}A_HG_{2t} = u_{2t},$$

$$u_{1t}c_{1t}\phi_J = \beta E_t V_{s_{J-1},t+1},$$

$$u_{1t}c_{1t}(1-\theta) - \theta\beta E_t\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^\alpha)V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}.$$

Here $\widetilde{V}_{d,t+1}$ and ζ_{Dt} are defined as in the main text; that is, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$ and $\zeta_{Dt} \equiv \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right) / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$, where $\widetilde{d}_t \equiv d_t/p_{t-1}$. The first-order condition for s_{Jt} is accompanied by Benveniste-Scheinkman conditions for s_{jt} $(j = J-1, ..., 2)$, s_{1t}, and k_{Mt}, respectively,

$$V_{s_jt} = -u_{1t}c_{1t}\phi_j + \beta E_t V_{s_{j-1},t+1}, \quad j = J-1, ..., 2,$$

$$V_{s_1t} = -u_{1t}c_{1t}\phi_1 + \beta E_t V_{k_M,t+1},$$

$$V_{k_M,t} = u_{1t}c_{1t}\left[(1-\tau_r)r_t + \tau_r\delta_M\right] + \beta(1-\delta_M)E_t V_{k_M,t+1}.$$

The first-order condition for x_{Ht} has four Benveniste-Scheinkman conditions, for d_t, δ_{Dt}, R_t, and k_{Ht}. These are, respectively,

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\frac{R_t + \delta_{Dt}}{1+\pi_t} + \beta\frac{1-\delta_{Dt}}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\delta_{Dt}^\alpha - \kappa)V_{\delta_D,t+1} + \zeta_{xt}(R_t - i_t)V_{R,t+1}\right],$$

$$V_{\delta_D,t} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \left[\zeta_{xt}(\kappa - \delta_{Dt}^\alpha) + \frac{(1-\delta_{Dt})\alpha\delta_{Dt}^{\alpha-1}}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}}\right]\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t V_{\delta_D,t+1}$$

$$- \left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t\widetilde{V}_{d,t+1} + \zeta_{xt}(i_t - R_t)\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t V_{R,t+1},$$

$$V_{Rt} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \frac{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}}\beta E_t V_{R,t+1},$$

$$V_{k_Ht} = u_{1t}c_{2t}A_{Ht}G_{1t} + \beta E_t V_{k_H,t+1}(1-\delta_H),$$

where ζ_{xt} is defined as in the main text: $\zeta_{xt} \equiv \theta x_{Ht} / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$. Notice that the terms involving $\widetilde{V}_{d,t+1}$, $V_{\delta_D,t+1}$, and $V_{R,t+1}$ appear only in the first-order condition for x_{Ht}, as claimed in the main text. These terms drop out if $\theta = 0$. In this case the optimal decisions are characterized by the same conditions as in GKR, implying the same allocations and prices.

The equilibrium is computed by combining the linear-quadratic approximation methods of Hansen and Prescott (1995) and Benigno and Woodford (2006). Specifically, after transforming the model so that it is specified in terms of stationary variables π_t and $\widetilde{d}_t \equiv d_t/p_{t-1}$ (instead of nonstationary variables p_t and d_t), the home production function (2) and the budget constraint (9), with l_t and m_t substituted out from equations (8) and (10), are substituted in the period utility function $u(.,.)$. The utility function is then used to form a Lagrangian that has the nonlinear laws of motion (11)-(13) as constraints.

This Lagrangian forms the return function in the Bellman equation to be approximated with a linear-quadratic form around a nonstochastic steady state, with the variables expressed as percentage deviations from steady state. The steps for computing equilibria of distorted linear-quadratic economies, described by Hansen and Prescott (1995), then follow; with a vector of exogenous state variables $\Omega_t = [z_t, ..., z_{t-n}]$, a vector of endogenous state variables $\Phi_t = [s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, \widetilde{d}_t, \delta_{Dt}, R_t]$, and a vector of decision variables $\Upsilon_t = [h_{Mt}, h_{Ht}, x_{Ht}, s_{Jt}, \widetilde{d}_{t+1}, \delta_{D,t+1}, R_{t+1}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}]$, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ are Lagrange multipliers for the non-linear constraints (11)-(13).[35] The use of the Lagrangian ensures that second-order cross-derivatives of the nonlinear laws of motion (11)-(13), evaluated at steady state, appear in equilibrium decision rules (Benigno and Woodford, 2006). The alternative procedure of substituting out $\widetilde{d}_{t+1}$, $\delta_{D,t+1}$, and R_{t+1} from these laws of motion into the period utility function is not feasible here as these three variables are interconnected in a way that does not allow such substitution. The Lagrangian is

$$L_t = u\left(c(c_{Mt}, c_{Ht}), 1 - h_{Mt} - h_{Ht}\right) + \lambda_{1t}\left[d_{t+1} - (1 - \delta_{Dt})d_t - l_t\right]$$

$$+ \lambda_{2t}\left[\delta_{D,t+1} - (1 - \nu_t)\delta_{Dt}^\alpha - \nu_t\kappa\right] + \lambda_{3t}\left[R_{t+1} - (1 - \nu_t)R_t - \nu_t i_t\right],$$

with the remaining constraints of the household's problem substituted in the consumption aggregator $c(.,.)$, as mentioned above. For our calibrations the steady-state values of the Lagrange multipliers $(\lambda_{1t}, \lambda_{2t}, \lambda_{3t})$ are positive, implying that the above specification of the Lagrangian is correct in the neighborhood of the steady state.

The Lagrange multipliers are instrumental for computing the wedge, τ_{Ht}. Notice from equation (16) that the wedge depends on conditional expectations of the derivatives of the value function. The multipliers, which are obtained as an outcome of the solution method, provide a straightforward way of computing these expectations. The mapping between the multipliers and the expectations is obtained from the first-order conditions for d_{t+1}, $\delta_{D,t+1}$, and R_{t+1} in the household's problem. Forming the Bellman equation

$$V\left(z_t, ..., z_{t-n}, s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right)$$

$$= \max\left\{L_t + \beta E_t V\left(z_{t+1}, ..., z_{t-n+1}, s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\right\},$$

the respective first-order conditions are

$$\lambda_{1t} + \lambda_{2t}\left[\frac{(1 - \delta_{Dt})\delta_{Dt}^\alpha d_t + p_t\theta\kappa x_{Ht}}{d_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1 - \delta_{Dt})d_t R_t + p_t\theta i_t x_{Ht}}{d_{t+1}^2}\right] + \beta E_t V_{d,t+1} = 0,$$

$$\lambda_{2t} + \beta E_t V_{\delta_D,t+1} = 0,$$

$$\lambda_{3t} + \beta E_t V_{R,t+1} = 0.$$

When the model is transformed so that it is specified in terms of π_t and $\widetilde{d}_t$, rather than p_t

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[35]In the version with residential time to build, the $n_{\iota t}$'s become a part of Φ_t and n_t^* becomes a part of Υ_t, but with q_t^* being its counterpart in the aggregate counterpart to Υ_t.

and d_t, the first of these conditions changes to

$$\widetilde{\lambda}_{1t} + \lambda_{2t}\left[\frac{(1-\delta_{Dt})\delta_{Dt}^\alpha\widetilde{d}_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta\kappa x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1-\delta_{Dt})\widetilde{d}_t R_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta i_t x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \beta E_t\widetilde{V}_{d,t+1} = 0,$$

where $\widetilde{\lambda}_{1t} \equiv p_t\lambda_{1t}$.

Appendix C: VAR estimates

The exogenous VAR process used in Section 5 is estimated on U.S. data for logged and linearly detrended Solow residual, the interest rate on the conventional 30-year FRM, and the CPI inflation rate. The estimation period is 1984.Q1-2006.Q4. The series for the Solow residual is taken from data accompanying Gomme and Rupert (2007). The capital stock used for the construction of the residual is the sum of structures and equipment & software (current costs deflated with the consumption deflator), which is consistent with our mapping of k_{Mt} into the data in the rest of the calibration. The number of lags in the VAR is determined by the multivariate AIC. The point estimates (ignoring the constant term) are

$$z_{t+1} = \begin{pmatrix} 0.933 & -0.543 & -0.283 \\ 0.023 & 0.953 & 0.020 \\ 0.021 & 0.431 & 0.246 \end{pmatrix} z_t + \begin{pmatrix} 0.118 & -0.070 & 0.183 \\ -0.016 & -0.134 & 0.036 \\ 0.111 & -0.249 & 0.164 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.147 & 0.633 & 0.117 \\ 0.036 & -0.011 & 0.043 \\ -0.084 & -0.197 & 0.187 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.0049 & 0 & 0 \\ 0.0002 & 0.0009 & 0 \\ -0.0011 & 0.0009 & 0.0026 \end{pmatrix} \epsilon_{t+1},$$

where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$ and $\epsilon_{t+1} \sim N(0,I)$. These point estimates are used to solve the model and run the computational experiments in Sections 5 and 6.

In Section 6, a four-variable VAR is also used. Here, $z_t = [\log A_{Mt}, i_t^{FRM}, \pi_t, i_t^{ARM}]^\top$, where i_t^{FRM} is, as before, the interest rate on the conventional 30-year FRM and i_t^{ARM} is the yield on a 3-month Treasury bill. Here, the AIC criterium dictates four lags. The point estimates are

$$z_{t+1} = \begin{pmatrix} 0.858 & 0.014 & -0.157 & -1.232 \\ 0.044 & 0.849 & 0.042 & 0.008 \\ 0.085 & 0.172 & 0.241 & 0.554 \\ 0.049 & 0.127 & 0.021 & 1.362 \end{pmatrix} z_t + \begin{pmatrix} 0.070 & -0.221 & 0.192 & 2.122 \\ -0.020 & -0.070 & 0.048 & -0.006 \\ 0.103 & -0.023 & 0.162 & -0.721 \\ -0.041 & -0.107 & -0.010 & -0.346 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.302 & -0.168 & 0.036 & -2.277 \\ 0.005 & 0.051 & 0.045 & -0.036 \\ -0.140 & 0.097 & 0.204 & 0.406 \\ -0.004 & 0.123 & -0.032 & -0.090 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.231 & 1.124 & 0.053 & 0.615 \\ 0.027 & -0.189 & -0.062 & 0.178 \\ 0.012 & -0.458 & -0.153 & 0.060 \\ 0.032 & -0.219 & -0.010 & 0.063 \end{pmatrix} z_{t-3}$$

$$+\begin{pmatrix} 0.0042 & 0 & 0 & 0 \\ 0.0003 & 0.0008 & 0 & 0 \\ -0.0009 & 0.0008 & 0.0025 & 0 \\ 0.0001 & 0.0003 & 0.0001 & 0.0006 \end{pmatrix} \epsilon_{t+1},$$

where $\epsilon_{t+1} \sim N(0, I)$.

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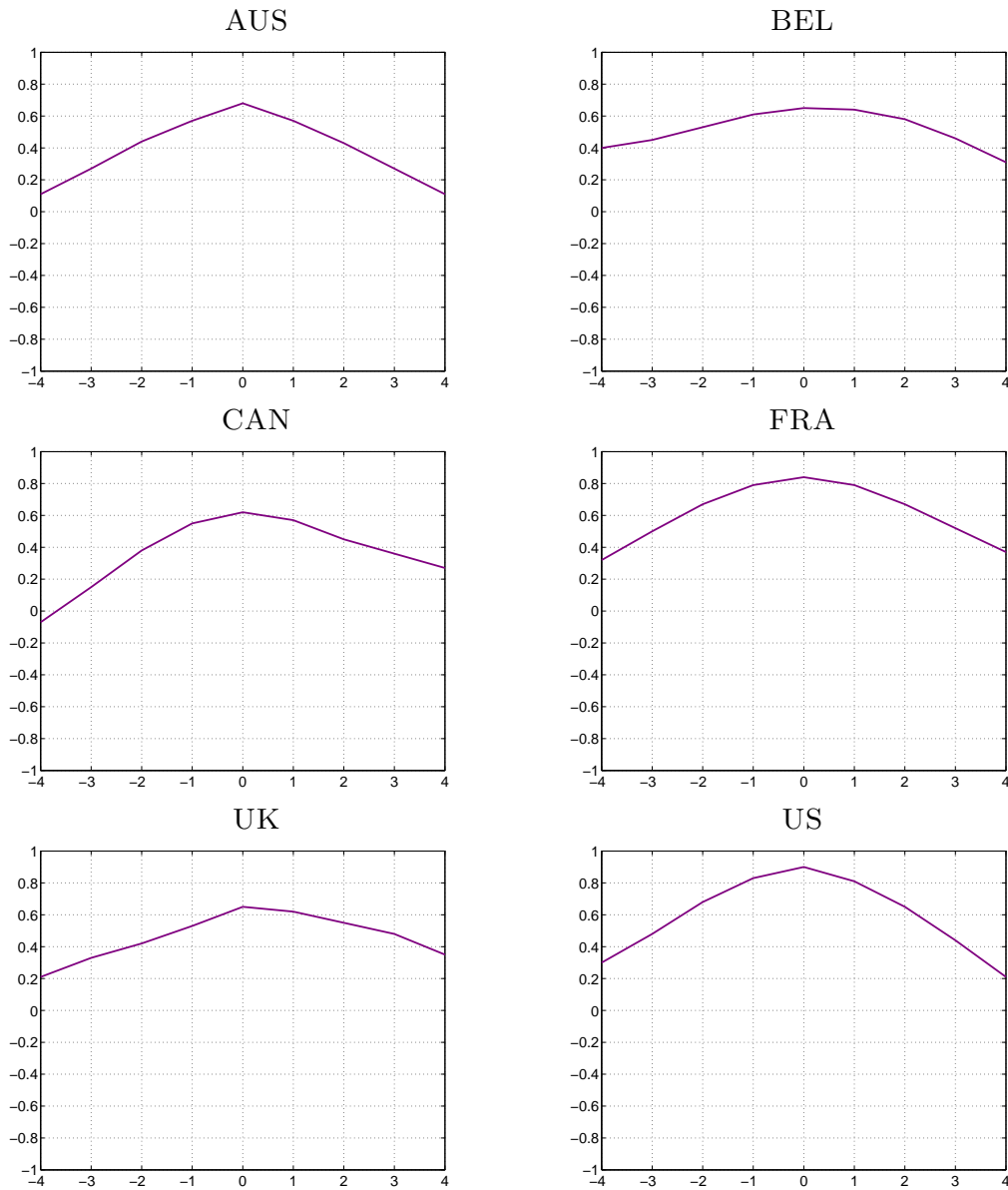
Figure 1: Cyclical dynamics of total fixed investment (gross fixed capital formation). The plots are correlations of real investment in $t + j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter. The volatility of total fixed investment (measured by its standard deviation relative to that of real GDP) is: AUS = 3.98, BEL = 3.93, CAN = 3.32, FRA = 2.65, UK = 2.55, US = 3.23.
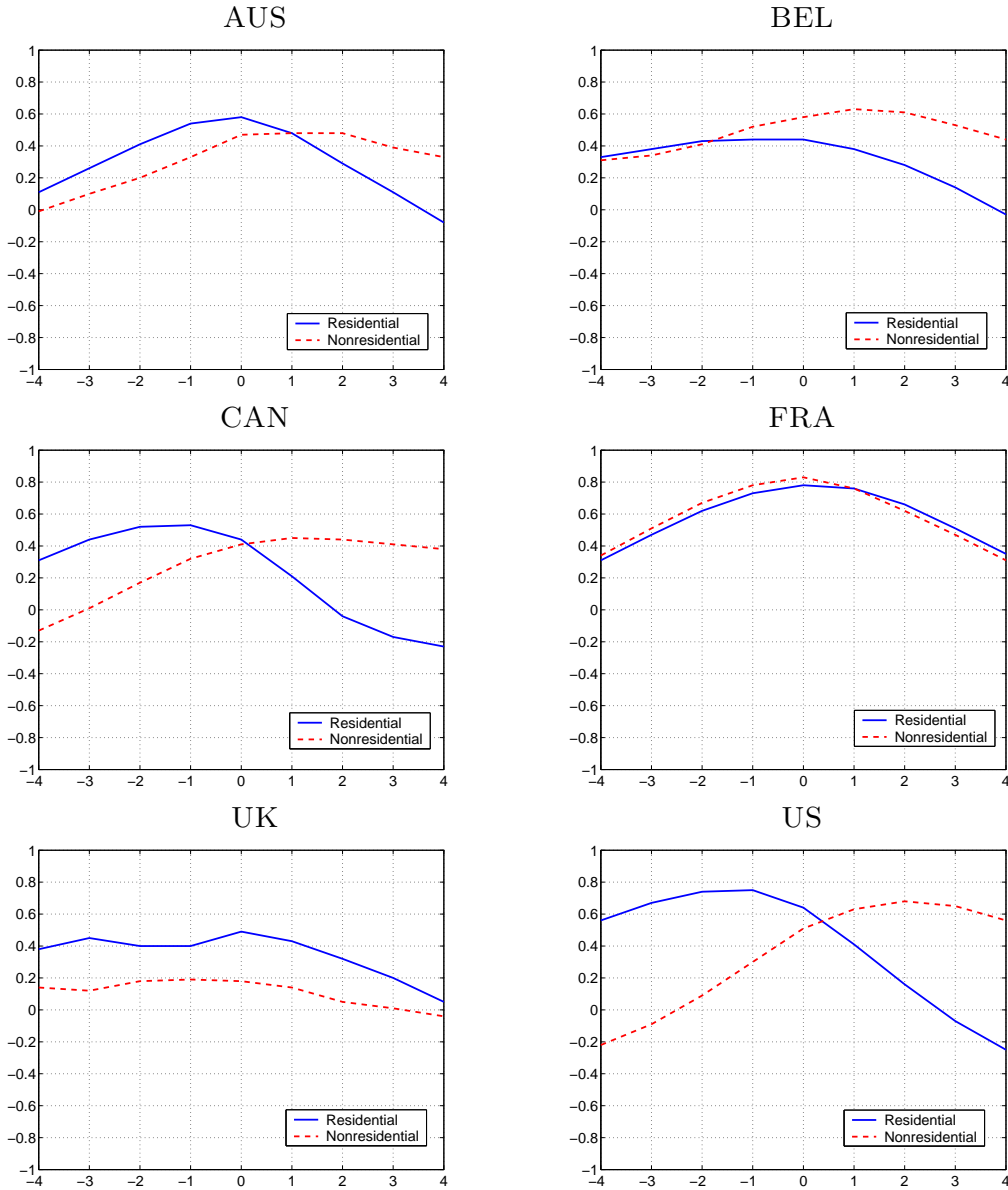
Figure 2: Cyclical dynamics of residential and nonresidential structures. The plots are correlations of real investment in $t+j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter (in the case of BEL and FRA nonresidential is the sum of structures and equipment). The volatility of residential (nonresidential), relative to that of real GDP, is: AUS = 5.95 (6.96), BEL = 7.97 (4.36), CAN = 4.39 (3.97), FRA = 3.05 (3.24), UK = 5.02 (3.24), US = 6.42 (3.40).
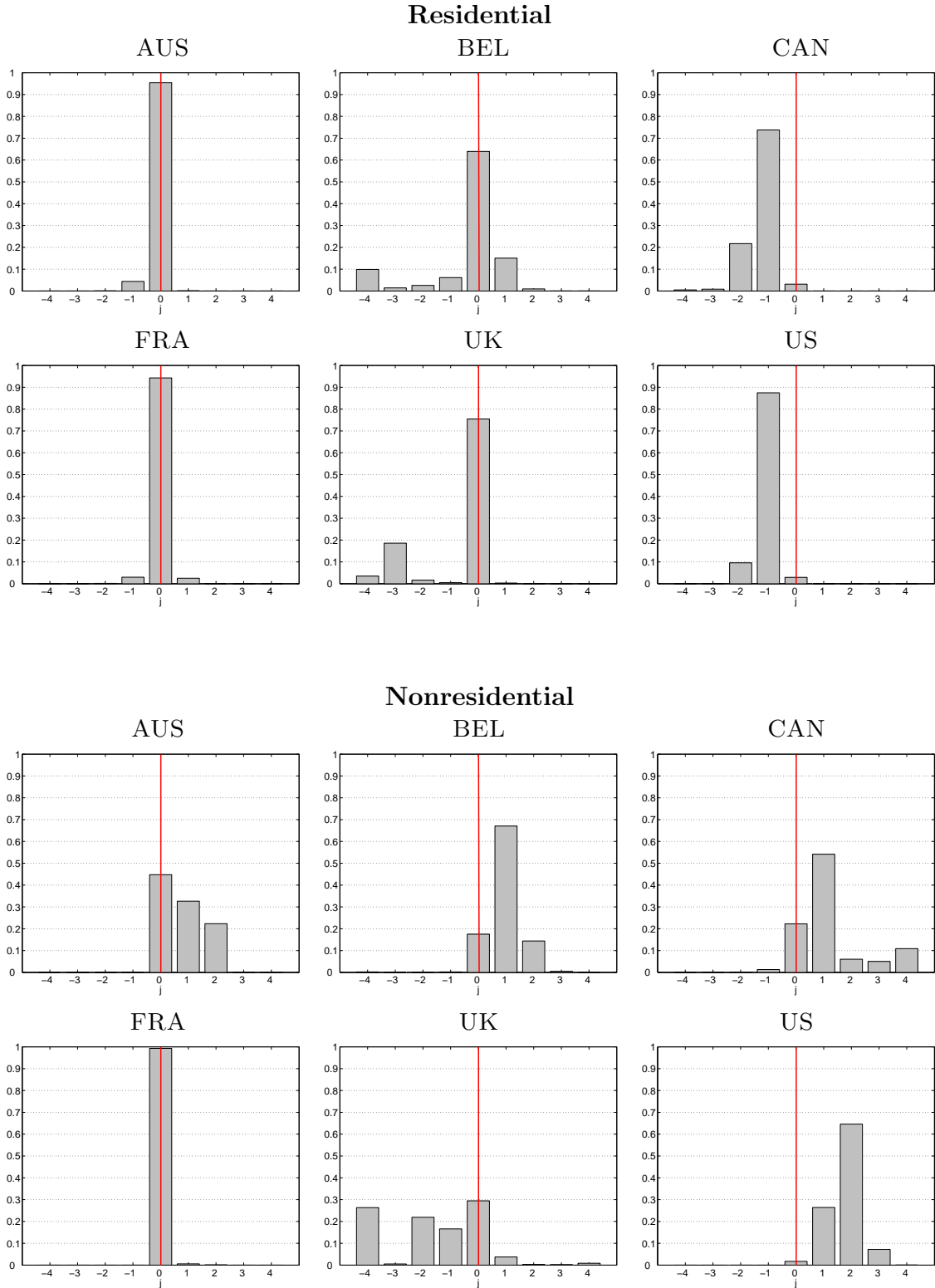
Figure 3: Statistical significance of leads and lags in structures dynamics. Histograms show the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data (in each case a series is block-bootstrapped and then logged and HP filtered; a cross-correlogram is then computed for the HP-filtered series).
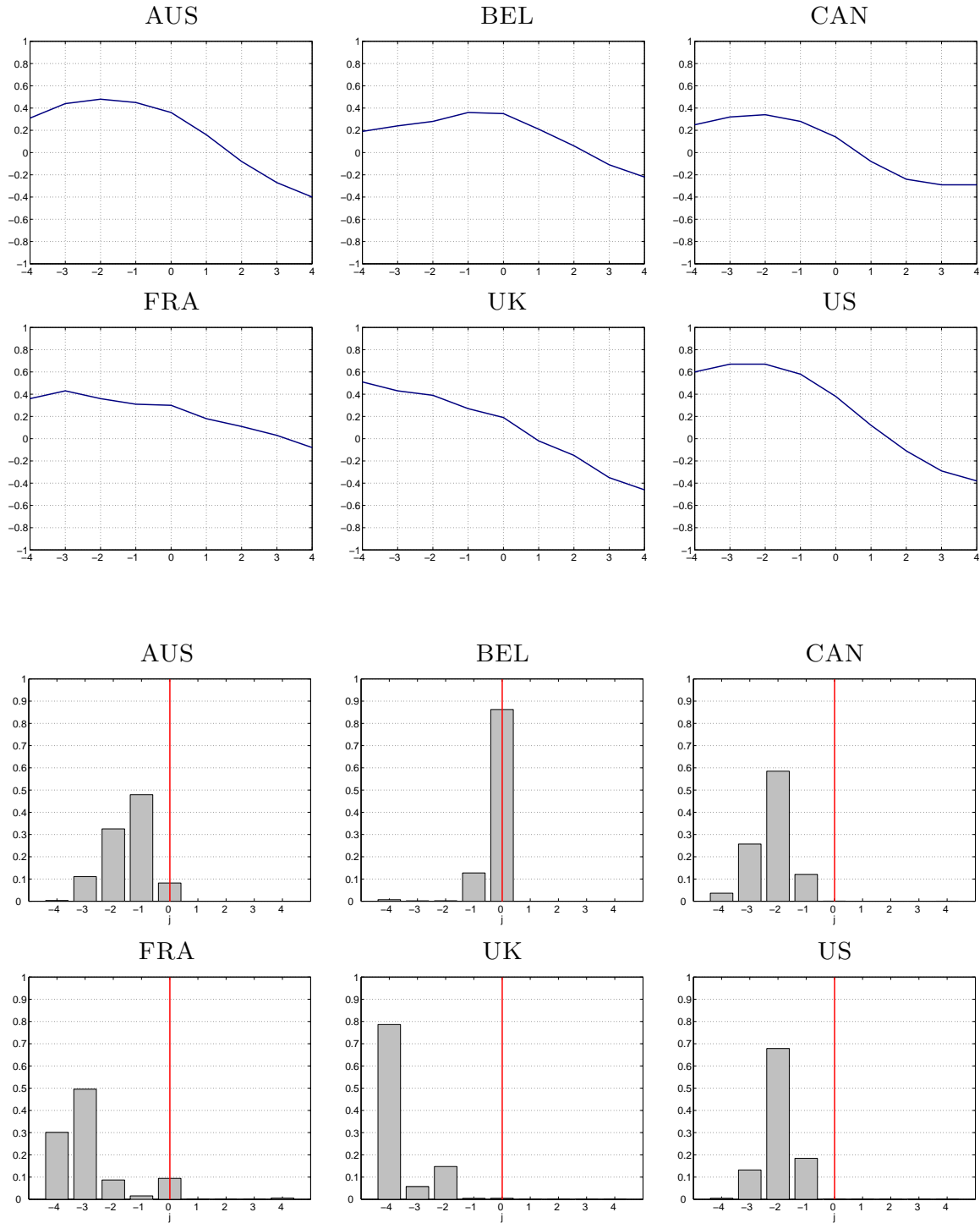
Figure 4: Housing starts. The top six charts plot cross-correlations in the historical data (logged and HP-filtered); the bottom six charts show the statistical significance of leads and lags in housing starts dynamics; i.e., the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data. The volatility of housing starts in the actual data, relative to that of real GDP, is: AUS = 8.80, BEL = 11.67, CAN = 9.95, FRA = 6.24, UK = 7.86, US = 9.72. Note: due to a relatively short length of starts data for the U.K., residential building permits are used instead as a proxy (the two series co-move very closely during the period for which both are available).

Table 1: Residential investment—further details[a]

	Relative std. dev.[b]	$j = -4$	-3	-2	-1	0	1	2	3	4
		Correlations of real GDP in t with a variable in $t+j$:								
United States: 59.Q1–83.Q4										
Residential structures										
Single family	8.84	0.58	0.65	0.73	0.72	**0.62**	0.39	0.14	-0.11	-0.30
Multifamily	11.40	0.13	0.25	0.38	0.48	**0.51**	0.46	0.34	0.21	0.07
Starts										
1 unit	8.85	0.61	0.68	0.70	0.61	**0.39**	0.12	-0.12	-0.33	-0.42
5+ units	14.16	0.39	0.52	0.61	0.60	**0.50**	0.30	0.10	-0.08	-0.22
Completions[c]										
1 unit	7.33	0.66	0.74	0.80	0.81	**0.78**	0.60	0.37	0.16	-0.05
5+ units	9.56	-0.02	0.12	0.27	0.42	**0.58**	0.75	0.77	0.73	0.65
Mortgages[d]										
Single family	14.22	0.45	0.56	0.69	0.63	**0.49**	0.30	0.15	-0.10	-0.23
Multifamily	17.41	0.43	0.42	0.46	0.45	**0.39**	0.31	0.16	0.03	-0.11
United States: 84.Q1–06.Q4										
Residential structures										
Single family	8.40	0.51	0.57	0.60	0.57	**0.48**	0.28	0.05	-0.13	-0.25
Multifamily	10.42	-0.02	-0.01	0.07	0.14	**0.22**	0.27	0.31	0.32	0.30
Starts										
1 unit	9.32	0.42	0.47	0.44	0.35	**0.23**	-0.01	-0.17	-0.29	-0.37
5+ units	16.43	0.05	0.16	0.27	0.40	**0.44**	0.40	0.33	0.21	0.13
Completions										
1 unit	6.51	0.36	0.43	0.52	0.50	**0.45**	0.33	0.15	-0.02	-0.16
5+ units	13.71	0.06	0.04	0.06	0.14	**0.23**	0.31	0.40	0.43	0.39
Mortgages[d]										
Single family	18.55	0.16	0.21	0.14	0.11	**0.10**	0.04	0.09	0.11	0.05
Excl. MEW[e]	20.83	0.18	0.21	0.11	0.06	**0.03**	-0.01	0.04	0.08	0.04
Multifamily	68.83	0.25	0.24	0.13	0.09	**0.03**	0.02	-0.03	-0.03	-0.01
Share of ARMs[f]	12.98	0.20	0.25	0.32	0.39	**0.41**	0.37	0.23	0.07	-0.06
Canada										
Residential structures[g]										
Single family	7.21	0.33	0.44	0.48	0.44	**0.27**	0.01	-0.29	-0.44	-0.42
Multifamily	6.60	-0.08	-0.13	-0.16	-0.10	**-0.08**	-0.05	0.03	0.10	0.05
United Kingdom[h]										
Starts	8.35	0.28	0.28	0.26	0.18	**0.10**	-0.08	-0.22	-0.38	-0.41
Completions	5.14	0.22	0.31	0.41	0.44	**0.48**	0.36	0.28	0.13	-0.01

[a] The series are logged (except for shares and multifamily mortgages) and filtered with Hodrick-Prescott filter; multifamily mortgages are expressed as a ratio to their mean due to negative values in the data.
[b] Standard deviations are expressed relative to that of a country's real GDP.
[c] 1968.Q1-1983.Q4.
[d] Net change in home and multifamily mortgages, deflated with GDP deflator (home = 1-4 family properties, multifamily = 5+ family properties). The fraction of new construction accounted for by 2-4 family structures is small, home mortgages are therefore a good proxy for single family housing mortgages, for which data are not available.
[e] MEW = mortgage equity withdrawal (home equity loans).
[f] 1985.Q1-2006.Q4.
[g] 1981.Q1-2006.Q4.
[h] 1990.Q1-2006.Q4.

Table 2: Cyclical dynamics of mortgage rates[a]

		Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$:								
			$j = -4$	-3	-2	-1	0	1	2	3	4
Mortgage rates[c]											
AUS	ARM	0.59	-0.29	-0.22	-0.16	-0.03	**0.12**	0.25	0.39	0.48	0.50
BEL	FRM 10 yrs	0.89	-0.17	0.01	0.19	0.38	**0.56**	0.63	0.60	0.53	0.41
CAN	FRM 5 yrs	0.77	-0.52	-0.41	-0.24	-0.04	**0.19**	0.38	0.45	0.45	0.43
FRA	FRM 15 yrs	0.87	-0.10	-0.02	0.10	0.20	**0.30**	0.36	0.35	0.31	0.27
UK[d]	ARM	1.29	-0.68	-0.52	-0.31	-0.06	**0.17**	0.36	0.49	0.55	0.56
US	FRM 30 yrs	0.55	-0.59	-0.55	-0.46	-0.29	**-0.07**	0.09	0.16	0.21	0.23
Government bond yields[e]											
AUS	3-m	1.07	-0.19	-0.06	0.10	0.24	**0.34**	0.44	0.52	0.45	0.34
BEL	10-yr	0.75	-0.01	0.20	0.33	0.49	**0.53**	0.50	0.43	0.31	0.19
CAN	3-5-yr	0.73	-0.42	-0.25	-0.06	0.17	**0.39**	0.52	0.54	0.50	0.41
FRA	10-yr	0.86	-0.12	-0.02	0.10	0.21	**0.29**	0.31	0.28	0.25	0.24
US	10-yr	0.53	-0.45	-0.39	-0.29	-0.11	**0.04**	0.09	0.10	0.12	0.09
	3-m	0.88	-0.45	-0.30	-0.10	0.17	**0.39**	0.48	0.51	0.49	0.46
Inflation rates[f]											
AUS		1.96	-0.19	-0.09	-0.02	0.12	**0.15**	0.21	0.31	0.28	0.17
BEL		1.80	0.15	0.19	0.13	0.15	**0.15**	0.17	0.17	0.16	0.15
CAN		1.44	-0.16	-0.03	0.05	0.16	**0.26**	0.35	0.32	0.35	0.35
FRA		1.72	-0.23	-0.13	-0.03	0.11	**0.20**	0.27	0.30	0.32	0.31
UK		2.80	-0.28	-0.22	-0.12	-0.01	**0.03**	0.18	0.23	0.29	0.27
US		1.28	-0.27	-0.13	-0.01	0.18	**0.37**	0.45	0.47	0.50	0.49

[a] GDP is in logs; all series are filtered with Hodrick-Prescott filter; time periods differ across countries due to different availability of mortgage rate data: AUS (59.Q3-06.Q4), BEL (80.Q1-06.Q4), CAN (61.Q1-06.Q4), FRA (78.Q1-06.Q4), UK (65.Q1-06.Q4), US (71.Q2-06.Q4).

[b] Standard deviations are expressed relative to that of a country's real GDP.

[c] Based on a typical mortgage for each country, as reported by Calza et al. (forthcoming) and Scanlon and Whitehead (2004). ARM = adjustable rate mortgage (interest rate can be reset within one year), FRM = fixed rate mortgage (interest rate can be at the earliest reset only after 5 years). The number of years accompanying FRMs in the table refers to the number of years for which the mortgage rate is typically fixed.

[d] U.K. mortgage rate data are available only from 1995.Q1. 3-m T-bill rate is used as a proxy for the adjustable mortgage rate for the period 1965.Q1-1994.Q4; the correlation between the two interest rates for the period 1995.Q1-2006.Q4 is 0.97. As the 3-m T-bill rate is used for this purpose, it is omitted from the next panel of the table.

[e] Constant maturity rates.

[f] Consumer price indexes, q-on-q percentage change at annual rate.

A. Quarterly payments

B. Balance

C. Composition of payments

D. Approximation error
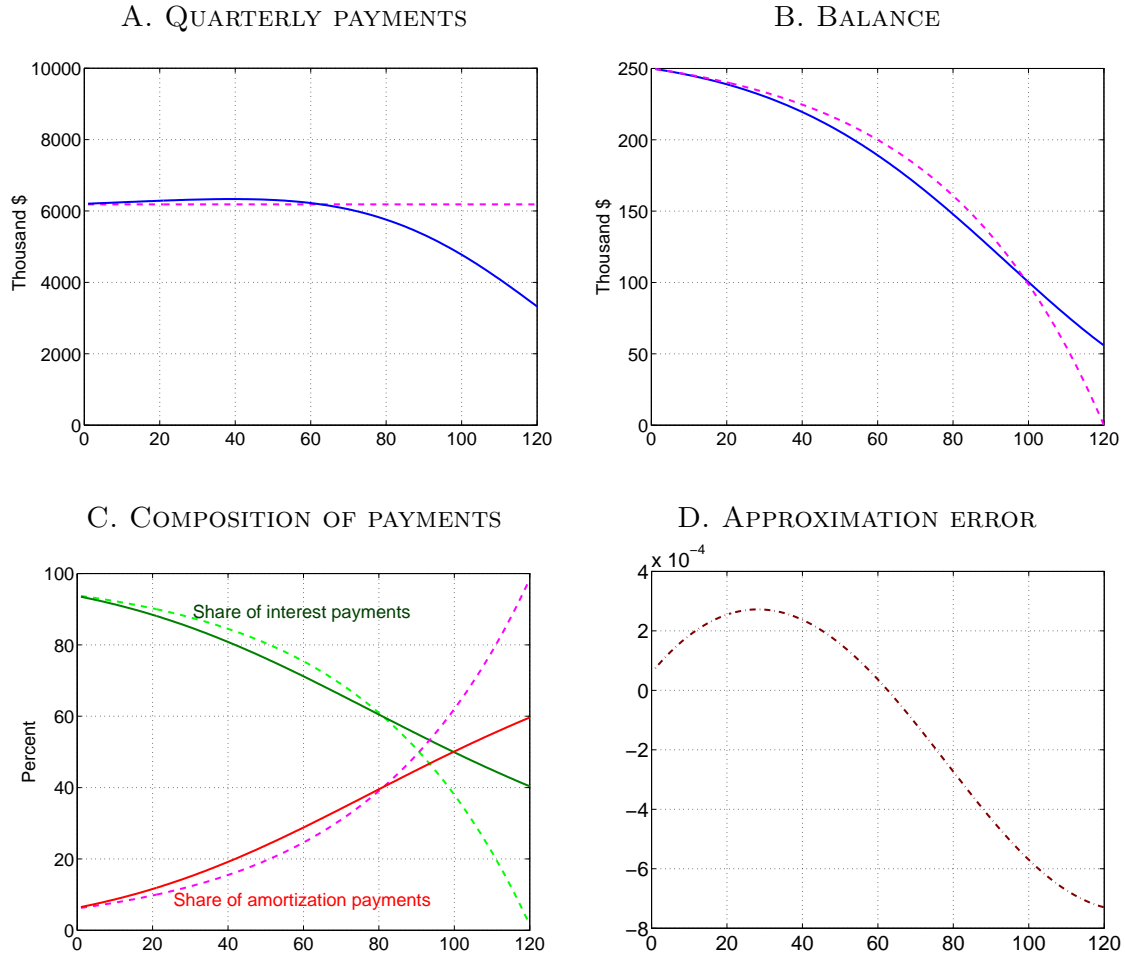
Figure 5: Mortgage: model vs real-world calculator. Solid line=model, dashed line=mortgage calculator. Here, $l_0 = \$250,000$, $4 \times i = 9.28\%$, $\alpha = 0.9946$, and $\kappa = 0.00162$. The approximation error is expressed as the present value (using $1/i$) of the difference between payments in the model and in the mortgage calculator (panel A), divided by the size of the loan.

Table 3: Calibration

Symbol	Value	Definition
Preferences		
β	0.988	Discount factor
ω	0.472	Consumption share in utility
ψ	0.692	Share of market good in consumption
Home technology		
δ_H	0.0115	Depreciation rate
η	0.305	Capital share in production
Nonresidential time to build		
J	4	Number of periods
ϕ_j	0.25	Fraction completed at stage j
Market technology		
δ_M	0.0248	Depreciation rate
ϱ	0.283	Capital share in production
σ	6.4	PPF curvature parameter
Tax rates		
τ_w	0.243	Tax rate on labor income
τ_r	0.612	Tax rate on capital income
Mortgages		
θ	0.76	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Adjustment factor
Other		
i	0.0232	Steady-state mortgage rate
π	0.0113	Steady-state inflation rate

Note: The parameters of the exogenous stochastic process are contained in Appendix D.

Table 4: Cyclical behavior of the model economy[a]

v_{t+j}	Rel. st.dev.[b]	Correlations of y in period t with variable v in period $t+j$:								
		$j = $ -4	-3	-2	-1	0	1	2	3	4
Model—main aggregates and hours										
y	1.01	-0.03	0.19	0.48	0.75	**1.00**	0.75	0.48	0.19	-0.03
h_M	0.56	0.10	0.31	0.57	0.76	**0.89**	0.68	0.41	0.07	-0.21
c_M	0.48	-0.21	-0.09	0.13	0.38	**0.70**	0.52	0.38	0.29	0.28
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
Model—investment components and wedge										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
Data—investment components[c]										
x_H	8.40	0.51	0.57	0.60	0.57	**0.48**	0.28	0.05	-0.13	-0.25
x_M	4.53	0.22	0.38	0.56	0.72	**0.84**	0.88	0.84	0.74	0.58

[a] Calibration is as in Table 3. The entries are averages for 200 runs of the length of 92 periods each, the same as the number of periods for 1984.Q1-2006.Q4. All variables are in percentage deviations from steady state, except the wedge, which is in percentage point deviations from steady state. Before computing the statistics for each run, the artificial series were filtered with the HP filter.
[b] Standard deviations are measured relative to that of y; the standard deviation of y is in absolute terms.
[c] Data: 1984.Q1-2006.Q4, y=GDP, x_H=single-family structures, x_M=structures plus equipment & software; all logged and HP-filtered.

Table 5: Impact of various model features on investment dynamics

v_{t+j}	Rel. st.dev.	Correlations of y in period t with variable v in period $t+j$:								
		$j = -4$	-3	-2	-1	0	1	2	3	4
Benchmark[a] (FRM, 30 years, $\theta = 0.76$)										
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
No mortgage finance ($\theta = 0$)										
x	4.21	0.08	0.27	0.52	0.76	**0.98**	0.75	0.46	0.15	-0.10
x_H	0.78	-0.07	0.06	0.30	0.55	**0.84**	0.55	0.37	0.28	0.34
x_M	5.79	0.09	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.14
τ_H	–	–	–	–	–	–	–	–	–	–
No mortgage finance and linear PPF ($\theta = 0$, $\sigma = 0$)										
x	4.77	0.14	0.29	0.50	0.72	**0.99**	0.69	0.43	0.19	0.04
x_H	14.66	-0.19	-0.08	0.02	0.20	**0.54**	0.51	0.52	0.48	0.50
x_M	6.32	0.36	0.41	0.54	0.59	**0.52**	0.22	-0.07	-0.29	-0.48
τ_H	–	–	–	–	–	–	–	–	–	–
1-period loan ($\alpha = 1$, $\kappa = 1$)										
x	4.30	0.07	0.27	0.52	0.76	**0.98**	0.75	0.47	0.17	-0.11
x_H	0.83	-0.08	0.10	0.35	0.63	**0.86**	0.66	0.45	0.43	0.17
x_M	5.85	0.08	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.13
τ_H	0.21	-0.01	-0.04	-0.08	-0.13	**-0.04**	-0.15	-0.09	-0.22	0.23
FRM (4-variable VAR)										
x	4.95	0.18	0.33	0.56	0.77	**0.95**	0.74	0.48	0.20	0.01
x_H	8.46	0.35	0.49	0.56	0.55	**0.48**	0.31	0.16	-0.04	-0.23
x_M	5.55	-0.08	-0.01	0.23	0.50	**0.80**	0.69	0.50	0.31	0.23
τ_H	2.87	-0.34	-0.49	-0.51	-0.47	**-0.34**	-0.22	-0.10	0.08	0.26
ARM (4-variable VAR)										
x	4.68	0.17	0.27	0.50	0.73	**0.97**	0.74	0.46	0.18	0.02
x_H	2.59	0.32	0.22	0.05	-0.17	**-0.43**	-0.54	-0.60	-0.60	-0.54
x_M	7.56	0.09	0.20	0.44	0.69	**0.96**	0.77	0.53	0.29	0.13
τ_H	1.01	-0.26	-0.20	0.02	0.27	**0.61**	0.58	0.59	0.56	0.58
Residential time to build										
x	4.32	0.08	0.28	0.54	0.77	**0.95**	0.69	0.40	0.08	-0.17
x_H	6.51	0.18	0.32	0.47	0.57	**0.60**	0.42	0.14	-0.16	-0.40
n_4	8.89	0.33	0.40	0.50	0.48	**0.38**	-0.10	-0.33	-0.40	-0.34
n_0	8.88	-0.05	-0.02	0.06	0.18	**0.33**	0.40	0.50	0.48	0.38
x_M	4.11	-0.13	0.05	0.31	0.60	**0.90**	0.80	0.62	0.38	0.14
τ_H	3.17	-0.22	-0.34	-0.43	-0.42	**-0.29**	-0.16	-0.02	0.18	0.34

[a] Calibration as in Table 3.
Note: n_4 = housing starts (houses that in period t are four periods away from completion), n_0 = housing completions (houses that in period $t-1$ were one period away from completion and that in period t become a part of the housing stock).